Chapter 7

Magnetism—Source of the Field

7.1 INTRODUCTION

In the previous chapter, we learned about the effect of a magnetic field on a moving charge (or on a current carrying wire). In this chapter, we will discuss the origin of the magnetic field. We recall that the source of a gravitational field, which exerts a force on one mass, is another mass, and the source of an electric field, which exerts a force on one charge, is another charge. We will therefore not be surprised to find that one source of a magnetic field, which exerts a force on a moving charge, is another moving charge. Indeed, the basic origin of a magnetic field is a moving charge or an equivalent current in a wire. In a later chapter, we will learn that there is another basic source for a magnetic field, namely an electric field that varies with time. In this chapter we will develop the concepts and equations needed to understand the magnetic fields produced by moving charges.

7.2 FIELD PRODUCED BY A MOVING CHARGE

To obtain the magnetic field produced by a charge, q, moving with velocity **v**, at a point located at a displacement **r** from the charge, we need a mathematical expression for the field in terms of q, **v** and **r**. The geometry is shown in Fig. 7-1. In this figure, a charge, q, located at point a is moving with velocity **v**, as shown. We seek the magnitude and direction of the magnetic field at point b, displaced from point a by the vector **r**. Thus, the point b is at a distance r from point a along a line that makes an angle ϕ with the velocity **v**. As was the case with the force exerted by the magnetic field, we are looking for a vector (in this case **B**) which is formed from some combination of two vectors (in this case **v** and **r**) and a scalar q. And once again we will discuss separately the magnitude and the direction of this vector **B**. The results we express here were determined from a wide array of experimental studies of magnetic fields and their sources.

Magnitude of the Field

The formula for the magnitude of the field is:

$$|B| = (\mu_0/4\pi)|qv \sin \phi/r^2| \tag{7.1}$$

where $(\mu_0/4\pi)$ is a constant which, for our system of units, is equal to 10^{-7} T · m/A. (The 4π is included for later convenience.) This formula, together with the prescription for finding the direction of the field, is known as the **Law of Biot and Savart**.

We have used absolute value signs, since the magnitude is always positive. The magnitude of the field **B** does not depend on the sign of the charge nor on the sign of $\sin \phi$ (which in any case is positive between 0° and 180°). The direction of the field will, however, be dependent on the sign of q.

This formula tells us that the field is zero if ϕ is zero. This occurs if the point b lies along the line of **v**, i.e. if the present path of the charge would carry it through the point b. In order for a magnetic field

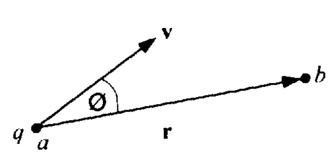

Fig. 7-1

to be produced at a point b, that point must lie at some non-zero distance from the extended line of **v**. The largest magnetic field is produced when ϕ is 90°. This occurs when the point b is located along the line perpendicular to **v** at point a.

The magnitude of **B** decreases as $1/r^2$ with the distance from point a. This is reminiscent of the dependence of **g** and of **E** on the distance from their respective sources. As expected, the field increases with both q and v. Thus the field gets bigger for charges which move fast and for those that have a lot of charge. The field decreases as one goes to points that are further away from the charge and for those at smaller angles to the line along which the charge moves.

Problem 7.1.

(*a*) A charge of 2×10^{-6} C is moving with a velocity of 3×10^4 m/s when passing point a in Fig. 7-1. What is the magnitude of the field at point b if that point is at a distance of 2×10^{-3} m from point a at an angle ϕ of 30°?

(*b*) What is the magnitude of the field if the charge were -2×10^{-6} coulomb?

(*c*) What is the magnitude of the field if the angle ϕ were 150°?

Solution

(*a*) Substituting $q = 2 \times 10^{-6}$ coulomb, $v = 3 \times 10^4$, $\phi = 30°$ and $r = 2 \times 10^{-3}$ into Eq. (*7.1*), we get $|B| = (10^{-7})(2 \times 10^{-6})(3 \times 10^4)(\sin 30°)/(2 \times 10^{-3})^2 = 7.5 \times 10^{-4}$ T.

(*b*) Since only the absolute value of each variable enters, the answer is the same as for part (*a*).

(*c*) Since $\sin 150° = \sin 30°$, the answer is still the same.

Problem 7.2. A charge of 3.0×10^{-5} C is at the origin in Fig. 7-2, moving in the positive x direction with velocity 2.0×10^6 m/s. The length of each side of the cube is 2.0×10^{-3} m. Calculate the magnitude of the field at (*a*) point B; (*b*) point E; (*c*) point H; (*d*) point C; and (*e*) point F.

Solution

In all five cases, $(\mu_0/4\pi)\, qv = (10^{-7})(3.0 \times 10^{-5})(2.0 \times 10^5) = 6.0 \times 10^{-7}$. The difference between each case is the value of r and of $\sin \phi$. Thus, the solution for each case is

(*a*) $\phi = 0$, $\sin \phi = 0$ and therefore $|B| = 0$.

(*b*) $\phi = 90°$, $\sin \phi = 1$ and $r = 2.0 \times 10^{-3}$. Therefore $|B| = 6.0 \times 10^{-7}(1)/(2.0 \times 10^{-3})^2 = 0.15$ T.

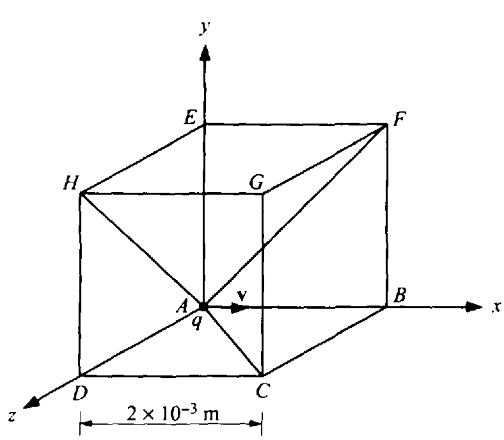

Fig. 7-2

(*c*) $\phi = 90°$, $\sin\phi = 1$ and $r = 2.0 \times 10^{-3}(\sqrt{2})$. Therefore $|B| = 0.075$ T.

(*d*) $\phi = 45°$, $\sin\phi = 0.707$ and $r = 2.0 \times 10^{-3}(\sqrt{2})$. Therefore $|B| = 0.053$ T.

(*e*) $\phi = 45°$, $\sin\phi = 0.707$ and $r = 2.0 \times 10^{-3}(\sqrt{2})$. Therefore $|B| = 0.053$ T.

Direction of the Field

The direction of the field is perpendicular to both **v** and **r**, and it is therefore perpendicular to the plane containing both **v** and **r**. This is illustrated in Fig. 7-3. Here, we call θ the angle between **v** and **r**. Again, there are two possible directions which are perpendicular to this plane, and we need a rule to select the correct direction. Once again this is the right-hand rule. In this case we apply the rule by placing our fingers in the direction to rotate **v** into **r**, and our thumb will then point in the direction of **B**. In Fig. 7-3, we draw the plane of **v** and **r**, and the perpendicular to that plane. The two possible perpendicular direction are up and down. Using the right-hand rule selects the downward direction as the correct one. This is the correct answer if the charge q is positive. For a negative charge, the direction of **B** is reversed.

There is a nice way to visualize this geometry. In Fig. 7-4 we draw the same vectors **v** and **r**. At the tip of **r** (point b) we draw the plane through b that is perpendicular to the line of the vector **v**, cutting that line at point O. Thus the line from b to O is perpendicular to line aO, and equals $r\sin\theta$. The magnetic field at b lies in this plane and is perpendicular to bO at b. In fact, if one draws a circle in this plane, whose center is at O and whose circumference passes through point b, the direction of the magnetic field at point b is tangent to the circle at that point. The circle through b is shown in the figure. It

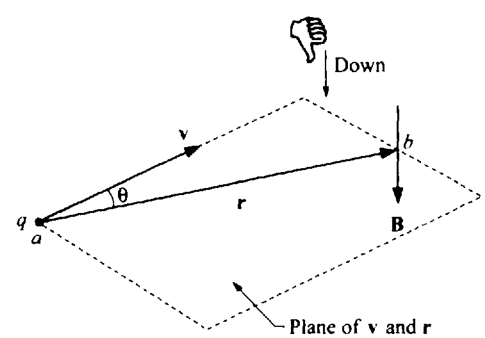

Fig. 7-3

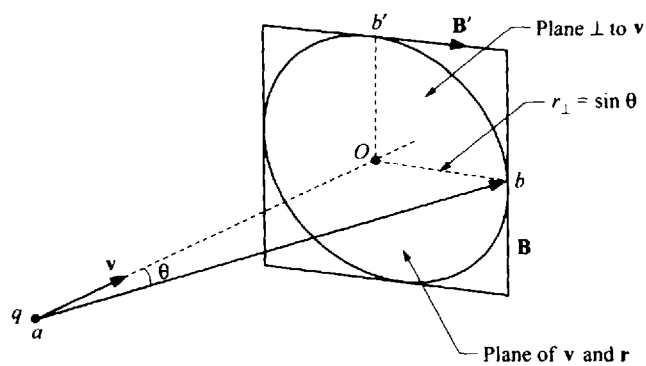

Fig. 7-4

is not hard to see that the magnetic field at any point on this circle, has a common magnitude and a direction tangent to the circle at that point. To determine which way the magnetic field points along the tangent (for instance, at b the direction could be either up or down), we use an equivalent right-hand rule to that defined above. Put the thumb of your right hand along the direction of **v**, and your fingers will circle around that line in the direction of **B**. In the case depicted, the direction of **B** at point b is down. At point b' the tangent to the circle is horizontal, and the magnetic field points to the right as can be seen by applying the right-hand rule. This picture is often very useful, since it shows that if one traces the magnetic field lines, (in a manner similar to tracing electric field lines), they form concentric circles around the direction of **v**. The direction of this circling is determined by the right-hand rule. Again, if the charge q is negative, we reverse the direction of **B**, which means that that the magnetic field lines will now be circling in the opposite direction. It is often useful to view the field lines by looking toward the charge along the direction of the velocity. This is shown in Fig. 7-5, where we view the charge coming out of the paper. The magnetic field lines are circles in the plane of the paper, with centers on the line of **v**. The direction of the field is tangent to the circle at any point, with the sense determined by the right-hand rule. In the case shown, the field lines circle in the counter-clockwise direction for a positive charge, and clockwise for a negative charge.

Problem 7.3. Determine the direction of the field in Problem 7.2.

Solution

(*a*) Since the field is zero at B, there is obviously no direction to determine.

(*b*) Here **r** is the vector from A to E. The plane containing **v** and **r** is the x–y plane, and the perpendicular to that plane is the $\pm z$ direction. If we place our fingers in the direction rotating **v** into **r**, our thumb points in the $+z$ direction, which is therefore the direction of **B**.

Alternatively, we could draw the view as seen by looking toward **v**, i.e. by looking down the x axis. This view is shown in Fig. 7-6(*a*). We draw a circle through E, with its center at A. The tangent to this circle at any point is the direction of **B** at that point. By the right-hand rule we know that the field lines circle in the counter-clockwise direction (for the positive charge). At E, the tangent to the circle, and therefore the direction of **B**, points toward H, or in the positive z direction.

(*c*) Here it is simplest to use the alternate approach discussed in part (*b*). The same Fig. 7-6(*a*) also contains the point H. The circle through H, with center at A, is also shown in the figure. The tangent to this circle at H is perpendicular to AH and pointing in the direction ED, i.e. 45° below the positive z axis, as shown.

(*d*) Again, we draw the plane that we see as we look toward **v**, but this time we draw the plane through point C [see Fig. 7-6(*b*)]. The point B is where the extension of the vector **v** pierces this plane. The circle centered on B and passing through C is shown in the figure, again with the field lines circling counter-clockwise. At C, the tangent to the circle is in the $-y$ direction, which is the direction of **B**.

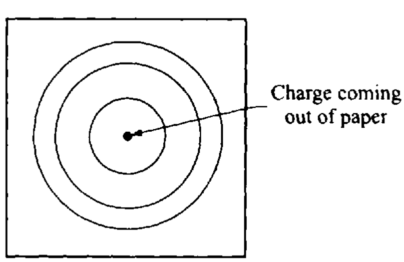

Fig. 7-5

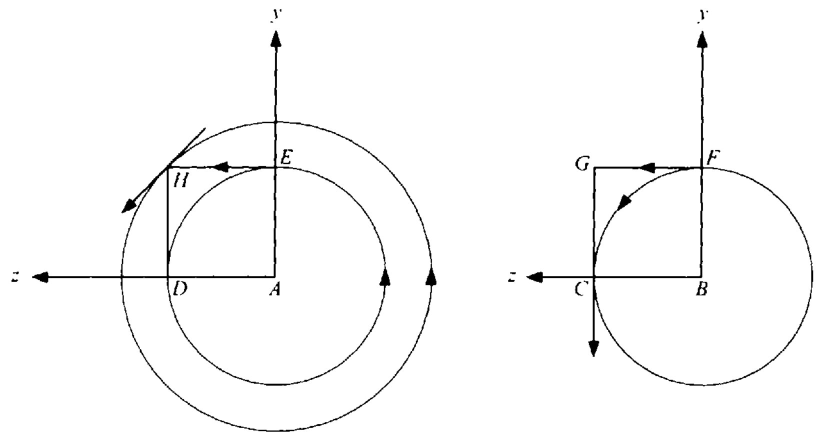

Fig. 7-6

(e) The same drawing that we used for part (d) can be used for this part. The circle through F has a tangent at F which points in the positive z direction which is therefore the direction of $\mathbf{B}$.

Problem 7.4. A charge q_1 is moving with velocity $\mathbf{v}_1$ along the positive x axis. Another charge q_2 is moving with velocity $\mathbf{v}_2$ parallel to $\mathbf{v}_1$ at a distance d from q_1 as shown in Fig. 7-7(a). Find an expression for the magnitude of the magnetic force that q_1 exerts on q_2, in terms of q_1, q_2, $\mathbf{v}_1$, $\mathbf{v}_2$, and d, and find the direction of this force.

Solution

Charge q_1 produces a magnetic field at the position of q_2 as we just learned. This magnetic field exerts a force on q_2 as we learned in Chap. 6. We therefore solve this problem in two steps. First we calculate the field $\mathbf{B}_{1 \text{ at } 2}$ produced by q_1 at the position of q_2, and then we calculate the force that this field exerts on q_2.

To calculate the field produced by q_1 at the position of q_2, we first calculate its magnitude. This is given by $|\mathbf{B}_{1 \text{ at } 2}| = \mu_0 q_1 |\mathbf{v}_1| \sin\theta/4\pi r^2$, where $r = d$ and θ is the angle between $\mathbf{v}_1$ and $\mathbf{r}$, which is 90° ($\mathbf{r}$ is the vector from point 1 to point 2). Thus,

$$|\mathbf{B}_{1 \text{ at } 2}| = (\mu_0/4\pi)(q_1 |\mathbf{v}_1|/d^2) \tag{7.2}$$

To determine the direction of $\mathbf{B}_{1 \text{ at } 2}$ we draw the view with $\mathbf{v}_1$ coming out of the paper, as shown in Fig. 7-7(b). The circle centered on the line of $\mathbf{v}_1$ and going through q_2 is shown on the figure. The tangent to this circle at q_2 is in the $-y$ direction, which is the direction of $\mathbf{B}_{1 \text{ at } 2}$.

We now have to calculate the force exerted by $\mathbf{B}_{1 \text{ at } 2}$ on q_2. This is also done by calculating separately the magnitude and the direction of the force. We recall from Chap. 6 that the magnitude of the force is, in general, given by Eq. (6.1):

$$|F| = |qvB \sin\phi|$$

In our case, $q = q_2$, $v = v_2$, $B = B_{1 \text{ at } 2}$, and $\phi = 90°$ since the angle between $\mathbf{B}$ and $\mathbf{v}_2$ is 90°. Substituting for $B_{1 \text{ at } 2}$ in the equation, we get that

$$|F| = |q_2 v_2(\mu_0/4\pi)(q_1 v_1/d^2)| = |(\mu_0/4\pi)q_1 q_2 v_1 v_2/d^2| \tag{7.3}$$

To get the direction of the force, we recall that $\mathbf{F}$ is perpendicular to both $\mathbf{v}_2$ and to $\mathbf{B}_{1 \text{ at } 2}$. Since $\mathbf{B}_{1 \text{ at } 2}$ is in the $-y$ direction and $\mathbf{v}_2$ is in the $+x$ direction, the plane containing both these vectors is the x-y plane. Using the right-hand rule (rotate $\mathbf{v}_2$ into $\mathbf{B}$), we find that the direction of $\mathbf{F}$ is $-z$. Thus q_2 is being attracted to q_1 via the magnetic force.

The above calculation was performed for the case of two positive charges moving parallel to each other in the same direction. In this case, we got an attractive force on q_2 due to q_1. We can deduce

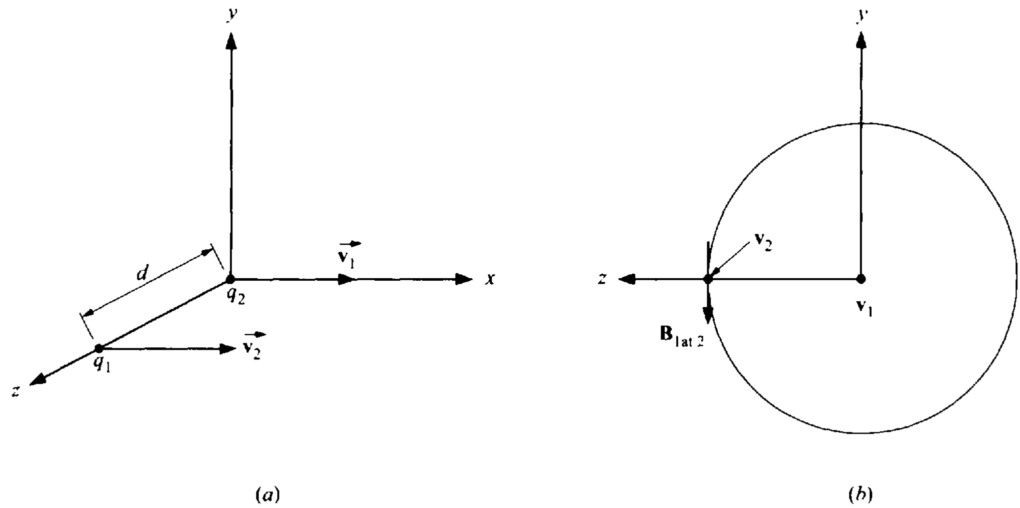

Fig. 7-7

several generalizations from this example. Firstly, if we had calculated the force that q_2 exerts on q_1, we would also have found an attractive force of the same magnitude. Secondly if *one* of the charges had been negative, then the force would have become repulsive, since either $\mathbf{B}_{1 \text{ at } 2}$ (if q_1 were negative) or the right-hand rule for F (if q_2 were negative) would have been reversed. However, if both charges were negative, the force would still have been attractive. Thirdly, if the velocities of the charges had been in parallel, but *opposite* directions, then, for charges of the same sign, the force would be repulsive, since *one* of the velocity directions would be reversed.

It is also important to note that, in addition to the magnetic force between the charges, there is also an electric force between the charges. In fact, the electric force will generally be much greater than the magnetic force, unless the velocities of the charges are comparable to the velocity of light. This is more fully explored in one of the supplementary problems.

7.3 FIELD PRODUCED BY CURRENTS

As we learned in Chap. 6, current flowing in a wire is equivalent to moving charge. There we discussed the force exerted on a current flowing in a small length of wire. We showed there that we could use the same formula that we used for a charge q moving with a velocity v, if we replaced $q\mathbf{v}$ with $I\mathbf{L}$, where $\mathbf{L}$ is the small, directed length of wire carrying current I. The same is fundamentally true when one wishes to calculate the field produced by a current element $\mathbf{L}$ carrying current I. However, any current element must be part of a continuous circuit, and each part of that circuit produces its own magnetic field at every point. The actual field at any point is the vector sum of the contributions from all elements of the circuit. This will clearly pose calculation problems, which we will have to discuss.

To calculate the magnetic field produced by a current I flowing in a wire of length ΔL (since we are talking about a small portion of a longer wire we use the designation ΔL), we use the same formula that we developed in Sec. 7.2, replacing $q\mathbf{v}$ with $I\Delta\mathbf{L}$. Thus, the magnitude of the part of the field produced by this current element at a point located at a distance r from the element is (see Fig. 7-8)

$$|\Delta B| = (\mu_0/4\pi)\,|I\Delta L \sin\phi/r^2| \tag{7.4}$$

where the terms in the formula have the same meaning as previously, and as labeled in the figure. The direction of the field is calculated in the same manner as for the moving charge, and would be into the paper for the case in Fig. 7-8.

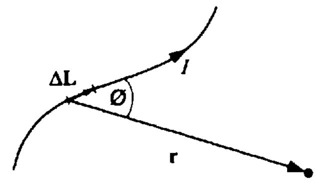

Fig. 7-8

In order to calculate the field at this point, we would have to add together, vectorially, the contributions from the entire length of the wire. This can be done for certain special cases without the use of advanced mathematics, but for the more general case, the use of calculus is needed. Let us discuss some special cases which are often used in practice.

Field at the Center of a Current Carrying Ring

Consider a ring, of radius R, which carries a current I, flowing in the clockwise direction in Fig. 7-9. We want to calculate the field at the center of the ring due to this current. We proceed by calculating the field produced by the segment ΔL at the center (point O). The magnitude of the field is $\Delta B = (\mu_0/4\pi)\,|I\Delta L/R^2|$, since the angle between $\Delta \mathbf{L}$ and $\mathbf{R}$ is 90°. This magnitude is the same for any segment of the ring, independent of the location of ΔL along the ring. The direction of the field is perpendicular to the plane of $\Delta \mathbf{L}$ and $\mathbf{R}$, which means that it is in or out of the plane of the ring. Using the right-hand rule, we determine that it is into the paper in the figure. Again, this direction is independent of the location of ΔL along the ring, so that every ΔL along the wire contributes the same ΔB, and in the same direction. To get the total magnetic field from all the ΔL, we have to add vectors which are all in the same direction, so we have only to add the magnitudes. The total field will therefore be $B = (\mu_0/4\pi)\,|IL/R^2| = (\mu_0/4\pi)I(2\pi R)/R^2 = \mu_0 I/2R$. Thus, in general, the field at the center of a ring is given by:

$$B = \mu_0 I/2R \tag{7.5}$$

The direction (into the paper) can be deduced from the following right-hand rule. Wrap the fingers of your right hand around the circle in the direction of the current, and your thumb points in the direction of the field.

Problem 7.5.

(*a*) Calculate the magnitude and direction of the magnetic field at the center of the circle in Fig. 7-10, whose radius is 1.5 m and which carries a current of 2 A in the direction shown.

(*b*) Suppose that instead of a single circle we had a tightly wound coil of $N = 50$ turns (same current, same radius); find B.

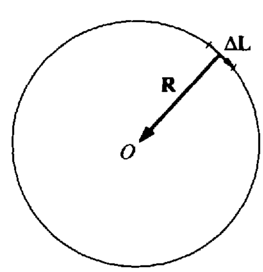

Fig. 7-9

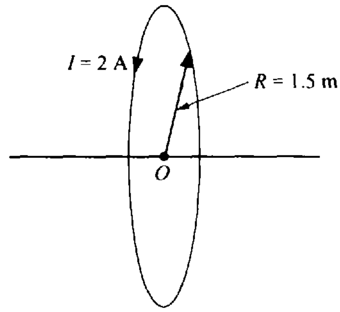

Fig. 7-10

Solution

(*a*) Using Eq. (7.5), $B = \mu_0 I/2R = \mu_0(2)/2(1.5) = 4\pi \times 10^{-7}/1.5 = 8.38 \times 10^{-7}$ T. To get the direction, we wrap our fingers around the ring in the direction of I, and determine that the direction is to the right.

(*b*) If the coil consists of N turns, all of the same radius and carrying the same current, we just add the contributions of each loop of the coil:

$$B = N\mu_0 I/2R = 50(8.38 \times 10^{-7}) = 4.19 \times 10^{-5}\text{T} \tag{7.6}$$

Field Along the Axis of a Ring

In Fig. 7-11, we draw a ring, of radius R, carrying a current I in the direction shown. The current is out of the paper at the top, and into the paper at the bottom. Along the axis of the ring (the line OP), we seek the magnetic field produced by this current at an arbitrary distance, x, from the center of the ring, O. We choose a current element ΔL at the top of the ring, where the current is coming out of the paper. The field at P due to this element is

$$|\Delta B| = (\mu_0/4\pi)|I\Delta L \sin \phi/r^2| \tag{7.7}$$

Here ϕ is the angle between $\Delta\mathbf{L}$ (out of the paper) and $\mathbf{r}$, which is 90°, and $r = \sqrt{R^2 + x^2}$. Thus

$$|\Delta B| = (\mu_0/4\pi)I\Delta L/(R^2 + x^2) \tag{7.8}$$

The direction of the field is more difficult to visualize. It is shown if Fig. 7-11. This direction is clearly perpendicular to $\mathbf{r}$ and to $\Delta\mathbf{L}$, and conforms to the right-hand rule. Note that θ, the angle between $\mathbf{R}$ and $\mathbf{r}$, is also the angle between $\mathbf{B}$ and the x axis. If we now consider an equivalent current element at the bottom of the ring, we get a field of the same magnitude and with a direction below the axis at the

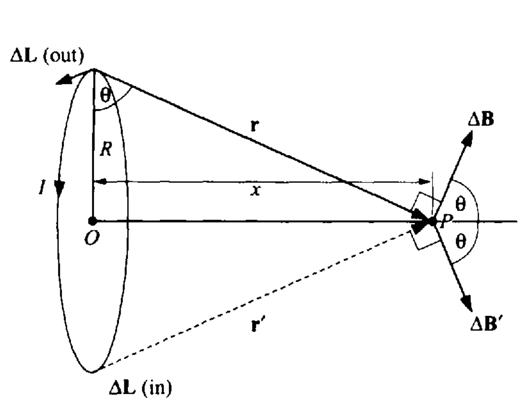

Fig. 7-11

same angle as this ΔB (see Fig. 7-11). If we add those two fields together, the vertical components will cancel and we will be left with only the horizontal components. This horizontal component will equal $\Delta B \cos\theta$

$$|\Delta B_x| = (\mu_0/4\pi)I\Delta L \cos\theta/(R^2 + x^2),$$

and with $\cos\theta = R/\sqrt{R^2 + x^2}$, we get

$$|\Delta B_x| = (\mu_0/4\pi)I\Delta LR/(R^2 + x^2)^{3/2} \tag{7.9}$$

Problem 7.6. Find an expression for the field due to a current carrying coil at any point along its axis of symmetry.

Solution

Using Eq. (7.9), and noting that for every segment in the upper part of the ring, there will be an opposite segment in the lower part which will cancel other components than the horizontal component, we have only to add together the horizontal components due to all the ΔLs around the loop. Since the magnitude of B_x is the same for all the segments, we can add them together very easily. Adding all the ΔL together gives the circumference of the circle, $2\pi R$, and therefore

$$B = (\mu_0/4\pi)I(2\pi R)R/(R^2 + x^2)^{3/2} = \mu_0 IR^2/2(R^2 + x^2)^{3/2} \tag{7.10}$$

Again, for N turns in the coil, the field is multiplied by N. The direction is along the axis to the right for the direction of current chosen. In general, the direction along the axis can be determined by the same right-hand rule as for the center of the circle.

Note. Obtaining the magnetic field of the loop off the x axis is much harder, since we don't have the symmetry that allowed us to solve the on axis problem.

Problem 7.7. Calculate the magnitude and direction of the magnetic field at point P in Fig. 7-11, if $R = 1.5$ m, $I = 2$ A and $x = 2$ m.

Solution

Using Eq. (7.10), $B = 4\pi \times 10^{-7}(2)(1.5)^2/2(1.5^2 + 2^2)^{3/2} = 1.8 \times 10^{-7}$ T. The direction is to the right.

Problem 7.8. Two identical coils, each having 1000 turns, are separated by a distance of 1.6 m along a common axis, as in Fig. 7-12. For each coil $R = 1.5$ m and $I = 2$ A. Calculate the field at a point on the axis midway between the coils.

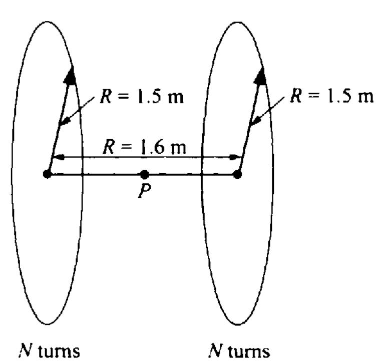

Fig. 7-12

Solution

For each coil, we can use Eq. (*7.10*) multiplying by N for the number of turns. Therefore, for each coil, $B = 10^3\,(4\pi \times 10^{-7})(2)(1.5)^2/2(1.5^2 + 0.8^2)^{3/2} = 5.8 \times 10^{-4}$ T. The direction is to the right. For both coils together, the field is 1.16×10^{-3} T.

Field of a Long Straight Wire

Suppose that a long straight wire carries a current I to the right, as in Fig. 7-13(*a*). We want to know the magnitude and direction of the field produced by the current in this wire at a point located at a distance R from the wire. The method required is to take segments of length ΔL along the wire, calculate the field ΔB produced by that segment at the point and add the contributions from each segment together vectorially. We will carry out part of this process, then indicate how to complete it and then give the final answer. Choose a segment ΔL as shown. At point P, we calculate the magnitude of ΔB to be

$$\Delta B = (\mu_0/4\pi)I\Delta L \sin\theta/r^2$$

The direction of the field is perpendicular to $\mathbf{\Delta L}$ and to $\mathbf{r}$, i.e. perpendicular to the paper, and by the right-hand rule the direction is into the paper. This direction is the same for all segments of the wire, so that we can deduce that the direction of the field of the entire wire will be in this same direction. However, the magnitude of the field from each segment will vary since r and $\sin\theta$ is different for each segment. To add up all the contributions from the segments requires the use of calculus. When we do this, we come up with the result that

$$B = (\mu_0/4\pi)2I/R \tag{7.11}$$

In this equation, I is the current in the wire and R is the perpendicular distance of the point P from the wire. The magnitude of the field depends only on these two variables. To get the direction of the magnetic field, we make use of the same picture that we developed for the direction of the magnetic field produced by a moving charge. The field lines are circles about the axis of the long wire. This is easiest to visualize if we draw the straight wire as coming out of or going into the paper, as in Fig. 7-13(*b*). The magnetic field lines are then circles in the plane of the paper, with their center at the wire. The magnetic field at any point is tangent to the circle through that point and the direction of circling is obtained from the right-hand rule, i.e. put your thumb in the direction of the current and the fingers circle in the direction of the magnetic field.

Problem 7.9. A current of 4 A flows in a long, straight wire along the x axis in Fig. 7-14. Calculate the magnitude and direction of the magnetic field at the following corners of the cube, whose side is 0.8 m: (*a*) corner d; (*b*) corner f; and (*c*) corner h.

Solution

(*a*) It is useful to draw a picture of the situation looking at the current coming out of the paper. We therefore draw Fig. 7-14(*b*), with the current coming out at the origin, and the sides *bcgh* and *adhe* in

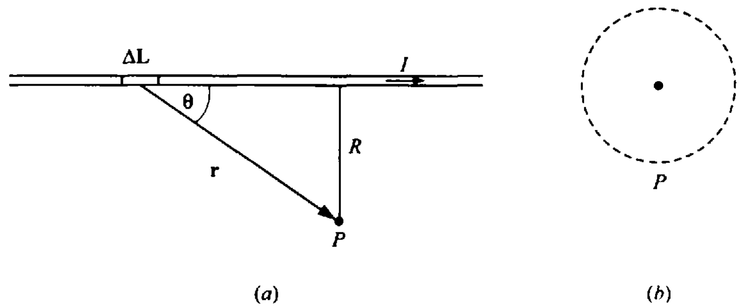

Fig. 7-13

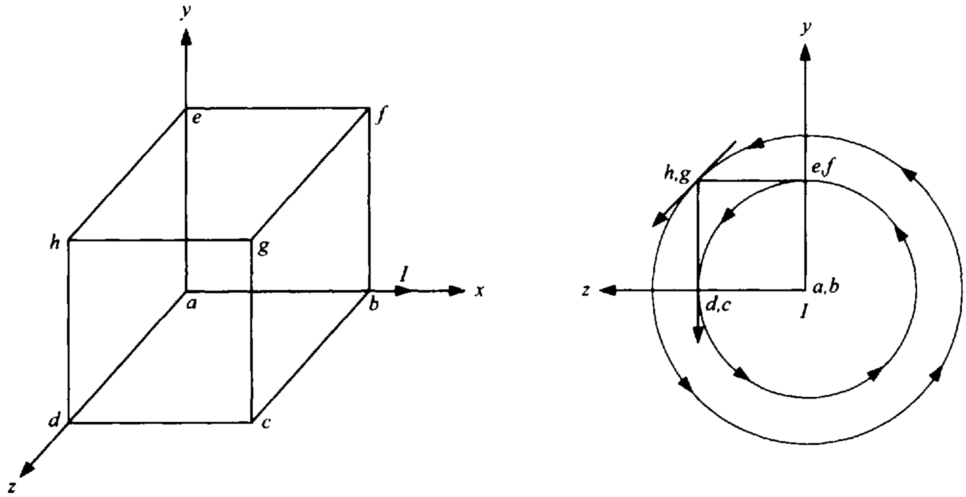

Fig. 7-14

the plane of the paper. We know that the magnitude of the field at any point is given by $B = (\mu_0/4\pi)2I/R$, where R is the perpendicular distance from the point to the line of current, which in our case means to the x axis.

For corner d, $|B| = 10^{-7}(2)(4)/0.8 = 10^{-6}$ T, and the direction is tangent to the circle drawn with center at a and going through point d. This circle is shown on the figure. It is seen from the figure that this tangent is in the $\pm y$ direction at point d. To choose between these two possibilities, we use the right-hand rule which tells us that the field lines circle the axis in a counter-clockwise direction. Therefore the magnetic field at c is in the $-y$ direction.

(*b*) For corner f the magnitude of B is also $B = 10^{-6}$ T, since R is the same as for part (*a*), and the same circle drawn for part (*a*) also goes through point e. Note that the fact that point e is further out along the x axis is totally irrelevant to this calculation. The tangent to the field line at this point is in the $+z$ direction.

(*c*) For corner h the perpendicular distance to the x axis is $0.8\sqrt{2}$, and the magnetic field is $B = 7.07 \times 10^{-5}$ T. The circle through h is also shown on the figure, and the tangent to that circle at h is at an angle of 45° below the $+z$ axis (the direction from e to d). This direction is also shown on the figure.

Problem 7.10. A current of 4 A flows in a long, straight wire out of the paper, as in Fig. 7-15. A charge of 2×10^{-4} C is located at point P, a distance of 3 mm from the wire. This charge is moving with a velocity of magnitude 4×10^{6} m/s. Calculate the force exerted on the charge if the direction of its velocity is (*a*) into the paper; (*b*) to the right; and (*c*) upward.

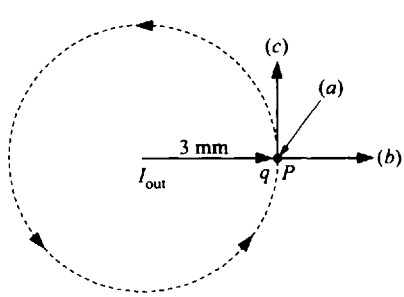

Fig. 7-15

Solution

(*a*) The force exerted on the charge comes from the magnetic field produced by the current in the long straight wire. We will therefore first calculate the magnetic field produced by the wire at the position of the wire, i.e. at point *P*. The magnitude of the field is $B = 10^{-7}\,(2)(4)/3 \times 10^{-3} = 2.67 \times 10^{-4}$ T. Drawing the circle through *P* around the current, we find the direction of **B** to be upward. Note that this magnetic field exists at *P* whether or not there is a charge at this point. Since a moving charge does exist at *P*, the field exerts a force on this moving charge. This force depends on the magnitude and direction of **v**.

The magnitude of the force is given by $F = |qvB \sin \phi|$. If **v** is into the paper, then $\phi = 90^\circ$, and $F = 2 \times 10^{-4}\,(4 \times 10^6)(2.67 \times 10^{-4}) = 0.21$ N. The direction of the force is perpendicular to the plane of **v** and **B**, so that it is either toward or away from the current line. Using the right-hand rule (rotating **v** into **B**) we find that the force is away from the current.

(*b*) Using the same value for **B** that we calculated in (*a*), and noting that $\phi = 90^\circ$, we get that $F = 0.21$ N again. The direction is now either into or out of the paper (perpendicular to **v** and **B**), and the right-hand rule chooses out of the paper.

(*c*) Here v is in the same direction as the magnetic field, so that $\phi = 0$. The force is therefore zero.

Field in a Long Solenoid

We now consider the field produced within a long, tightly wound **solenoid**. This case is depicted in Fig. 7-16. A wire is continuously wound around a long pipe with adjacent windings close to each other. This is similar to the case of the field produced by a ring along its axis, except that we have to add together the fields of many parallel rings. This calculation can be performed using the calculus. Furthermore, we want to know the field everywhere within the solenoid, not just on its axis. This is an extremely difficult calculation, but can be derived using Ampere's law—see the next section. When this calculation is performed, we find that the field within a long solenoid is the same at any point within the solenoid, and is zero (or very small) outside the solenoid. The magnitude of the field inside the solenoid is given by $|B| = \mu_0 nI$, where n is the number of turns per meter. The direction of the field is parallel to the axis with the direction given by the same right-hand rule used for the ring (the fingers circle in the direction of the current and the thumb points in the direction of the field). The result is the reason that solenoids are so very useful for producing magnetic fields. The field produced is *uniform*, with the same magnitude and direction everywhere within the solenoid. Furthermore, this **uniform field** does not depend on the radius of the solenoid, only on the number of windings per unit length. One can, for instance, wind several layers of turns, one on top of the other, to increase n, and each layer will contribute the same field, independent of the radius (as long as the solenoid is truly long).

Problem 7.11. Calculate the magnetic field produced by the solenoid in Fig. 7-16, if the current is 25 A, the radius of the winding is 3 cm, and if there are 700 turns per meter.

Solution

The magnitude of the field is $B = \mu_0 nI = (4\pi \times 10^{-7})(700)(25) = 0.022$ T. The radius did not enter into the calculation. The direction, using the right-hand rule, is to the right.

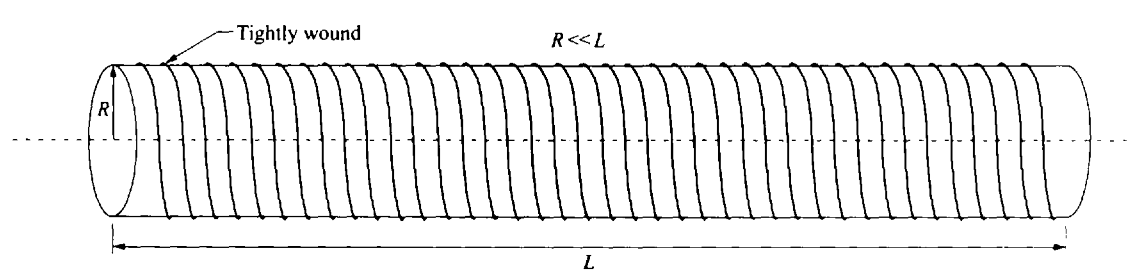

Fig. 7-16

If the solenoid is not infinitely long, but the length is much greater than the radius, then the above result is still nearly true as long as one is not too near to the end of the windings. The field lines inside the solenoid are straight lines, parallel to the axis, until one approaches the ends. Outside the solenoid, the field is no longer zero, and the field lines are as shown in Fig. 7-17. This happens to be the same field line configuration as for a permanent bar magnet, which we have already described at the end of Sec. 6.5 as consisting of many circulating currents. It is therefore not surprising that a bar magnet produces the same field as that of a solenoid.

Note again that **field lines** form closed loops, unlike electric field lines that begin or end at a point charge. The fact that magnetic field lines don't converge to or diverge from a point is a fundamental property of the magnetic field and can be stated as a general law: "**Magnetic field lines** never converge to a point or diverge from a point".

Composite Fields

If several wires each produce magnetic fields, then the actual **magnetic field** at any point is the vector sum of the fields produced by each wire. This is illustrated by the following examples.

Problem 7.12. Calculate the magnetic field produced by the two long parallel wires, each carrying a current of 25 A, which are separated by 0.6 m, as in Fig. 7-18.

(*a*) At point P, between the two wires and at a distance of 0.2 m from the first wire.

(*b*) At point Q, to the right of the wires, and at a distance of 0.4 m from the second wire.

Solution

(*a*) The magnitude of the field from each wire is $B = (\mu_0/4\pi)(2I/R)$. Therefore,

$$B_1 = 10^{-7}(2)(25)/(0.2) = 2.5 \times 10^{-5}\text{T},$$

and
$$B_2 = 10^{-7}(2)(25)/(0.4) = 1.25 \times 10^{-5}\text{T}.$$

The direction of B_1 is into the paper, whereas the direction of B_2 is out of the paper. Adding the two vectorially, we get that $B = (2.5 - 1.25) \times 10^{-5}$ T into the paper.

(*b*) Here $B_1 = 10^{-7}(2)(25)/(1) = 5 \times 10^{-6}$ T, and $B_2 = 10^{-7}(2)(25)/(0.4) = 1.25 \times 10^{-5}$ T. The direction of both B_1 and B_2 is into the paper. Thus $B = (1.25 + 0.5) \times 10^{-5}$ T $= 1.75 \times 10^{-5}$ T, into the paper.

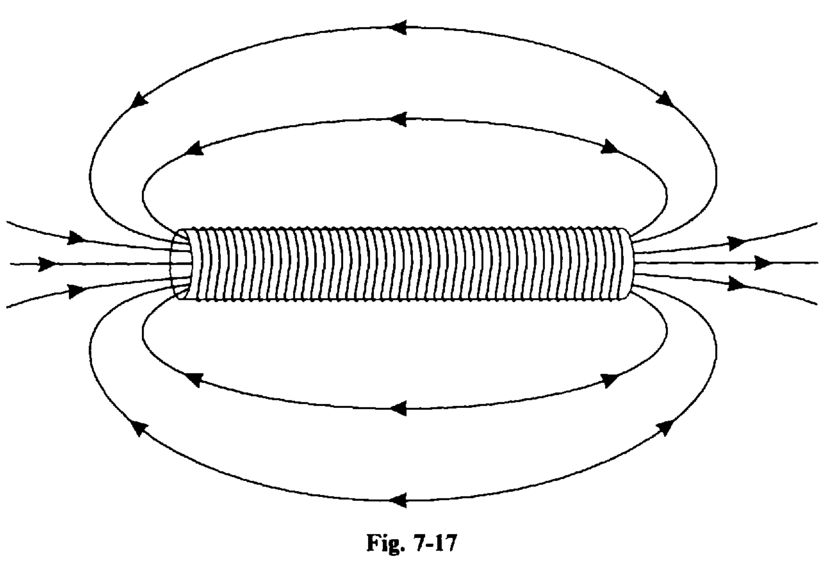

Fig. 7-17

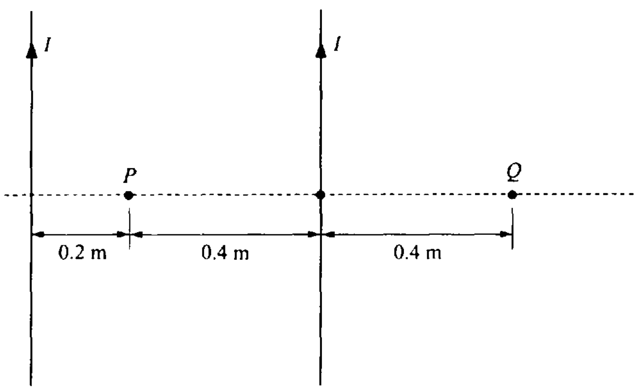

Fig. 7-18

Problem 7.13. Calculate the magnetic field produced by the long parallel wire carrying a current of 25 A, and the ring carrying a current of 2.5 A, at the center of the ring. The center of the ring is 0.5 m from the wire, and the ring has a radius of 0.2 m. The currents are in the direction shown in Fig. 7-19.

Solution

The magnitude of the field from the wire is $B = (\mu_0/4\pi)(2I/R)$. Therefore,

$$B_w = 10^{-7}(2)(25)/(0.5) = 10^{-5}\text{T}.$$

The magnitude of the field from the ring at its center is $B = \mu_0 I/2R$, and therefore

$$B_R = 4\pi \times 10^{-7}(2.5)/(2)(0.2) = 7.9 \times 10^{-6}\text{T}.$$

The direction of B_w is into the paper, whereas the direction of B_R is out of the paper. Adding these together vectorially gives $B = (7.9 - 1) \times 10^{-6} = 6.9 \times 10^{-6}$ T out of the paper.

7.4 AMPERE'S LAW

In Sec. 7.3, we learned how to calculate the field produced by a current. In that formulation, we added together the contributions of the various segments of the wire to get the total field. As we saw, except for the simplest situations, obtaining the field is very difficult. There is a powerful general law relating the magnetic field and the current, which often gives insight into the behavior of the magnetic field, and, in certain circumstances, allows for the compete determination of the field without lengthy calculation. This relationship is given by **Ampere's law**. This mathematical relationship between the current and the magnetic field is similar in spirit to Gauss's law in electrostatics, but quite different mathematically. We will first show the basis for arriving at this law in a very special case, then state the law in general and apply it to calculating the magnetic fields of special current configurations.

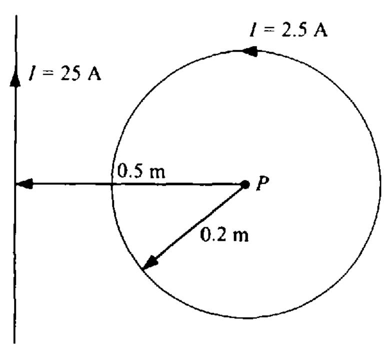

Fig. 7-19

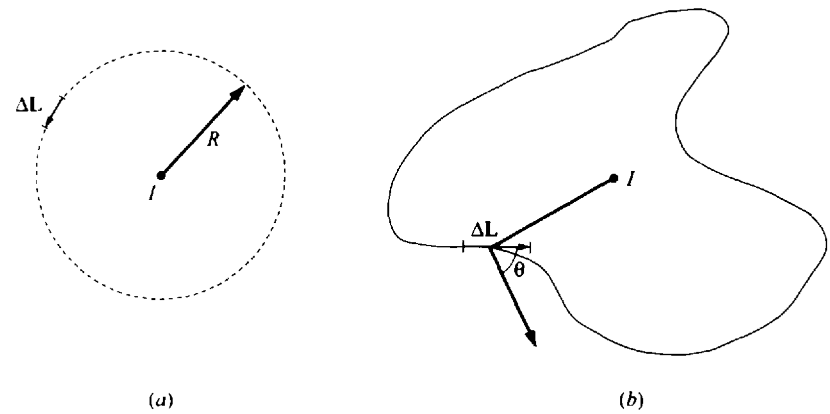

Fig. 7-20

Consider the case of a long straight wire, as drawn in Fig. 7-20(*a*). We have already shown that the magnetic field circles around the wire. If we draw a circular path around the wire as shown, at a radius R, the magnetic field at any point on this wire is the same as at any other point and has the value $B = (\mu_0/4\pi)2I/R$. Furthermore, the direction of **B** is tangent to the circle, and parallel to the **ΔL** segments making up the circumference. If we add up all the $B\Delta L$ products around the circle, we get: $[(\mu_0/4\pi)2I/R](2\pi R) = \mu_0 I$. What makes this interesting is the fact that it is generalizable to any shape closed path contour surrounding the wire, such as that of Fig. 7-20(*b*). As shown in this figure, **B** is no longer tangent to the contour and indeed varies both in magnitude and in angle to the curve from point to point. Nonetheless, if we multiply the tangential component of **B** by **ΔL** for each infinitesimal segment, and add them up around the contour, the result is still $\mu_0 I$. This is analogous to calculating the work done by a variable force as a particle moves along its path (Chapter 6, Section 6.2). We thus calculate the quantity ($B \cos \theta\Delta L$) for each segment of the path, where θ is the angle between **B** and **ΔL**. As you recall, this is the same as taking the component of **B** along **ΔL** at any point, and multiplying by ΔL. The contribution from each segment can be positive or negative, depending on whether θ is less than or greater than 90°. To get Ampere's law, we assume that the **ΔL** directions obey the right-hand rule with thumb pointing in the direction of the current and fingers circling in the direction of the **ΔL**'s. If we now add the contributions to ($B \cos \theta\Delta L$) from all the segments of the path, then the sum will still equal $\mu_0 I$. For any path around the wire, as long as it is a closed path, and it is directed by the right-hand rule, we get that the sum of ($B \cos \theta\Delta L$) for the entire closed path will equal $\mu_0 I$. If several currents flow in different wires going through the area enclosed by the path, each will contribute its own $\mu_0 I$, and the sum of ($B \cos \theta\Delta L = B_t \Delta L$) for the entire closed path will equal $\mu_0 I_{total}$, where I_{total} is the total current flowing through the area enclosed by the path. Using the terminology of the calculus, we say that the *line integral* of $B \cos \theta\Delta L$ around a closed path equals the total current flowing through the area enclosed by that path. This very important result is **Ampere's law**!

$$\lim \Delta L \to 0, \quad \sum_{\text{closed loop}} B_t \Delta L = \mu_0 I_{total} \tag{7.12}$$

Ampere's law is a very general result, valid in all circumstances of magnetostatics. It depends on the fact that, in contrast to the case of electric field lines in electrostatics, magnetic field lines do not start or end on charges. There are no point sources of magnetic field lines. Instead, magnetic field lines close upon themselves. The amount of magnetic field along these closed paths depends on the enclosed current, in the manner given by Ampere's law.

Note. If we were calculating the work due to a conservative force around a closed loop, we would get zero, in contrast to the non-zero result of Ampere's law. In this regard, B behaves like a non-conservative "force".

In order to be able to use Ampere's law to evaluate magnetic fields, one has to be able to evaluate the sum of ($B \cos \theta \Delta L$) along some closed path. This is usually possible only for cases of special symmetry, where one knows that the field has the same value at every point along the path. In that case, the sum is just equal to the value of B times the length of the path. This is similar to the case of Gauss's law which can be used to evaluate the electric field in cases of special symmetry. We will discuss several such symmetry cases, where Ampere's law is often used to evaluate the magnetic field.

Long Straight Wire

This case was actually used by us in deriving the simplest special case of Ampere's law. We now use Ampere's law to derive the field in this case just to introduce the technique that is generally used in applying Ampere's law.

In Fig. 7-20(*a*), the current in the wire is coming out of the paper. We draw the circle around this wire at a radius R, and will use this circle as the path for adding up all the contributions to ($B \cos \theta \Delta L$). The symmetry of the problem immediately tells us that B has the same value at all points on the path, since each point at the same R looks identical to the wire. Furthermore, there cannot be a radial component to the magnetic field, or a component in or out of the paper, because the magnetic field has to be perpendicular to both the current direction and the displacement from a current element to a point of interest on the circle. Thus B must be everywhere tangent to the circle and $B \cos \theta$ will equal the constant B at every point on the path. The sum of ($B \cos \theta \Delta L$) along the whole circle will therefore be $B(2\pi R)$, and, by Ampere's law, this equals $\mu_0 I$, or

$$B(2\pi R) = \mu_0 I, \qquad \text{or} \qquad B = \mu_0 I/2\pi R,$$

which is the correct result.

Coaxial Cable

A **coaxial cable** consists of an inner solid conductor of radius R_1, carrying current I out of the paper, and an outer, concentric hollow cylinder, of radius R_2, carrying the same current I into the paper, the current being distributed uniformly around the cylinder, as in Fig. 7-21(*a*). We will calculate the magnetic field produced by these currents in the region between the conductors ($R_1 < r < R_2$), and in the region outside both conductors ($r > R_2$). Again, because of the circular symmetry, the field will be the same at all points at the same distance from the wires. Therefore, if we choose as our path a circle of radius r, centered on the axis of the wires, the field will be the same at all points on the path. Also, for the same reason discussed in the previous section the field is confined to the plane of the circle. In

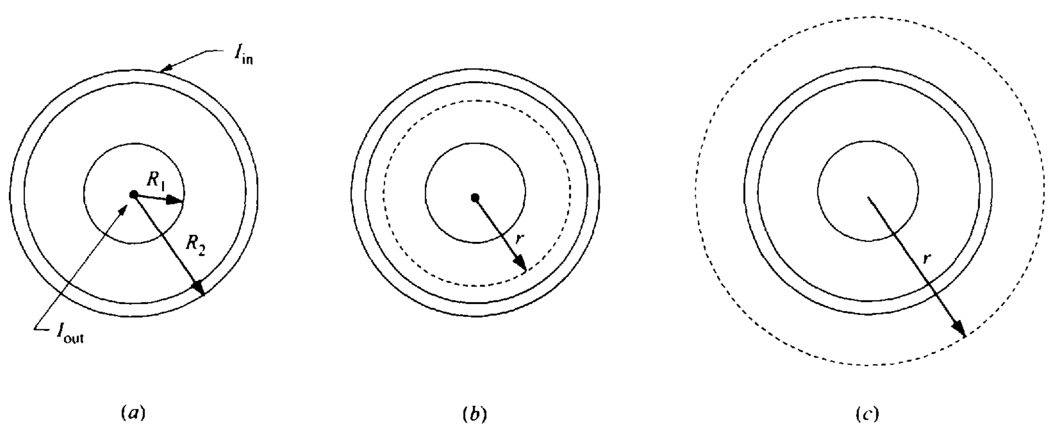

Fig. 7-21

addition the field will not have a radial component since, by symmetry, this would imply field lines meeting at a point at the center of the circle which, as noted earlier, violates a key law of magnetic fields. Thus $\cos\theta = 1$ everywhere along the circle.

To evaluate the magnetic field between the wires, we draw a circular path of radius r between the wires ($R_1 < r < R_2$), as in Fig. 7-21(*b*). Evaluating ($B \cos\theta\Delta L$) along this path gives $B(2\pi r)$, which must equal $\mu_0 I_{total}$, where I_{total} is the current flowing through the area enclosed by the path. Since $r < R_2$, the only current flowing through this area is that in the inner conductor, which is I. This, once again, gives that $B = \mu_0 I/2\pi r$.

To evaluate the field outside of both cylinders, we draw a circular path with radius greater the $R_2(r > R_2)$, as in Fig. 7-21(*c*). Again, the sum of ($B \cos\theta\Delta L$) along this path gives $B(2\pi r)$, which must equal $\mu_0 I_{total}$, where I_{total} is the current flowing through the area enclosed by the path. Now, however, the circular path encloses both cylinders, and both currents flow through the enclosed area. Since the currents flow in opposite directions, the total current will be zero. Therefore, we find that $B = 0$ in this region.

Problem 7.14. A coaxial cable consists of an inner conductor of radius 0.02 m and a thin outer conductor of radius 0.06 m. What current is needed in this cable to produce a magnetic field of 10^{-5} T at a point located at a distance of 0.03 m from the axis of the cable?

Solution

Since this point is located between the cylinders ($0.02 < 0.03 < 0.06$), we use the formula derived for that case $B = \mu_0 I/2\pi r$. Thus $B = (4\pi \times 10^{-7})(I)/2\pi(0.03)$, or $10^{-6} = 2 \times 10^{-7}(I)/0.03$, which gives $I =$ 0.15 A.

Problem 7.15. A special coaxial cable consists of an inner conductor of radius 0.02 m and an outer conductor of radius 0.06 m. The inner conductor carries a current of 2 A out of the paper, while the outer conductor carries a current of only 1.5 A into the paper. What is the magnetic field at (*a*) a point between the cylinders, at a distance r from the axis; and (*b*) a point outside both cylinders at a distance r from the axis?

Solution

Since the two currents are not equal in magnitude, we cannot just use the previous result. Instead, we have to start with Ampere's law and apply it to this case. We still have the circular symmetry, and can use a circular path to evaluate the sum of ($B \cos\theta\Delta L$).

(*a*) For this case we use a circle with radius between 0.02 m and 0.06 m, with radius r. The evaluation of the sum is identical to our previous case. The I_{total} in this case is just the current in the inner conductor. This results in $B = \mu_0 I/2\pi r = 4 \times 10^{-7}/r$.

(*b*) For this case we use a circle with radius greater than 0.06 m, with radius r. The evaluation of the sum is identical to our previous case. The I_{total} in this case, however, is the current in both the inner and the outer conductor. The total current is therefore (2.0 − 1.5)A. This results in $B = \mu_0 I_{total}/2\pi r = 10^{-7}/r$.

Long Solenoid

A different application of Ampere's law is in the case of a very long **solenoid**, pictured in cross-section in Fig. 7-22, where we assume $R \ll L$, and we are interested in the part of the solenoid far from the ends. Here the current in the coils circling the solenoid, I, is coming out at the top and going in at the bottom. The symmetry that we have for this long solenoid requires that the field does not depend on the distance along the axis, since, for a long solenoid, every point along its length is identical. Furthermore, if one draws a circle around the axis, the field does not depend on where one is on the circle. In fact it can depend only on the distance from the axis, r (we will see that it actually turns out to be

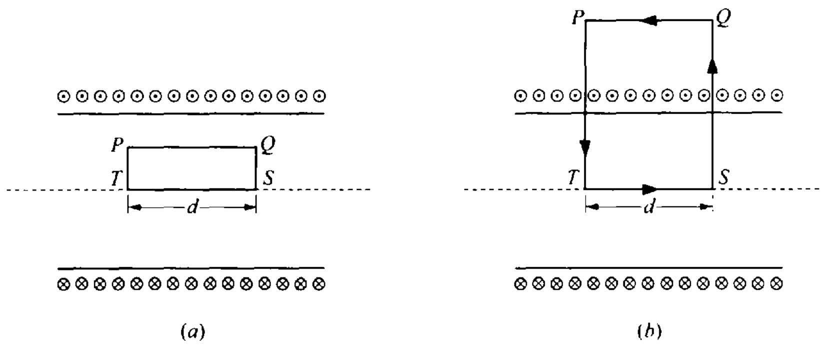

Fig. 7-22

independent of r as well). At any point P, at a distance r from the axis, the field cannot have a radial component for the same reason mentioned when we discussed a long straight wire. Similarly it cannot have a component tangent to the circle since, by symmetry, that would be the same everywhere along the circle and Ampere's law would give a non-zero result even though no current flows through the circle. It has only a component along the direction of the axis. Thus the field lines within the solenoid are parallel to the axis of the solenoid.

Problem 7.16. Use Ampere's law to show that the magnetic field anywhere within the solenoid is the same as that along the axis of the solenoid, and therefore is uniform.

Solution

To show that the field is uniform, we choose a rectangular path $PQST$ shown in Fig. 7-22(*a*), where TS is along the axis. Since there is no radial component of the field, then, along the segments TP and QS, there is no component of B along the path, and the contribution from these segments will be zero. Along the segments PQ and ST, the field is parallel to the path, let us assume to the right. Then for segment PQ the contribution to ($B \cos \theta \Delta L$) will be $B_r d$, where B_r is the field at a distance r from the axis, and d is the length of the path PQ. For the segment ST, the contribution will $-B_0 d$, where B_0 is the field at the axis, and the minus sign comes from the fact that we are moving along that path in a direction opposite to the field. The sum over the entire path is therefore $(B_r - B_0)d$, which, by Ampere's law must equal $\mu_0 I_{total}$, where I_{total} is the current going through the area of $PQST$. There is no current going through this area, since the only current in the problem exists in the wires circling the solenoid. Thus,

$$(B_r - B_0)d = 0, \qquad \text{and} \qquad B_r = B_0.$$

This shows that at all r within the solenoid the field is the same and equal to the value on the axis.

Problem 7.17. Use Ampere's law to calculate the value of the uniform field within the solenoid.

Solution

To calculate the magnitude of the field, we note that the field *outside* the solenoid will be very small as we go far away from the solenoid. We choose a path $PQST$, shown in Fig. 7-22(*b*), which has current flowing through it in the direction out of the paper due to the coil lines at the top of the solenoid. We choose the segment PQ to be very far from the solenoid, so the contribution from that segment will be zero. The direction around the loop is chosen counter-clockwise by the right-hand rule. Again, the contribution from TP and QS will be zero, and the only segment contributing a non-zero value is ST. From this segment we get $B_0 d$ as in the previous paragraphs. This must equal $\mu_0 I_{total}$, where I_{total} is the current flowing through the area of $TPRS$. The total current equals the number of wires between U and V times I, the current in each wire. This is ndI, where n is the number of turns per meter. We therefore conclude that

$$B_0 d = \mu_0 ndI, \qquad \text{or} \qquad B_0 = \mu_0 nI,$$

which is the result we quoted in Sec. 7.3.

Problem 7.18. A solenoid is made from 2000 windings on a length of 2 m. The radius of the solenoid is 0.3 m. If the windings carry a current of 3 A, what is the magnetic field near the middle of the solenoid?

Solution

Since the length of the solenoid is much greater than the radius, it can be considered to be a long solenoid when one calculates the field far from the ends of the solenoid. Then the field is $B = \mu_0 nI = 4\pi \times 10^{-7}(2000/2)(3) = 3.77 \times 10^{-3}$T.

Toroidal Solenoid

A **toroidal solenoid** consists of wires wound around a toroid, which is a doughnut shape usually with a circular cross-section, as in Fig. 7-23. The mean radius of the toroid is r. We assume that the current flows into the paper on the outside of the toroid, and out of the paper on the inside of the toroid. In order to use Ampere's law, we draw a circular path through the toroid at its mean radius, r. We will go around this circle in the counter clockwise direction, since the positive direction for current going through the area of this circle is out of the paper. Every point on this path is identical to any other point on the path, so the magnetic field along this direction will not vary as we move along the path. Furthermore, by arguments similar to those for the long solenoid, there is no component of B in the plane of any cross-section perpendicular to the solenoid. When we add all the contributions to the sum, we get $B(2\pi r)$, where B is the component of the field along the path at the radius r. This must equal μ_0 times the current through the area. The only current in the problem is the current in the wires wound around the toroid. Only the wires on the inside of the toroid go through the area of our path, so the total current through the path is NI, where N is the total number of windings. Furthermore, this current is positive, since it is coming out of the paper, which is our positive direction. Equating the sum to the total current gives

$$B(2\pi r) = \mu_0 NI, \qquad \text{or} \qquad B = \mu_0 NI/(2\pi r)$$

This is the field within the toroid at a point located at a distance r from the axis of the toroid. Note that $N/2\pi r$ is the number of turns per unit length, if the length is measured at the center of the ring. For a case where the radius of the cross-sectional area of the toroid is much less than the mean radius of the toroid, the toroid is nearly like a long solenoid, and the formula for the magnetic field is identical with the one for the solenoid.

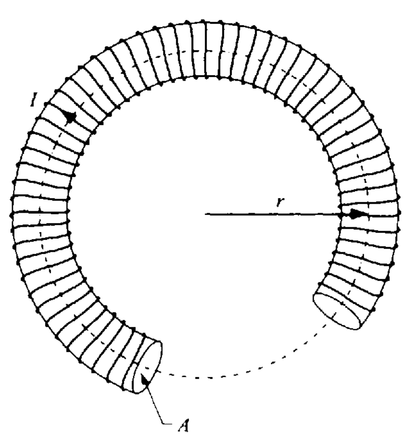

Fig. 7-23

Problem 7.19. A toroid has a mean radius of 1.5 m, and a circular cross section with a radius of 5 cm. There are 3000 windings on the toroid, carrying a current of 2 A. What is the magnetic field at (*a*) the center of the cross-section; and (*b*) within the toroid, just at the inner edge?

Solution

(*a*) The field is given by $B = \mu_0 NI/(2\pi r) = 4\pi \times 10^{-7}(3000)(2)/(2\pi)(1.5) = 9 \times 10^4\text{T}$.

(*b*) Here, the radius of the path is $(1.5 - 0.05)$ m $= 1.45$ m, so that $B = 9.3 \times 10^{-4}\text{T}$. This does not differ too much from the field at the center since the radius of the cross-section is small compared to the mean radius of the toroid.

This case of the toroid illustrates the usefulness of Ampere's law for calculating the magnetic field, since, for this case, any other kind of calculation would be very difficult.

Problems for Review and Mind Stretching

Problem 7.20. In Fig. 7-24, a charge of -6×10^{-4} C is moving up (in the $+y$ direction) with a velocity of 3×10^6 m/s. The charge is instantaneously at the point on the y axis at a distance of 1.5 m from the origin. What magnetic field does this charge produce (*a*) on the positive x axis, at a distance of 2 m from the origin; and (*b*) on the positive z axis, at a distance of 3.6 m from the origin?

Solution

(*a*) The formula for the magnitude of the field is

$$|B| = (\mu_0/4\pi)|qv \sin\phi/r^2| \tag{7.1}$$

Here $r = (1.5^2 + 2^2)^{1/2} = 2.5$, and $\sin\phi = \sin(180^\circ - \phi) = 2/2.5 = 0.8$. Thus

$$B = 10^{-7}(6 \times 10^{-4})(3 \times 10^6)(0.8)/(2.5)^2 = 2.3 \times 10^{-5}\text{T}$$

The direction is obtained by drawing a circle about the y axis (the line of **v**), going through the point, as in the figure. The tangent to the point on the x axis is in the $\pm z$ direction. Using the right-hand rule (thumb along **v** and the fingers curl around the circle), the direction is in $-z$, *if* q were positive. Since q is negative, the correct direction is in $+z$.

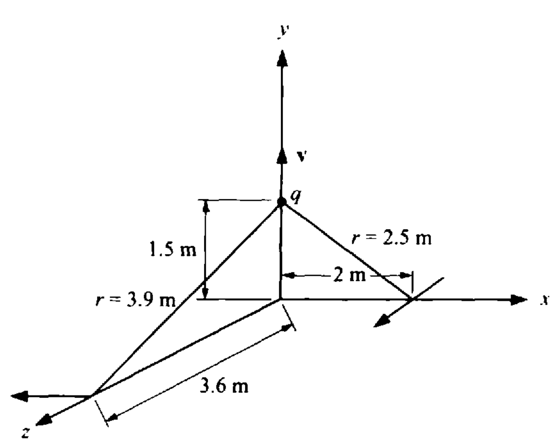

Fig. 7-24

(*b*) Here $r = (1.5^2 + 3.6^2)^{1/2} = 3.9$ m, and $\sin \phi = \sin(180° - \phi) = 3.6/3.9 = 0.923$. Thus

$$B = 10^{-7}(6 \times 10^{-4})(3 \times 10^6)(0.923)/(3.9)^2 = 1.1 \times 10^{-5}\text{T}$$

The direction is obtained by drawing a circle about the y axis (the line of **v**), going through the point, as in the figure. The tangent to the point on the z axis is in the $\pm x$ direction. Using the right-hand rule (thumb along **v** and the fingers curl around the circle), the direction is in $+x$, *if* q were positive. Since q is negative, the correct direction is in $-x$.

Problem 7.21. In Problem 7.4, calculate the electric force between the two charges, and compare it with the magnetic force between the same charges.

Solution

In Problem 7.4, we calculated the force between two charges, q_1 and q_2, moving parallel to each other with velocities v_1 and v_2, and separated by a distance d. The result was an attractive force of

$$|F_{\text{mag}}| = (\mu_0/4\pi)q_1 q_2 v_1 v_2/d^2 = 10^{-7} q_1 q_2 v_1 v_2/d^2 \qquad (i)$$

To calculate the electric force, we use Eq. (*3.1*),

$$|F_{\text{elec}}| = [1/(4\pi\varepsilon_0)]q_1 q_2/d^2 = 9 \times 10^9 q_1 q_2/d^2 \qquad (ii)$$

The ratio of these forces $F_{\text{mag}}/F_{\text{elec}} = 10^{-7}\, v_1 v_2/9 \times 10^9 = v_1 v_2/(3 \times 10^8)^2$. This is a small number, unless v_1 and v_2 are comparable to 3×10^8 m/s, which is the speed of light.

Problem 7.22. Two identical coils, each having 1000 turns, are separated by a distance of 1.6 m along a common axis, as in Fig. 7-25. For each coil $R = 1.5$ m and $I = 2$ A. The currents flow in *opposite* directions in the two coils, as shown. Calculate the field (*a*) at a point P_1 on the axis midway between the coils; and (*b*) at the center P_2 of the first coil.

Solution

(*a*) For each coil, we can use Eq. (*7.10*) multiplying by N for the number of turns. Therefore, for each coil, $B = 10^3\,(4\pi \times 10^{-7})(2)(1.5)^2/2(1.5^2 + 0.8^2)^{3/2} = 5.8 \times 10^{-4}$ T. The direction is to the right for the field from the first coil and to the left for the field from the second coil. For both coils together, the field is therefore zero.

(*b*) For the first coil, $B = 10^3\,(4\pi \times 10^{-7})(2)/2(1.5) = 8.38 \times 10^{-4}$ T, and points to the right. For the second coil, $B = 10^3(4\pi \times 10^{-7})(2)(1.5)^2/2(1.5^2 + 1.6^2)^{3/2} = 2.68 \times 10^{-4}$ T. The direction of this field is to the left. Therefore, the total field is $(8.38 - 2.68) \times 10^{-4} = 5.7 \times 10^{-4}$ T, to the right.

Problem 7.23. Two long, straight, parallel wires carry the same current, I, in the same direction. Calculate the force on a one meter length of the second wire due to the magnetic field produced by the first wire, if the wires are separated by a distance of 1 m.

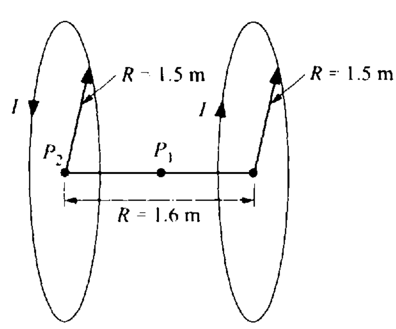

Fig. 7-25

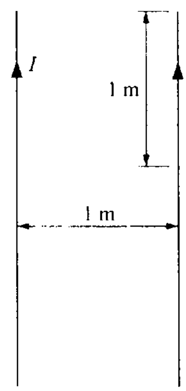

Fig. 7-26

Solution

We draw the situation in Fig. 7-26. The first wire produces a magnetic field of

$$B = (\mu_0/4\pi)2I/R = 2 \times 10^{-7}I, \text{ and the direction is into the paper} \qquad (7.11)$$

The force on the length ΔL $(=1)$ of the second wire is $I\Delta LB = 2 \times 10^{-7}I^2$, and the direction is toward the first wire. This is actually the way we define the unit of current (ampere), and from the ampere we define the unit of charge (coulomb). The ampere is defined as the current needed in this setup so that a force of 2×10^{-7} N is exerted on a 1 m length of the second wire.

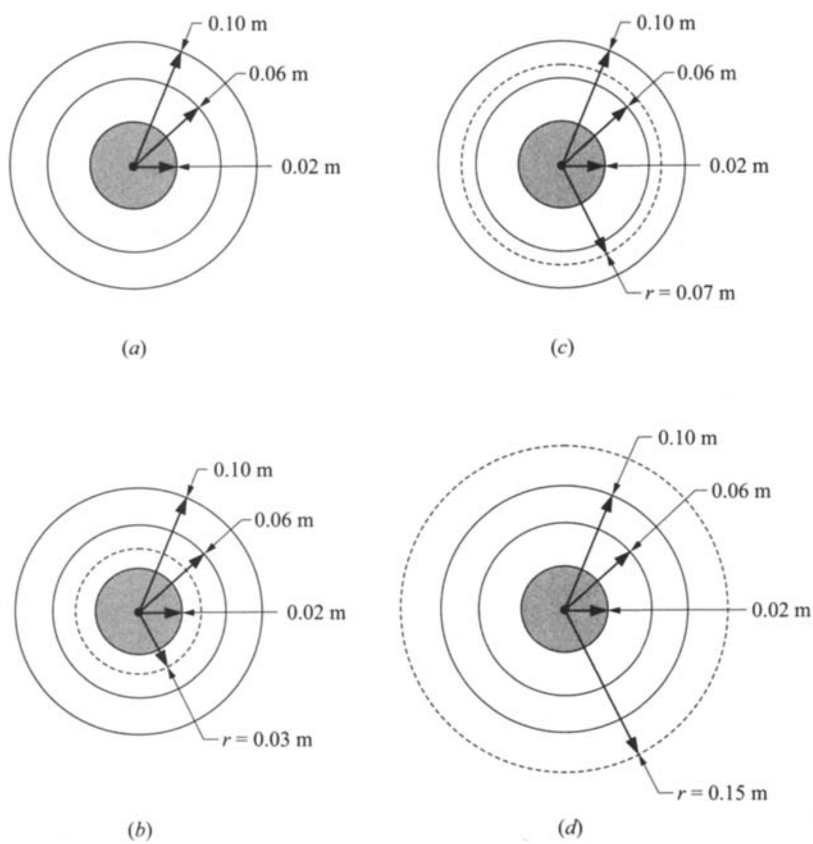

Fig. 7-27

Problem 7.24. A special coaxial cable consists of three concentric cylinders, as in Fig. 7-27(*a*). The inner cylinder is a solid conductor, of radius 0.02 m. The outer two cylinders are thin conducting hollow cylinders, with radii of 0.06 and 0.10 m, respectively. The inner cylinder carries a current of 2 A out of the paper, the second carries a uniformly distributed current of 1.5 A into the paper, and the third carries a uniformly distributed current of 0.5 A into the paper. Calculate the magnetic field produced at (*a*) $r = 0.03$ m; (*b*) $r = 0.07$ m; and (*c*) $r = 0.15$ m.

Solution

(*a*) We draw a circular path at $r = 0.03$ m, as in Fig. 7-27(*b*). This circle lies between the two inner conductors. The sum along the path gives $2\pi(0.03)B$, which we equate to $\mu_0 I_{total}$. Only the inner conductor carries current through the area of the path, so that $I_{total} = 2$ A. Thus $B = 4\pi \times 10^{-7}(2)/(2\pi)(0.03) = 1.33 \times 10^{-5}$ T, and points counter-clockwise about the symmetry axis.

(*b*) We draw a circular path at $r = 0.07$ m, as in Fig. 7-27(*c*). This circle lies between the two outer conductors. Again, the sum along the path, $2\pi(0.07)B$, equals $\mu_0 I_{total}$, where I_{total} is now the current in the two innermost conductors. This current equals $(2 - 1.5)$ A out of the paper. Therefore, $B = 4\pi \times 10^{-7}(0.5)/(2\pi)(0.07) = 1.41 \times 10^{-6}$ T.

(*c*) We draw a circular path at $r = 0.15$ m, as in Fig. 7-27(*d*). This circle lies outside of all the conductors. Again, the sum along the path, $2\pi(0.15)B$, equals $\mu_0 I_{total}$, where I_{total} is now the current in all three conductors. This current equals $(2 - 1.5 - 0.5) = 0$. Therefore, $B = 0$.

Supplementary Problems

Problem 7.25. A charge of 1.7×10^{-3} C is moving north with a velocity of 3×10^5 m/s. What magnetic field (magnitude and direction) does it produce at a point due east which is 1.2×10^{-2} m away?

Ans. 0.35 T, vertically down

Problem 7.26. A charge of -1.7×10^{-3} C is moving south with a velocity of 3×10^5 m/s. What magnetic field (magnitude and direction) does it produce at a point due west which is 1.2×10^{-2} m away?

Ans. 0.35 T, vertically up

Problem 7.27. A charge of -1.7×10^{-3} C is moving west with a velocity of 3×10^5 m/s. What magnetic field (magnitude and direction) does it produce at a point, *P*, which is reached by going north 1.2×10^{-2} m and then west by 0.9×10^{-2} m? (See Fig. 7-28.)

Ans. 0.18 T, vertically up

Problem 7.28. An elevator, carrying a charge of 0.2 C, is moving down with a velocity of 4×10^3 m/s. The elevator is 10 m from the bottom and 3 m horizontally from point *P* in Fig. 7-29. What magnetic field does it produce at point *P*?

Ans. 2.1×10^{-5} T, out

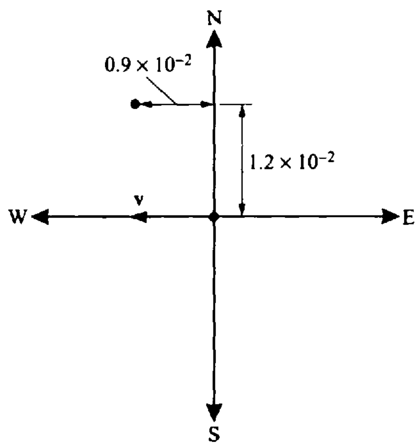

Fig. 7-28

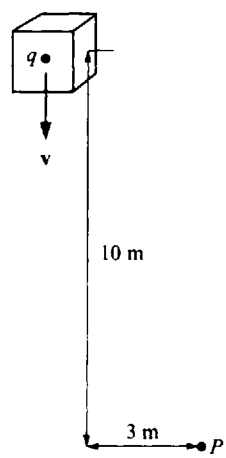

Fig. 7-29

Problem 7.29. One charged particle of -1.3×10^{-6} C is moving north with a velocity of 5×10^6 m/s. Another charged particle, of -2×10^{-6} C is moving south, on a parallel path, with a velocity of 3×10^6 m/s, at a distance of 0.11 m (see Fig. 7-30). What force is exerted between the particles?

Ans. 3.22×10^{-4} N, repulsion

Problem 7.30. A magnetic field at the center of a ring of radius 0.6 m, due to the current in the ring, is 1.2×10^{-4} T. If there are 175 turns in the ring, what current is flowing in the ring?

Ans. 0.65 A

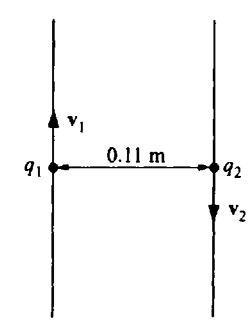

Fig. 7-30

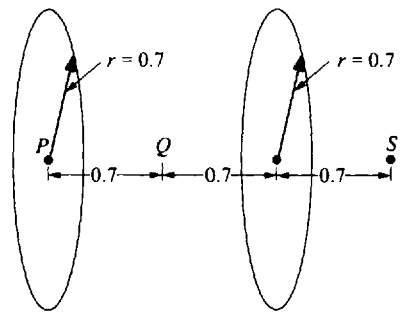

Fig. 7-31

Problem 7.31. Two identical coils, of radius 0.7 m, and having 1200 turns are parallel to each other on the same common axis, as in Fig. 7-31. They each carry a current of 0.8 A in the direction shown. Calculate the magnetic field produced by the two coils (*a*) at point P; (*b*) at point Q; and (*c*) at point S.

Ans. (*a*) 9.38×10^{-4} T; (*b*) 6.09×10^{-4} T; (*c*) 3.32×10^{-4} T

Problem 7.32. An overhead electric transmission line, supplying current to the houses on the street, carries a current of 2000 A. What is the magnetic field that this current produces on the street, 4 m below the line?

Ans. 10^{-4} T

Problem 7.33. A long wire carries current into the paper at the center of a rectangle of sides 6 m × 8 m (Fig. 7-32). The current in the wire is 6 A. What magnetic field (magnitude and direction) is produced at (*a*) point P; (*b*) point Q; and (*c*) point S?

Ans. (*a*) 4×10^{-7} T, in $-x$ direction; (*b*) 3×10^{-7} T, in $-y$ direction; (*c*) 2.4×10^{-7} T, at an angle of 53° below the $-x$ direction

Problem 7.34. Two long wires carry currents of 1.2 A into the paper. The wires are 0.2 m apart, as in Fig. 7-33. Calculate the magnetic field (magnitude and direction) that the wires produce at (*a*) point P; (*b*) point Q; and (*c*) point S.

Ans. (*a*) 0; (*b*) 3.2×10^{-6} T, in $-y$ direction; (*c*) 2.4×10^{-6} T, in $+x$ direction

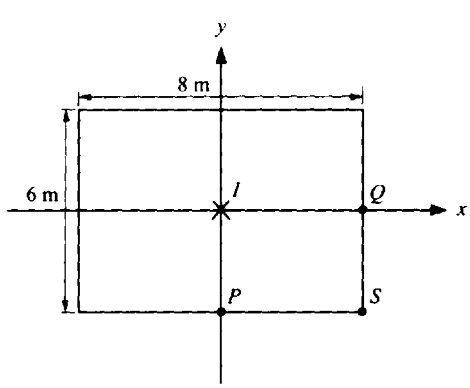

Fig. 7-32

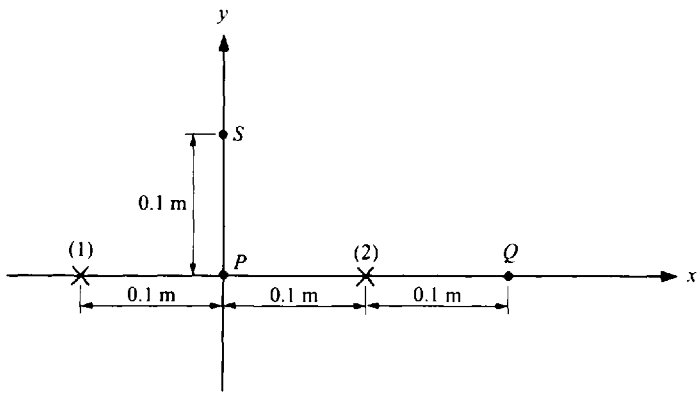

Fig. 7-33

Problem 7.35. Two long wires carry currents of 1.2 A, the first into the paper, and the second out of the paper. The wires are 0.2 m apart, as in Fig. 7-34. Calculate the magnetic field (magnitude and direction) that the wires produce at (*a*) point *P*; (*b*) point *Q*; and (*c*) point *S*.

Ans. (*a*) 4.8×10^{-6} T, in $-y$ direction; (*b*) 1.6×10^{-6} T, in $+y$ direction; (*c*) 2.4×10^{-6} T, in $-y$ direction

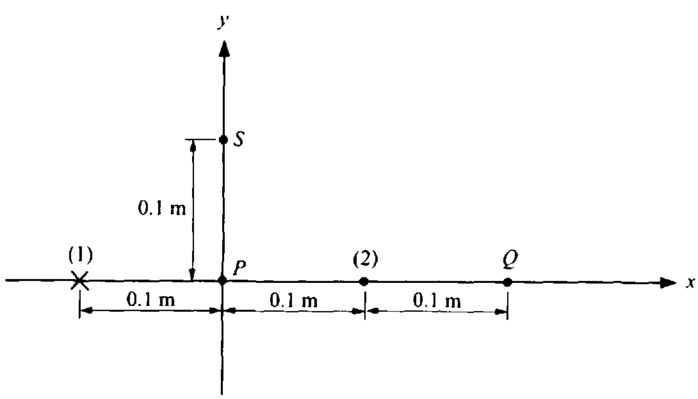

Fig. 7-34

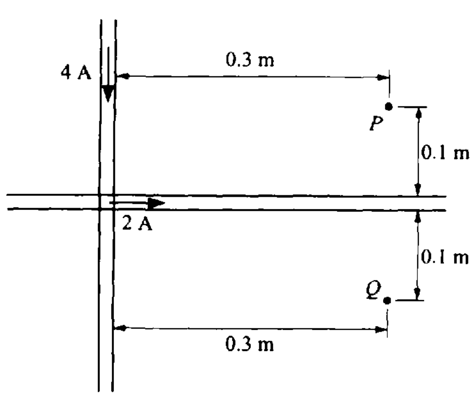

Fig. 7-35

Problem 7.36. Two perpendicular wires carry currents of 2 A and 4 A, respectively, as in Fig. 7-35. Calculate the magnetic field (magnitude and direction) at points P and Q from the two wires.

Ans. 6.67×10^{-6} T out for P; 1.33×10^{-6} T in for Q

Problem 7.37. A long solenoid is made by winding 1500 turns per meter on a radius of 0.3 m, and a second winding of 3500 windings per meter on a radius of 0.5 m, as in Fig. 7-36. Each winding carries a current of 2 A, and the direction of the current in the inner winding is shown on the figure.

(*a*) Calculate the field inside the inner coil if (*i*) the currents flow in the same direction in both windings; and (*ii*) the currents flow in opposite directions in both windings.

(*b*) Calculate the field in the region between the windings if (*i*) the currents flow in the same direction in both windings; and (*ii*) the currents flow in opposite directions in both windings.

Ans. (*a*) (*i*) 12.57×10^{-3} T to the left; (*ii*) 5.03×10^{-3} T to the right; (*b*) (*i*) 8.80×10^{-3} T to the left; (*ii*) 8.80×10^{-3} T to the right

Problem 7.38. A long solenoid has a length of 7 m and has 8400 windings on it. The field inside is 2×10^{-3} T. What current is flowing in the windings?

Ans. 1.33 A

Problem 7.39. A coaxial cable consists of a long, solid inner cylinder of radius 0.02 m and a long, concentric, hollow, conducting cylinder of inner radius 0.08 m. The current is 5 A in the opposite direction in the two conductors. What is the magnetic field at a radius of 0.03 m?

Ans. 3.33×10^{-5} T

Problem 7.40. A coaxial cable consists of a long, solid inner cylinder of radius 0.02 m and a long, concentric, hollow, conducting cylinder of inner radius 0.08 m. The current is 5 A in the *same* direction in the two conductors.

(*a*) What is the magnetic field at a radius of 0.03 m?

(*b*) What is the magnetic field outside of both conductors, at a radius of 0.10 m?

Ans. (*a*) 3.33×10^{-5} T; (*b*) 2×10^{-5} T

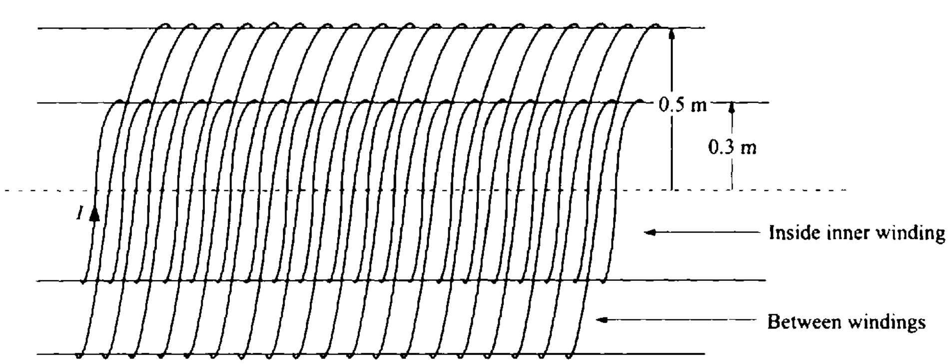

Fig. 7-36

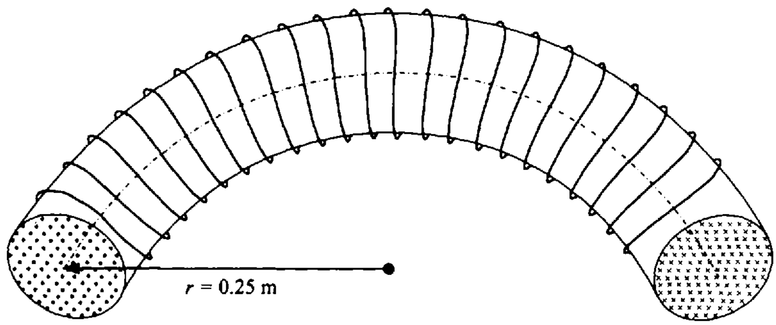

Fig. 7-37

Problem 7.41. A toroidal solenoid has a mean radius of 0.25 m. There are 800 windings around the solenoid, producing a magnetic field of 6×10^{-6} T at that radius. Figure 7-37 shows a cross-sectional view taken at the center of the toroid. The field is coming out at the left side, and going in at the right side. What current flows in the wire, and is the flow clockwise or counter-clockwise at the left?

Ans. 9.4×10^{-3} A, counter clockwise

Problem 7.42. A toroidal solenoid has a rectangular cross-section, of 0.01 m × 0.02 m, as in Fig. 7-38. The mean diameter of the toroid is 2.3 m. The current in the 7500 windings is 3 A, and flows clockwise around the left cross-section, as shown. Calculate the magnetic field (including the direction) at (*a*) the center of the rectangle (point *P*); and (*b*) the outer edge of the rectangle (point *Q*).

Ans. (*a*) 3.91×10^{-3} T, in at the left; (*b*) 3.88×10^{-3} T, in at the left

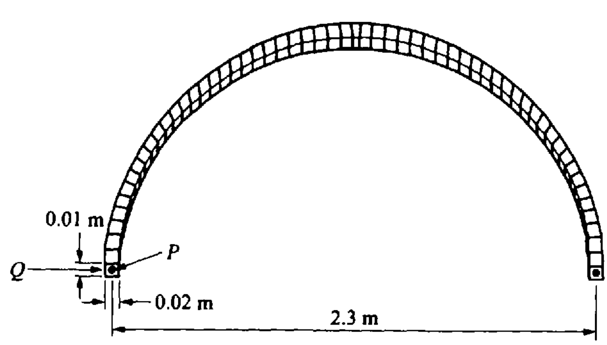

Fig. 7-38

Problem 7.43. What is the *direction* of the magnetic field at point P for the current configurations shown in Fig. 7-39?

Ans. to the right in all cases

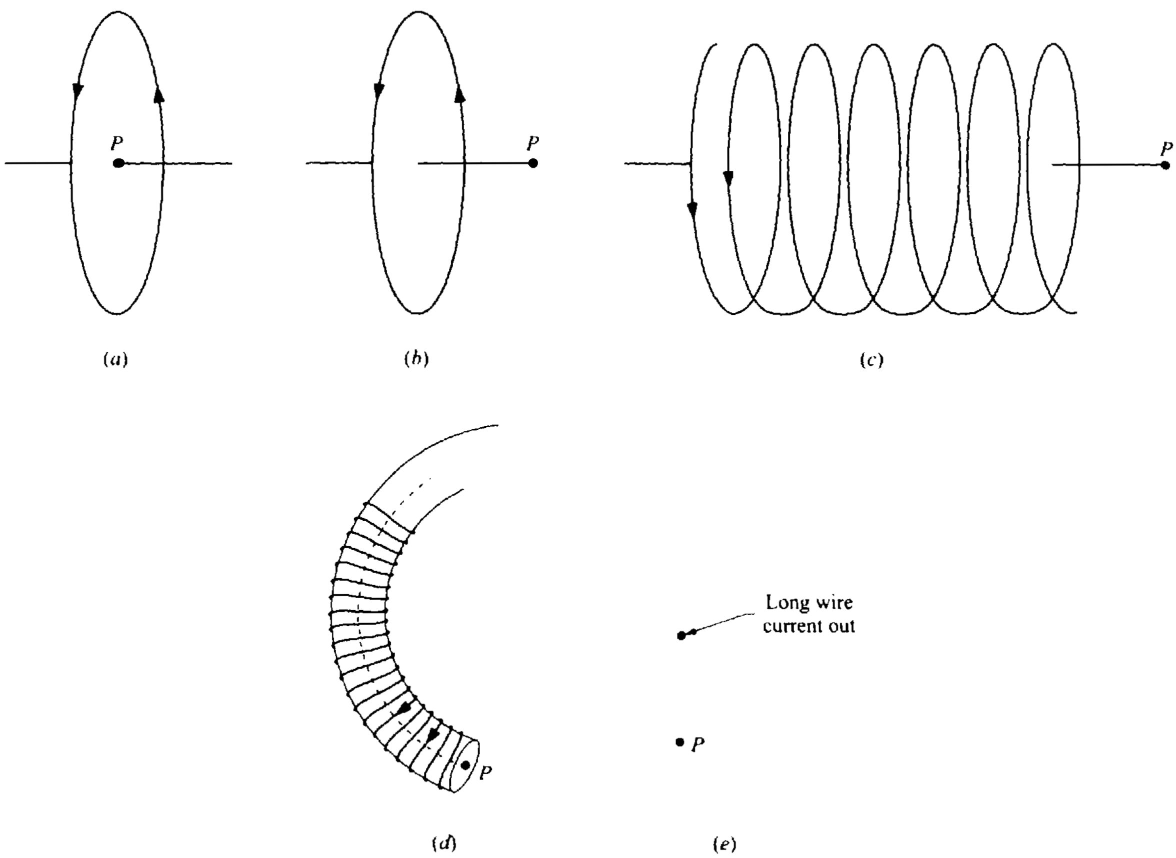

Fig. 7-39