Chapter 10

Inductance

10.1 INTRODUCTION

In the previous chapters, we learned about the creation of a magnetic field by a current in a wire, about magnetic flux, and about the EMF produced if the magnetic flux changes. It is clear that whenever a circuit carries a current, I, a magnetic field is produced in space, and specifically in the area surrounded by the circuit. Thus there will be a certain amount of magnetic flux through the circuit, due to the current in the circuit itself. This flux depends on the magnetic field produced by the current, as well as on the geometry of the circuit. This magnetic field is always proportional to the current in the circuit, and therefore the flux through the circuit is proportional to the current as well. The flux will therefore be given by some factor times the current, where that factor will depend on the detailed geometry of the circuit. That factor is given the name **self inductance**, L.

Similarly, if we have two circuits in close proximity, as in Fig. 10-1, the current in each will produce a magnetic field in the area of the other, and therefore a flux through the other circuit. The flux through each circuit is proportional to the current in the other circuit and the proportionality constants are called mutual inductances. We will first discuss self inductance and then address the issue of mutual inductance.

10.2 SELF INDUCTANCE

As was stated in the introduction, self inductance arises from the flux that a current circuit produces within its own area. The self inductance, which depends only on the geometry of the circuit, connects this flux with the current, and is defined as

$$L = \Phi/I, \text{ or } \Phi = LI \tag{10.1}$$

The terminology "self" inductance arises from the fact that it involves only the flux through a circuit caused by the current in that circuit itself. In general there may be additional flux through a circuit which originates from currents in other circuits or from permanent magnets. The self inductance is usually just called inductance, unless one wishes to distinguish it from the mutual inductance. The

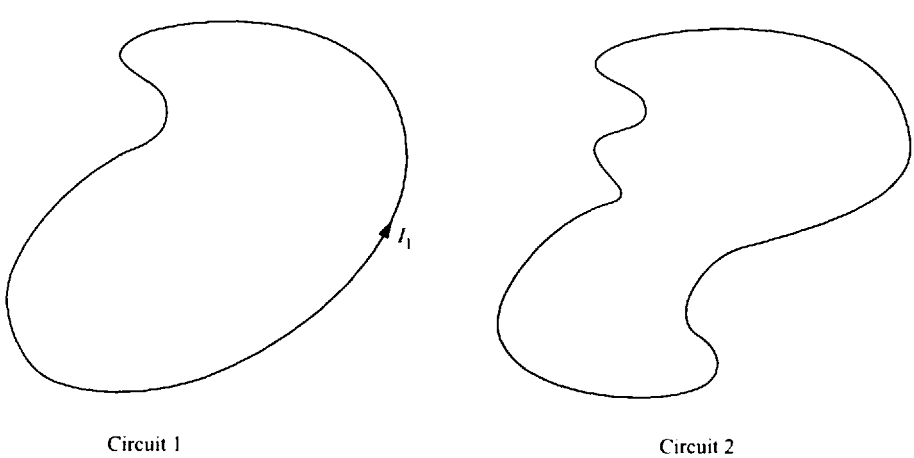

Fig. 10-1

unit for inductance is Wb/A, which is given the name **henry**. Practical circuits have inductance much smaller than one henry, more in the range of millihenries. The main use of the concept of inductance will be in circuits where the current changes, thus causing a proportional change in flux. This changing flux induces an EMF:

$$\text{EMF} = -\Delta\Phi/\Delta t = -L(\Delta I/\Delta t) \tag{10.2}$$

The fact that this EMF is "induced" by the changing flux is the source of the name inductance. The minus sign is a reminder of Lenz's law, that the induced current tries to oppose the change in current.

The procedure for calculating the self inductance is simple in principle, but in practice it may involve complicated calculations. The procedure is as follows. First, we calculate the magnetic field produced by a current, I, at every point within the area of the circuit. Then, using this field, we calculate the flux through the area of the circuit, taking account of the fact that the field is likely to vary from point to point in the area. Once we have calculated the flux through the area, we divide this flux by the current, resulting in the self inductance. The application of this procedure is best illustrated by some examples.

Solenoid

Suppose we have a long solenoid of length d which has n turns per meter, as in Fig. 10-2. We want to know the self inductance of the solenoid. Following the procedure outlined above, we first calculate the magnetic field inside the solenoid. This has previously been calculated to be $B = \mu_0 nI$. This field is uniform everywhere within the solenoid, and therefore the flux passing a single turn of the solenoid is just $\Phi_1 = BA\cos\theta = \mu_0 nIA$. In the length, d, of the solenoid, there are nd turns, so the total flux is $\Phi_T = \mu_0 n^2 IAd$ through the circuit. The self inductance, L, of the solenoid is therefore

$$L = \mu_0 n^2 Ad \tag{10.3a}$$

and the inductance per unit length is

$$L/d = \mu_0 n^2 A \tag{10.3b}$$

Problem 10.1.

(*a*) For the solenoid shown in Fig. 10-2, calculate the inductance per unit length if there are 180 turns/m, and the radius of the solenoid is 0.60 m.

(*b*) How does this answer change if the number of turns/m is 360?

Solution

(*a*) Using the formula that we just developed, we get $L/d = 4\pi \times 10^{-7}(180)^2(\pi)(0.60)^2 = 4.6 \times 10^{-2}$H.

(*b*) The inductance varies as the square of the number of turns/length, n; therefore $L/d = 4 \times$ (result of part a) $= 18.4 \times 10^{-2}$ H.

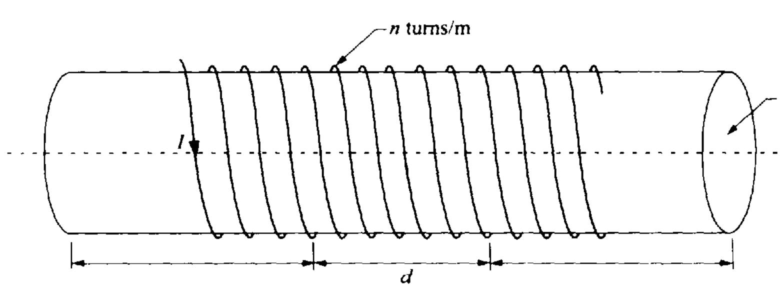

Fig. 10-2

Problem 10.2. For the solenoid shown in Fig. 10-2, calculate the inductance per unit length if there are 180 turns/m, the radius of the solenoid is 0.60 m and the solenoid is filled with a material of permeability of 5.2.

Solution

The field through the solenoid is now larger, as is the flux, so we expect an increase in the inductance. The formula that we just developed must be modified to account for the different material inside the solenoid. The magnetic field is modified by substituting μ for μ_0, and with this one change, we get $L/d = \mu n^2 A$. Thus $L/d = 4\pi \times 10^{-7}(5.2)(180)^2(\pi)(0.60)^2 = 2.4 \times 10^{-1}$ H.

Often an electric circuit will contain a solenoid, with the only significant magnetic field being produced by the solenoid itself. In that case a knowledge of the self inductance of the solenoid is necessary in order to fully understand the response of the circuit to changes in current.

Problem 10.3. A circuit contains a solenoid with an inductance of 3.0 mH, and carries a current of 2.0 A.

(*a*) How much flux passes through the solenoid?

(*b*) If the current is increased to 4.0 A in a 2 s interval, what is the average EMF induced in the circuit?

Solution

(*a*) We know that $\Phi = LI$, so therefore $\Phi = 3.0 \times 10^{-3}(2.0) = 6.0 \times 10^{-3}$ Wb.

(*b*) $\text{EMF}_{\text{AVE}} = -\Delta\Phi/\Delta t = -(6.0 \times 10^{-3}\ \text{Wb} - 3.0 \times 10^{-3}\ \text{Wb})/2.0\ \text{s} = -1.5 \times 10^{-3}$ V.

Problem 10.4. The circuit in the previous problem is changing its current at a steady rate of $\Delta I/\Delta t = 0.15$ A/s. How much EMF is induced in this circuit by this changing current?

Solution

We know that $\Phi = LI$, so therefore $\Delta\Phi = L\Delta I$, and $\Delta\Phi/\Delta t = L\Delta I/\Delta t$. Thus, $\text{EMF} = -\Delta\Phi/\Delta t = -L\Delta I/\Delta t = -3.0 \times 10^{-3}(0.15) = -4.4 \times 10^{-4}$ V. The minus sign reminds us that, in accordance with Lenz's law, the voltage is a "**back EMF**", opposing the change in current.

Problem 10.5. A circuit is changing its current at the rate of $\Delta I/\Delta t = 0.45$ A/m, and produces a back EMF of 3.0×10^{-5} V. What is the inductance of the circuit?

Solution

As in the previous problem, $\text{EMF} = -\Delta\Phi/\Delta t = -L\Delta I/\Delta t$. Therefore, $3.0 \times 10^{-3} = L(0.45)$ or $L = 6.7 \times 10^{-3}$H.

Toroid

To get the inductance of a toroid (Fig. 10-3), we follow the procedure developed previously. The field of a toroid at its mean radius, r, is $B = \mu_0 NI/2\pi r$ (see Chap. 7), where N is the total number of turns on the toroid. If the radius of the cross-sectional area is much less than the mean radius, r, then the field is practically uniform over the area of each turn. The flux through each turn is BA, and the total flux through all the turns of the toroid is $NBA = NA(\mu_0 NI/2\pi r) = \mu_0 N^2 IA/2\pi r$. Dividing by I gives

$$L = \mu_0 N^2 A/2\pi r \tag{10.4a}$$

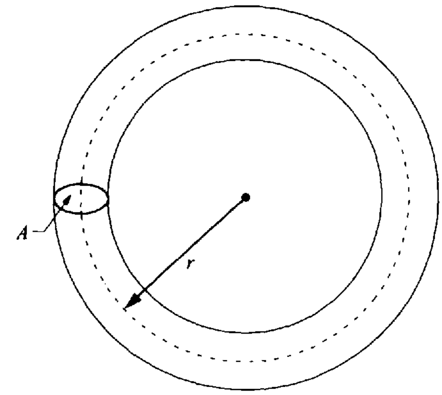

Fig. 10-3

as the inductance of the toroid. If the toroid is filled with material of permeability μ, then

$$L = \mu N^2 A/2\pi r. \tag{10.4b}$$

Problem 10.6. A toroid, of mean radius $r = 1.1$ m, has a cross-sectional area of 3.0×10^{-3} m^2.

(*a*) How many turns are needed if one wants to have an inductance of 8.0 mH?

(*b*) If the toroid were now stretched out to form a straight solenoid, what would the inductance be?

Solution

(*a*) We showed that $L = \mu_0 N^2 A/2\pi r$. Therefore $8.0 \times 10^{-3} = 4\pi \times 10^{-7}(N^2)(3.0 \times 10^{-3})/2\pi(1.1)$, or $N^2 = 1.47 \times 10^7$, or $N = 3.8 \times 10^3$ turns.

(*b*) The solenoid would have length $d = 2\pi r$, and the same small cross-section A. From Eq. (*10.4a*), $L_{SOL} = \mu_0 n^2 A/d$ with $n = N/d \Rightarrow L = \mu_0 N^2 A/d = \mu_0 N^2 A/2\pi r$ which is the same as the inductance of the toroid so the inductance is the same.

10.3 MUTUAL INDUCTANCE

Whenever one has two circuits near each other, it will be possible for a current which exists in one circuit to produce flux through the second circuit. We define a **mutual inductance** between the two circuits in the same way that we define the self inductance for a single circuit. If Φ_{12} is the flux in circuit 2 caused by a current I_1 in circuit 1, Then M_{12} is the factor that connect these two quantities, i.e.

$$\Phi_{12} = M_{12} I_1 \tag{10.5}$$

The exact value of M_{12} is determined by the geometrical relationship between the two circuits, just like the self inductance is determined by the geometry of the single circuit. If we manage to deduce the value of the mutual inductance, then we can always calculate the flux in circuit 2 produced by the current in circuit 1. Furthermore, if the current in circuit 1 changes, then the flux in circuit 2 changes proportionally, which means that $\Delta\Phi = M_{12}\Delta I$. Using Faraday's law, we know that this change in flux produces an EMF in circuit 2, given by

$$\text{EMF} = -\Delta\Phi/\Delta t = -M_{12}\Delta I_1/\Delta t \tag{10.6}$$

Therefore, the mutual inductance is also the link between the induced EMF and the changing current in the other circuit.

Note that if we were to consider the effect of a current in circuit 2 on circuit 1, we would obtain the analogous equations to Eqs. (*10.5*) and (*10.6*) by interchanging all subscripts 1 and 2. Then M_{21} would be the mutual inductance and Φ_{21} the flux in circuit 1 due to the current in circuit 2. It can be shown that

$$M_{12} = M_{21} \tag{10.7}$$

so that there is only one mutual inductance for the two circuits. We can measure the **mutual inductance** by measuring the induced EMF produced in one circuit by a known rate of change in current in the other circuit.

Problem 10.7. Two circuits are near each other, so that a current change in one circuit produces an induced EMF in the other circuit. The current in circuit 1 is 3.0 A, and changes to 3.5 A in a time of 2.0×10^{-2} s. The average EMF induced in the second circuit during this time is 3.4 V. What is the mutual inductance of the two circuits?

Solution

We know that EMF $= -M_{12}\,\Delta I_1/\Delta t$. Thus, $3.4 = M_{12}(0.5)/0.0020$, or $M_{12} = 0.014$ H.

To calculate the mutual inductance for a particular combination of circuits, the procedure is the same as for calculating the self inductance. First we calculate the magnetic field produced by the current I_1 in circuit 1 at the position of circuit 2. Using this field, we calculate the flux, Φ_{12}, enclosed by circuit 2. The mutual inductance is then Φ_{12}/I_1. We will follow this procedure in the examples below to calculate the mutual inductance for several special cases.

Coil on Solenoid

Problem 10.8. Consider a long solenoid, with n_1 turns/m wound on a radius r_1. This solenoid is part of circuit 1. Another coil is wound around the outside of the solenoid, with a total of N_2 turns. This is part of circuit 2. Find a formula for the mutual inductance between these circuits. The setup is shown in Fig. 10-4.

Solution

Following the definition of mutual inductance we first calculate the field produced in the region of circuit 2 by a current I_1 in circuit 1. This field is uniform and equal to $B_{12} = \mu_0 n_1 I_1$. Since the coil is wrapped tightly on the solenoid it has the same radius r_1. Each turn of circuit 2 thus has an area $\pi r_1{}^2$, and

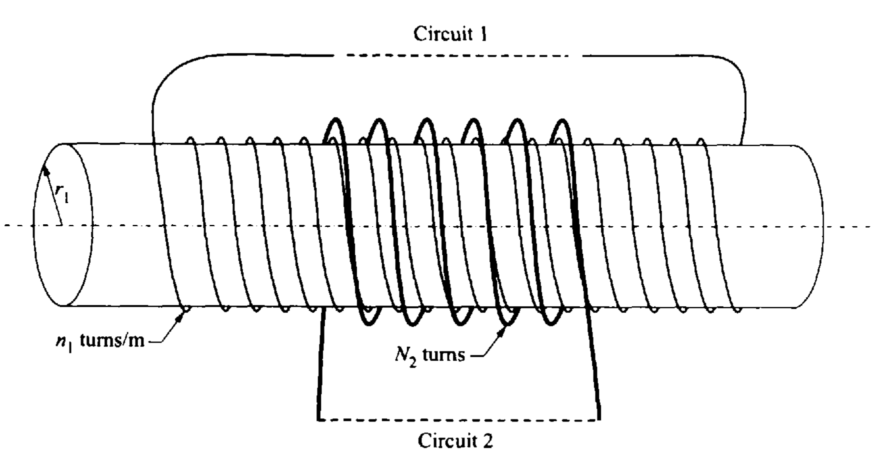

Fig. 10-4

encloses a flux of $B_{12}\pi r_1{}^2$. Therefore, the total flux through circuit 2 is $\Phi_{12} = N_2 B_{12}\pi r_1{}^2 = \mu_0 n_1 N_2 \pi r_1{}^2 I_1$. Finally, $M_{12} = \Phi_{12}/I_1 = \mu_0 n_1 N_2 \pi r_1{}^2$.

Note. The rest of circuit 1 would typically involve a single loop of wire and other circuit elements (not shown in the figure). The same is true of circuit 2. Some additional flux may pass through the single loop as well as other circuit elements further away. In general the flux through these is much smaller than that through the multiple wound elements that are close to each other, and we ignore these small additional contributions to flux in our considerations.

Problem 10.9. A long solenoid has 1800 turns/m, wound on a radius of 0.90 m. A second coil of 25 turns is wound on top of this solenoid. The first winding carries a current of 2.0 A.

(*a*) What is the flux through one turn of the second winding?

(*b*) What is the mutual inductance of the two circuits?

Solution

(*a*) The field produced by the current in circuit 1 is $B_{12} = \mu_0 n_1 I_1 = 4\pi \times 10^{-7}\,(1800)(2.0) = 4.52 \times 10^{-3}$ T. The flux through one turn of the second winding is $B_{12}\pi r^2 = 4.52 \times 10^{-3}(\pi)(0.90)^2 = 0.0115$ Wb.

(*b*) The total flux through the second circuit is $25(0.115) = 0.288$ Wb. The mutual inductance is $0.288/2 = 0.144$ H. Alternatively, we could have directly used the formula for the mutual inductance of this geometry from Problem 10.8: $M_{12} = \mu_0 n_1 N_2 \pi r_1^2 = 4\pi \times 10^{-7}\,(1800)(25)(\pi)(0.90)^2 = 0.144$ H.

Problem 10.10. Suppose the coil in Problem 10.9 has a current of 3.0 A. How much flux is produced in the solenoid due to this current?

Solution

We could proceed as in Problem 10.9 and find the magnetic field everywhere in the solenoid due to the current in the coil, but this would be difficult since, unlike the field due to the current in the solenoid, the field produced by the coil varies in magnitude and direction at different locations. Instead, we take advantage of Eq. (*10.7*): $M_{21} = M_{12}$. Then the flux through the solenoid (circuit 2) is $\Phi_{21} = M_{21} I_2 = M_{12} I_1 = 0.144(3.0) = 0.432$ Wb.

Problem 10.11. A long solenoid has 1800 turns/m, wound on a radius of $r_1 = 0.90$ m. A second coil of 25 turns is wound on top of this solenoid, but at a larger radius of $r_2 = 1.6$ m, as in Fig. 10-5. The first winding carries a current of 2.0 A.

(*a*) What is the flux through one turn of the second winding?

(*b*) What is the mutual inductance of the two circuits?

Solution

(*a*) The field produced by the current in circuit 1 is $B_{12} = \mu_0 n_1 I_1 = 4\pi \times 10^{-7}\,(1800)(2.0) = 4.52 \times 10^{-3}$ T. This field exists only within the first winding. Outside the radius of the first winding, the field is zero (see Sec. 7.4.3). The flux through one turn of the second winding is the sum of the fluxes within the radius r_1 and in the area between r_1 and r_2. Since the field is zero in that part of the area outside the first winding, this part of the area will not contribute anything to the sum. Therefore, the flux is just equal to the field inside the first winding, multiplied by the area of the first winding. This means that the flux equals $B_{12}\pi r_1{}^2 = 4.52 \times 10^{-3}(\pi)(0.90)^2 = 0.0115$ Wb, which is the same for whatever radius the second coil may have, provided it is greater than the radius of the first winding.

(*b*) The mutual inductance is the ratio of the total flux divided by the primary current. This equals $M_{12} = 25(0.0115)/2.0 = 0.144$ H.

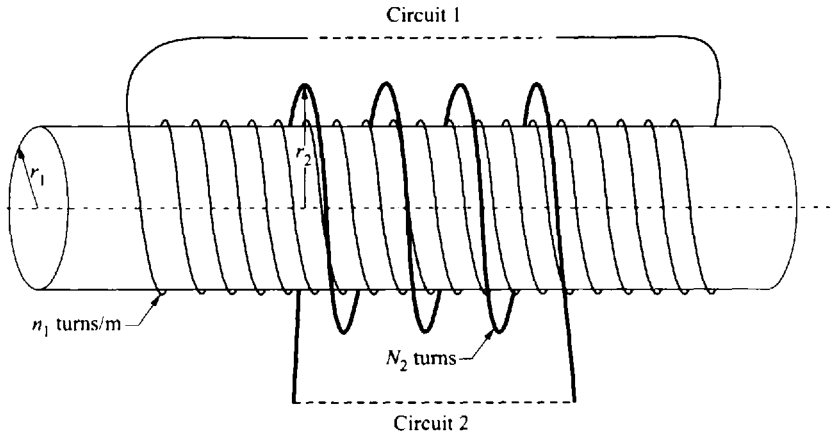

Fig. 10-5

Problem 10.12. A long solenoid has 1800 turns/m, wound around a material of relative permeability $\kappa_M = 150$, at a radius of 0.90 m. A second coil of 25 turns is wound on top of this solenoid. The first winding carries a current of 2.0 A.

(*a*) What is the flux through one turn of the second winding?

(*b*) What is the mutual inductance of the two circuits?

Solution

(*a*) The field produced by the current in circuit 1 is (recalling that $\mu = \chi_M \mu_0$) $B_{12} = \mu n_1 I_1 = 4\pi \times 10^{-7}(150)(1800)(2.0) = 0.68$ T. The flux through one turn of the second winding is $B_{12}\pi r^2 = 0.68(\pi)(0.90)^2 = 1.73$ Wb.

(*b*) $M_{12} = N_2/I_1 = 25(1.73)/2.0 = 21.6$ H.

Note. The results of (*a*) and (*b*) are just those of Problem 10.9 multiplied by the relative permeability, $\kappa_M = 150$.

Coil on Toroid

Consider the case of a toroid, which has a primary winding of N_1 turns, as in Fig. 10-6. The mean radius of the toroid is r, and the cross-sectional area of the toroid is A. A secondary winding of N_2 turns is wound on top of the primary, as shown in the figure. We wish to calculate the mutual inductance of these two circuits.

First we calculate the field produced by a current I_1 in the primary coil, in the region of the secondary coil. The field of a toroid at its mean radius is given by (see Sec. 10.2.2), $B = \mu_0 N_1 I/2\pi r$. As noted earlier, if the radius of the cross-sectional area, A, is small compared to the mean radius, r, then the field will be nearly uniform within the toroid. Then the flux through one turn of the secondary winding will be $\Phi = \mu_0 N_1 I_1 A/2\pi r$, and the total flux through the N_2 turns of the secondary winding will be $\Phi_{\text{total}} = \mu_0 N_1 N_2 I_1 A/2\pi r$. Therefore, the mutual inductance will be $M_{12} = \mu_0 N_1 N_2 A/2\pi r$. If the toroid is filled with material of permeability μ, then the mutual inductance will be

$$M_{12} = \mu N_1 N_2 A/2\pi r \tag{10.8}$$

Problem 10.13. A toroid has 550 turns, wound on a material of permeability 15, and has a mean radius of 2.5 m. A second coil of 25 turns is wound on top of this toroid. The cross-sectional area of the toroid is 0.56 m^2, and the first winding carries a current of 2.0 A.

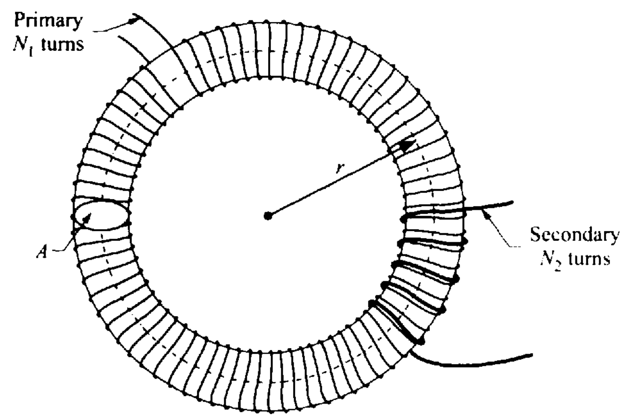

Fig. 10-6

(a) What is the flux through one turn of the second winding?

(b) What is the mutual inductance, M_{12}, of the two circuits?

(c) Find the flux in the toroid when a current of 7.0 A flows in the secondary winding.

Solution

(a) The field produced by the current in circuit 1 is $B_{12} = \mu N_1 I_1/2\pi r = 4\pi \times 10^{-7}$ (15)(550)(2.0)/(2π)(2.5) = 1.32×10^{-3} T. The flux through one turn of the second winding is $B_{12}A = 1.32 \times 10^{-3}(0.56) = 7.39 \times 10^{-4}$Wb.

(b) $M_{12} = N_2\Phi/I_1 = 25(2.32 \times 10^{-3})/2.0 = 9.24 \times 10^{-3}$ H.

(c) In the case we just discussed in this problem, we calculated M_{12}, which is the ratio of the flux in circuit 2 to the current in circuit 1. As in Problem 10.10, to calculate M_{21}, which is the ratio of the flux in circuit 1 to the current in circuit 2, would be much more difficult. Instead, we use the fact that $M_{21} = M_{12}$ to get $\Phi_{21} = M_{21}I_2 = 9.24 \times 10^{-3}(7.0) = 6.47$ Wb.

Coil Near Long Wire

Suppose the primary circuit involves a long straight wire, and the secondary circuit includes a small coil of area, A, with N_2 turns, located at a distance, r, from the wire (see Fig. 10-7). The long wire produces a magnetic field at the position of the small coil, and therefore, a flux through each turn of the coil. To calculate the mutual inductance of these two circuits, we again follow the prescribed procedure.

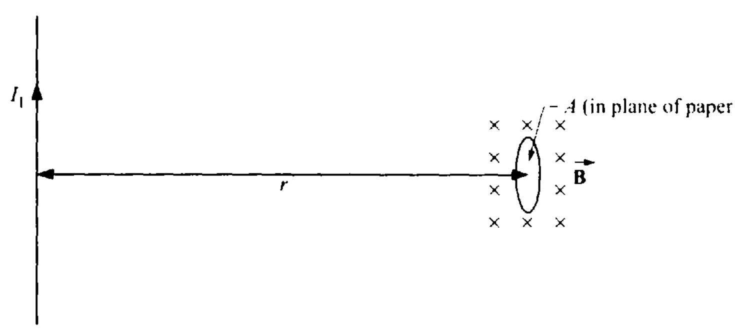

Fig. 10-7

First we calculate the field produced by the primary circuit (the long wire) at the position of the secondary circuit (the small coil). From Eq. (*7.11*) we know that this field is $B_{12} = (\mu_0/4\pi)\, 2I_1/r$. If the area of the coil is small enough so that all parts of the coil are at nearly the same distance, r, from the wire, then the field is uniform over this area, and the flux is just $\Phi_{12} = B_{12}A$. Then the mutual inductance will be $M_{12} = M_{21} = N_2\Phi_{12}/I_1 = (\mu_0/4\pi)2I_1N_2A/rI_1 = (\mu_0/2\pi)N_2A/r$.

Problem 10.14. A long straight wire carries a current of 3.0 A. A small rectangular coil, of sides 4.0 cm × 3.0 cm, is located at a distance of 1.3 m from the wire, and contains 246 turns.

(*a*) What is the flux through one turn of the rectangle?

(*b*) What is the mutual inductance of the two circuits?

Solution

(*a*) The field produced by the current in the long wire is $B_{12} = (\mu_0/4\pi)2I_1/r = 10^{-7}(2)(3.0)/(1.3) = 4.62 \times 10^{-7}$ T. The flux through one turn of the coil is $B_{12}A = 4.62 \times 10^{-7}(0.040)(0.030) = 5.54 \times 10^{-10}$ Wb.

(*b*) $M_{12} = N_2\Phi/I_1 = 246(5.54 \times 10^{-10})/3.0 = 4.54 \times 10^{-8}$ H.

Coil at Center of Loop

Problem 10.15. Suppose we have two concentric single loop circular coils, as in Fig. 10-8. If the small, inner coil (1) carries a current I_1, find an expression for the flux through the large, outer coil (2).

Solution

We realize that this flux can be calculated from $\Phi_{12} = M_{12}I_1$, if we know M_{12}. It would be very difficult to calculate M_{12} directly by the methods we used in the previous examples, since the magnetic field of the inner circuit varies considerably within the area of the outer circuit. However, since we know that $M_{12} = M_{21}$, we can calculate M_{21} instead, using the techniques we have used in the previous examples. To calculate M_{21} we first note that coil 1 is very small and right at the center of large coil 2. Therefore, to find the flux through coil 1 due to a current in coil 2, we need only find the magnetic field due to a current I_2 in coil 2 at the center of the coil. We have already done this in Chap. 7, and the result for a single loop is given by Eq. (7.5), $B_{21} = \mu_0 I_2/2r_2$. Since the radius of the small coil, r_1, is very small compared with r_2, the radius of the outer coil, the field is nearly uniform over the area of the small coil. The flux through the single loop small coil is then $\Phi_{21} = B_{21}\pi r_1^2 = \mu_0 I_2 \pi r_1^2/2r_2$, and $M_{21} = \mu_0 \pi r_1^2/2r_2 = M_{12}$. Finally we get $\Phi_{12} = M_{12}I_1 = \mu_0 \pi r_1^2 I_1/2r_2$.

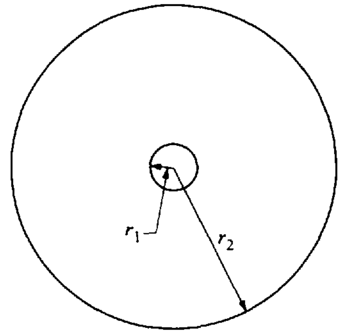

Fig. 10-8

Problem 10.16. Two concentric coils, of radii 0.80 m and 0.015 m, each have 250 turns. If the current in the inner coil changes at the average rate of 0.30 A/s, what is the average EMF induced in the outer coil?

Solution

The EMF in the outer coil is given by $N_2 \Delta\Phi_{12}/\Delta t = N_2 M_{12} \Delta I_1/\Delta t$. We have just shown (Problem 10.15) that for one turn of the inner coil, $M_{12} = \mu_0 \pi r_1^2/2r_2$; and therefore for N_1 turns, $M_{12} = N_1 \mu_0 \pi r_1^2/2r_2$. Then our EMF in the outer coil is: EMF $= N_2(N_1 \mu_0 \pi r_1^2/2r_2)\Delta I_1/\Delta t$. Using the values given, we get that EMF $= [250^2(4\pi \times 10^{-7})\pi(0.015)^2/2(0.80)](0.30) = 1.04 \times 10^{-5}$ V.

10.4 ENERGY IN AN INDUCTOR

Any circuit element that generates an inductance when current flows through it (e.g. a coil, a solenoid, a toroid) is called an inductor. Whenever one has an inductor which initially has no current, it takes energy to make current flow in the inductor. This can be seen from the fact that if one wants to increase the current, a **back EMF** is produced which attempts to stop the increase. In order to increase the current, an external driving voltage must be imposed on the circuit to overcome the back EMF, and this voltage will do work against the resisting EMF. The voltage will continue to do work until the current reaches its final value, at which time the current is no longer changing and no back EMF is being produced. During the time that the current is building up from zero to its final value, however, work must be done on the inductor. The work can be calculated if we remember that the **power** delivered to a system is the current at that time, I, times the voltage, V, at that same time. The power, $P = IV$, is the rate at which energy is being delivered to the system, $P = \Delta W/\Delta t$, where ΔW is the energy added to the system during the time interval, Δt.

The voltage imposed on the inductor is the negative of the back EMF. Since the back EMF equals $-L\Delta I/\Delta t$, the driving voltage must equal $L\Delta I/\Delta t$, and then $P = \Delta W/\Delta t = LI\Delta I/\Delta t$. We must solve this equation to get the total work needed, i.e. the total energy added to the system. By using the methods of calculus, one can show that the result is the same no matter how one changes the current in the inductor. It is always true that the energy stored in an inductor by virtue of the current that we have induced to flow in the inductor is

$$\text{Energy} = (\tfrac{1}{2})LI^2 \tag{10.9}$$

This result is similar to the case of storing energy in a capacitor by virtue of the charge that we have placed on the plates of the capacitor. There the energy was Energy $= (\frac{1}{2})Q^2/C$. We will make more use of these relationships in the future.

Problem 10.17. A solenoid, with an inductance of 55 mH stores an energy of 3.0 J. How much current is flowing in the solenoid?

Solution

The energy equals $(\frac{1}{2})LI^2 = 3.0 = (\frac{1}{2})(55 \times 10^{-3})I^2$, and therefore $I^2 = 109$, and $I = 10.4$ A.

Problem 10.18. A superconducting magnet carries a current of 500 A, and has a self inductance of 5.0 H. While the wires in the magnet are superconducting, the current does not decrease, since there is zero resistance and no energy is being dissipated. If the wires are heated and lose their superconductivity, the current rapidly becomes reduced to zero. How much energy would be released in the process of reducing the current to zero? Where does this energy go?

Solution

The energy released equals $(\frac{1}{2})LI^2 = (\frac{1}{2})(5.0)(500)^2 = 6.25 \times 10^5$ J. The energy would be dissipated as heat (RI^2) since the wires now have resistance.

We have shown that when current flows in an inductor, energy is stored, and we have interpreted this energy as being due to the current flow that we have induced in the inductor. There is another way to interpret this stored energy. Whenever current flows in an inductor, magnetic fields are set up in space. These magnetic fields are directly related to the currents, and the energy needed to set up the currents could equally well have been interpreted as the energy needed to set up the magnetic fields. This is analogous to the case of a capacitor, where the energy needed to charge the plates could equally well be interpreted as the energy needed to set up the electric fields due to these charges. Let us calculate the energy stored in the inductor in terms of the magnetic fields rather than in terms of the current.

To carry out this calculation we will take the case of a long solenoid, as in Fig. 10-2. In the case of a solenoid we know that the magnetic field, B, equals $\mu_0 nI$ inside the solenoid, and is zero outside. We also calculated previously that the inductance per unit length of the solenoid is $L/d = \mu_0 n^2 A$. The energy stored in length d is therefore $E = (\frac{1}{2})LI^2 = (\frac{1}{2})\mu_0 n^2 AdI^2$. But $I = B/\mu_0 n$, and therefore the energy equals $(\frac{1}{2})\mu_0 n^2 Ad(B/\mu_0 n)^2 = (\frac{1}{2})B^2(Ad)/\mu_0$, where Ad is the volume of the length d of the solenoid. Thus the energy density, or energy per unit volume, is

$$\text{Energy density} = (\tfrac{1}{2})B^2/\mu_0 \tag{10.10}$$

In this form, the energy stored in the solenoid is considered as being due to the magnetic fields that have been set up in space. At any point in space, where there is a magnetic field, a certain amount of energy is stored. This energy equals the energy density times the volume of space being considered. Although this calculation was for the special case of a solenoid, the result is true for any other configuration as well. We have previously shown that the same general consideration holds for electric fields as well and indeed the electric field energy density is given by $(\frac{1}{2})\varepsilon_0 E^2$. In other words, wherever electric or magnetic fields exist in space, energy is being stored in the form of these fields. The total energy density at any point in space is the sum of the electric and the magnetic field energy densities. Since the units for energy density are the same irrespective of their source, this offers a means of comparing the relative magnitudes of electric and magnetic fields. Electric and magnetic fields with the same energy density can be considered to be comparable to each other. In fact, we will see that in electromagnetic waves, which we will discuss in a later chapter, the electric and the magnetic fields associated with the wave have equal energy densities. These considerations lend credence to the idea that these fields are real physical quantities that actually exist in space, and are not merely mathematical contrivances that make it easier to calculate the forces exerted by the electric and magnetic interactions.

Problem 10.19. An electromagnetic wave in free space has an electric field of 100 V/m. If there is also a magnetic field associated with this wave, and the energy density of the magnetic field is the same as the energy density of the electric field, what is the magnitude of the magnetic field?

Solution

The energy density for the electric field is $(\frac{1}{2})\varepsilon_0 E^2$, and the energy density for the magnetic field is $(\frac{1}{2})B^2/\mu_0$. Equating these two expressions gives $(\frac{1}{2})B^2/\mu_0 = (\frac{1}{2})\varepsilon_0 E^2$, or $B^2 = \mu_0 \varepsilon_0 E^2 = (4\pi \times 10^{-7})(8.85 \times 10^{-12})(100)^2 = 1.12 \times 10^{-13}$. Thus, $B = 3.33 \times 10^{-7}$ T.

10.5 TRANSFORMERS

From what we have learned in the previous sections, it is clear that we can induce EMFs in one circuit by changing the current in another circuit. This forms the basis of the **transformer**, which is used to transform voltage in one circuit into a different voltage in a second circuit. We have already developed the ideas for this in our discussion of the mutual inductance of two windings on a solenoid in Sec. 10.3. In that case, all the magnetic flux established by the first winding, called the **primary coil**, passes through the turns of the other winding, called the **secondary coil**. In order to get large fluxes, it is useful to place ferromagnetic material within the solenoid that has a large permeability, such as iron. Using such a ferromagnetic material has another advantage. When one magnetizes the iron inside the solenoid by applying a current to the primary winding, that iron, being ferromagnetic, will cause the rest of the iron atoms to align their magnetic moments in the same direction, and become magnetized as well. Furthermore, most of the flux will be confined within the core, so that a wire wound around one part of the core will experience the same flux as that wound around another part. It is then possible to wind the secondary coil on a different part of the iron core, not necessarily on top of the primary winding. It is even possible to bend the iron into a different shape, such as the often used shape shown in Fig. 10-9. Here, the primary winding, with N_1 turns, is wound on one side of the rectangular ring, and the secondary winding, with N_2 turns, is wound on the other side of the ring. This is a typical transformer. If one changes the voltage in the primary circuit, the current in the primary circuit will change, and therefore the flux. For a perfect transformer, the flux through one turn of the secondary is the same as the flux through one turn of the primary. Therefore, the total EMF developed in each winding will depend on the number of turns in that circuit.

If one has DC in the primary, the current does not change, and there is no *change* in the flux. Then, there will be no EMF induced in the secondary. A transformer is useful only with currents that are changing, as with AC. In that case, it is possible to use a transformer to convert a voltage applied to the primary circuit into a larger or smaller voltage in the secondary circuit. This ability to easily convert (transform) voltages in AC, which is much more difficult for DC, is the main reason why AC is the primary source of power throughout the world.

By analyzing the transformer shown in Fig. 10-9 in more detail, one can relate the EMF induced in the secondary circuit and the applied voltage in the primary, V_p to the relative number of turns in these circuits. The result is that:

$$V_s/V_p = N_s/N_p \tag{10.11}$$

If $N_s > N_p$, then $V_s > V_p$, and we will have a step-up transformer. This is useful, for instance if one wants to use an appliance built for 220 volts in an area where only 110 volts are available. If $N_s < N_p$, then $V_s < V_p$, and we have a step-down transformer. This is used by power generating companies, who transmit power along transmission lines at very high voltages, and transform them down to safe levels before they enter one's home.

Problem 10.20. A power company generates electricity at a voltage of 12,000 V, and steps up this voltage to 240,000 V, using a transformer (transformer 1). The electricity is transmitted at this voltage to a substation, where it is stepped down to 8000 V (transformer 2) before being transmitted further. Before entering a house, the voltage is stepped down further to 240 V (transformer 3). What are the turns ratio of each of these five transformers?

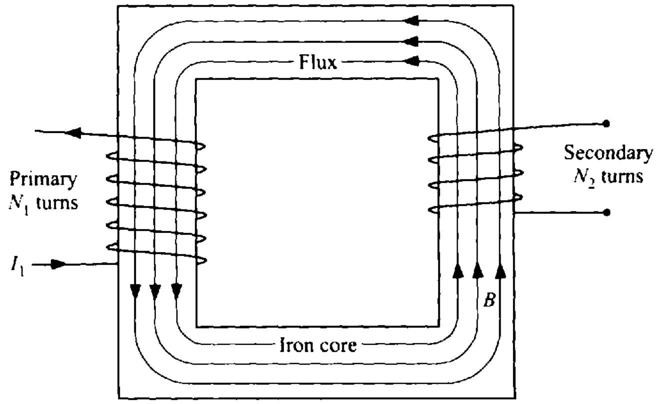

Fig. 10-9

Solution

The turn ratio $N_s/N_p = V_s/V_p$. Therefore, (*a*) for transformer 1, $N_s/N_p = 240{,}000/12{,}000 = 20$; (*b*) for transformer 2, $N_s/N_p = 8000/240{,}000 = 1/30$; and (*c*) for transformer 3, $N_s/N_p = 240/8000 = 3/100 = 0.030$.

Problems for Review and Mind Stretching

Problem 10.21. A coaxial cable consists of an inner conductor of radius, $R = 0.50$ m, separated from a hollow outer conductor by a distance, $\Delta R = 0.0020$ m, as in Fig. 10-10. The inner conductor carries a current, $I = 5.0$ A to the right, and the outer conductor carries the same current to the left. Consider the shaded region between the conductors, of length d. At any point in this region, r is between 0.500 m and 0.502 m.

(*a*) What is the magnetic field at a point in the shaded region, at a distance r from the axis?

(*b*) What is the flux through the shaded region of length d?

(*c*) What is the self inductance of the length d of the coaxial cable, and the inductance per unit length of the cable?

Solution

(*a*) In Sec. 7.4.2, we calculated the field produced by a coaxial cable in the region between the conductors. We found that $B = \mu_0 I/2\pi r$. Substituting in this equation gives, $B = (4\pi \times 10^{-7})(5.0)/2\pi r = 10^{-6}/r$ T. Since r is between 0.500 and 0.502 m, the field hardly varies in this region and we can substitute either value to get $B = 2.0 \times 10^{-6}$ T. The direction of the field is out of the paper in the shaded region, since the field lines circle about the center conductor in this direction (the right-hand rule).

(*b*) In general the flux through the area is $BA \cos\theta$, where θ is the angle between **B** and the normal to the area. For our case $\theta = 0°$, and $A = d\Delta R$. Thus, the flux, Φ, is $\Phi = Bd\Delta R = (\mu_0 I/2\pi R)\Delta R d = (\mu_0 I/2\pi)(\Delta R/R)d = 4.0 \times 10^{-9}$ d.

(*c*) The self inductance, L, is $\Phi/I = (\mu_0 I/2\pi R)\Delta R d/I = (\mu_0/2\pi R)\Delta R d = 8.0 \times 10^{-10}$ d. The inductance per unit length is $L/d = (\mu_0/2\pi R)\Delta R = (\mu_0/2\pi)(\Delta R/R) = 8.0 \times 10^{-10}$ H/m.

Note. A more accurate calculation, taking account of the variation of the field within the region between the inner and outer conductors of a coaxial cable yields the inductance per unit length to be $L/d = (\mu_0/2\pi) \ln (R_2/R_1)$; this is valid even if the difference between R_1 and R_2 is large.

Problem 10.22. An inductor with inductance L is connected in series with a resistor R. A battery completes the circuit with terminal voltage V_0 as shown in Fig. 10-11. At a certain instant, labeled $t = 0$,

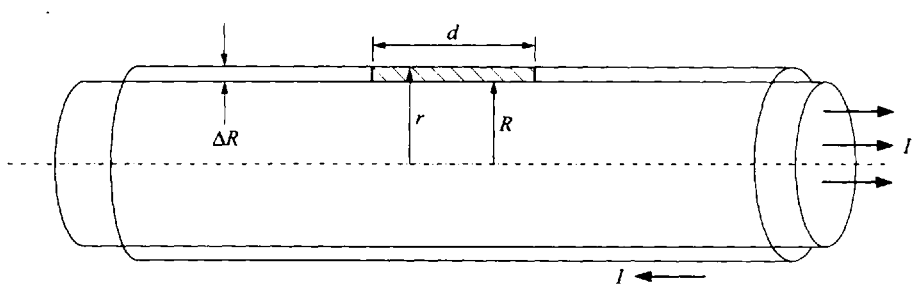

Fig. 10-10

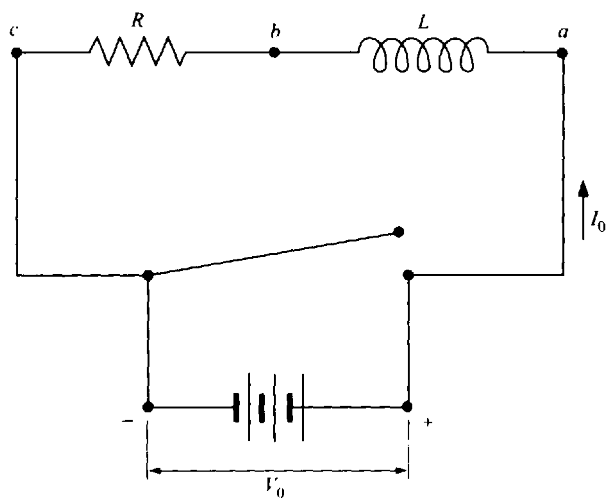

Fig. 10-11

the short circuit switch is closed eliminating the EMF of the battery. Assume the inductor coils have negligible resistance.

(*a*) What is the voltage drop from b to c, V_{bc}, across the resistance before the switch is closed, and what is the voltage drop from a to b, V_{ab}? Explain.

(*b*) At the instant the switch is closed explain qualitatively what happens to the current and voltages across the two elements.

Solution

(*a*) Since there is a steady current I_0, there is no change in flux and no EMF in the inductor. Therefore, since the inductor has zero resistance, V_{ab} is zero. Thus the entire terminal voltage of the battery appears across the resistance and we have $V_{bc} = V_0 = IR$ from Ohm's law.

(*b*) When the switch is closed there is no longer a voltage between points a and c. The current through the resistance starts to collapse. If not for the inductor this collapse, by Ohm's law, would be essentially instantaneous. Because of the inductor the collapse in the current is immediately opposed by an induced EMF in the inductor. This EMF opposes the change in the current and thus tries to maintain the status quo. The collapse in current is thus slowed down and occurs over a finite time interval.

Problem 10.23. Referring to Problem 10.22(*b*), and assuming ΔI is the change in current in an infinitesimal time interval Δt after the switch is closed and I is the current during that time interval:

(*a*) Following the reasoning behind Problem 10.22(*b*) what must the induced EMF in the inductor be in any infinitesimal time interval after the switch is closed? What is the direction of the EMF and what is the voltage V_{ab}?

(*b*) Find a relationship between L, R, ΔI, I and Δt.

Solution

(*a*) Since immediately after the switch closing the voltage drops around the circuit are zero, and the voltage across the resistor is still $V_{bc} = IR$, where I is the current at any time t after the switch closed, the induced EMF in the inductor must take the place of the battery to support the current. The direction of the EMF is from a to b, since it opposes the decrease in current in that direction (Lenz's law), and has the magnitude $\text{EMF}_{ab} = -L\Delta I/\Delta t$ for any infinitesimal time interval Δt.

Note. ΔI is negative from a to b so the EMF indeed points from a to b. The voltage from a to b is opposite to the EMF since the EMF points from higher to lower electrostatic voltage, as for a battery. In other words, the potential at b is higher than the potential at a, so $V_{ab} = -\text{EMF}_{ab} = L\Delta I/\Delta t$ (which is negative).

(*b*) We must have $V_{ab} + V_{bc} = 0$ at every instant. So:

$$L\Delta I/\Delta t + RI = 0 \qquad (i)$$

(In the infinitesimal time interval Δt immediately after the switch is closed, from $t = 0$ to $t = \Delta t$, this is $L\Delta I/\Delta t + RI_0 = 0$.) Turning the equation around we have, in general,

$$-(L/R)\Delta I/I = \Delta t \qquad (ii)$$

This equation can be solved using the calculus to give an expression for how the current falls to zero over time. While this will be discussed further in the next chapter, we note here that if L/R is large for a given Δt, $\Delta I/I$ will be small and the current will fall slowly; if L/R is small then for the same time interval Δt, $\Delta I/I$ will be larger and the current will fall more quickly. L/R is therefore called the time constant of the circuit.

Problem 10.24. An inductor, with inductance L, is connected in series with a resistor, R. A battery completes the circuit with terminal voltage V_0, and causes a current I_0 to flow in the circuit, as in Fig. 10-11. The voltage V_0 is increased by an amount ΔV_0. The current does not increase instantaneously since the inductor produces an induced EMF which tries to prevent any change in current. After a time Δt, however, the current has increased by ΔI, and the voltage across the resistor has increased by ΔV, where $\Delta V = R\Delta I$. This voltage, plus the voltage across the inductor, must equal the voltage ΔV_0.

(*a*) What is the magnitude of the average EMF induced in the circuit by the inductor during this time?

(*b*) What is the ratio of this average induced EMF in the inductor to the voltage increase that appears across the resistor after this time?

(*c*) If $L = 10$ mH and $R = 100\ \Omega$, at what value of Δt would the average induced EMF across the inductor equal the change in voltage ΔV?

Solution

(*a*) The average EMF induced by the inductor equals, in magnitude, $L\Delta I/\Delta t$.

(*b*) The change in voltage across the resistor is $\Delta V = R\Delta I$. Thus $\text{EMF}/\Delta V = L(\Delta I/\Delta t)/(R\Delta I) = (L/R)/\Delta t$.

(*c*) If $\text{EMF} = \Delta V$, then the ratio in part (*b*) is 1, and $L/R = \Delta t = 1.0 \times 10^{-4}$ s. Initially, all the increase in voltage of the battery appeared across the inductor, since the current has not yet changed, and there is therefore as yet no increase in voltage across the resistor. After a long time has elapsed, the current reaches its final constant value and there is no longer any change in current and therefore no voltage across the inductor. After a time equal to the time constant, the increase in voltage across the resistor approximately equals the voltage across the inductor.

The time constant, L/R, is thus a measure of how quickly the circuit approaches its final value. The time constant will be discussed further in the next chapter.

Problem 10.25. A long solenoid has 300 turns/m, and is wound on a radius of 0.90 m. A smaller coil, of radius 0.60 m, is inside the solenoid, with its plane perpendicular to the axis of the solenoid, as in Fig. 10-12.

(*a*) Determine the mutual inductance between the solenoid and the coil.

(*b*) If the coil is rotated by 90° in a time interval of 2.0×10^{-3} s while a current of 4.0 A is flowing in the solenoid, what is the average induced EMF in the coil during that time interval?

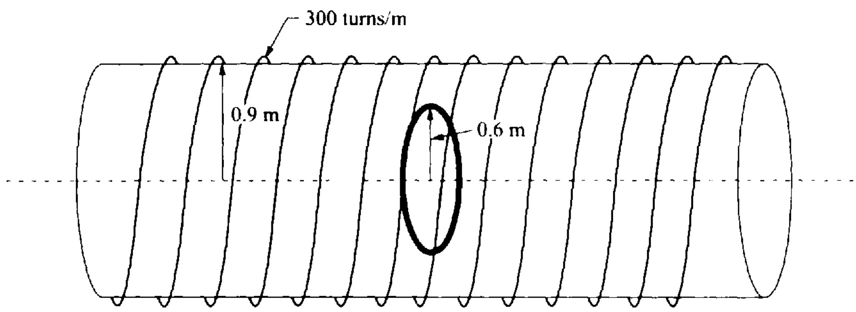

Fig. 10-12

Solution

(*a*) The field established by the long solenoid is $B = \mu_0 nI$, and is uniform within the solenoid. The flux through the small coil is $\Phi = BA = \mu_0 nI(\pi r^2)$, where r is the radius of the small coil. The mutual inductance is $M_{12} = M_{21} = M = \Phi/I = \mu_0 nI(\pi r^2)/I = \mu_0 n(\pi r^2) = (4\pi \times 10^{-7})(300)\pi(0.60)^2 = 4.26 \times 10^{-4}$ H.

(*b*) Here the flux drops to zero since the coil becomes parallel to the flux lines in the solenoid. EMF $= \Delta\Phi/\Delta t = \Phi/\Delta t = MI/\Delta t = 4.26 \times 10^{-4}(4.0)/2.0 \times 10^{-3} = 0.852$ V.

Problem 10.26. A toroid with a mean radius of 1.2 m and a cross-sectional area of 0.050 m^2, carries a current of 5.0 A in 750 turns.

(*a*) What is the self inductance of this toroid?

(*b*) How much energy is stored in the toroid?

(*c*) What is the energy density within the toroid?

Solution

(*a*) The inductance of a toroid was calculated in Sec. 10.2.2, and given by [Eq. (*10.4a*)] as $L = \mu_0 N^2 A/2\pi r$. Therefore, $L = (4\pi \times 10^{-7})(750)^2(0.050)/2\pi(1.2) = 4.7 \times 10^{-3}$ H.

(*b*) The energy stored is [Eq. (*10.9*)] $E = (1/2)LI^2 = 0.5(4.7 \times 10^{-3})(5.0)^2 = 0.059$ J.

(*c*) The energy density is [Eq. (*10.10*)] $(1/2)B^2/\mu_0 = (1/2)(\mu_0 NI/2\pi r)^2/\mu_0 = 0.155$ J/m^3. This result could also have been obtained by taking the total energy (0.059 J) and dividing by the volume of the toroid ($2\pi rA = 0.38$ m^3), or $0.059/0.38 = 0.155$.

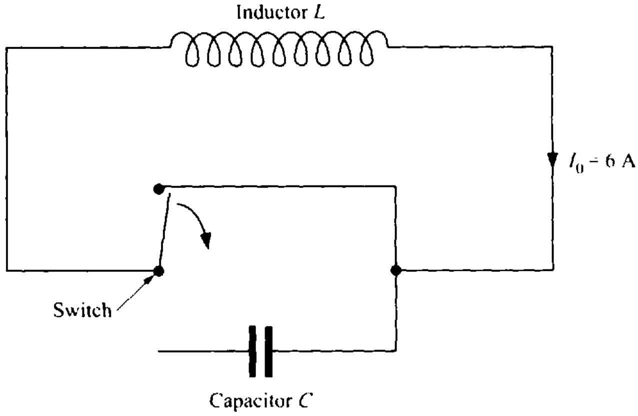

Fig. 10-13

Problem 10.27. A current of 6.0 A is initially flowing through an inductor which has an inductance of 0.15 H in a resistanceless circuit, as shown in Fig. 10-13. The switch is thrown introducing a capacitor into the circuit. The capacitor has a capacitance of 1.22×10^{-3} f, and is initially uncharged.

(*a*) What is the energy stored in the inductor initially?

(*b*) As the current flows into the capacitor, the capacitor becomes charged and the current decrease. When the current becomes zero, the capacitor is charged to its maximum. Assuming that no energy has been lost in this time, what is the voltage across the capacitor at this instant in time, and how much charge is stored on the capacitor?

Solution

(*a*) The energy stored in the inductor is $(1/2)LI^2$. Therefore Energy = $0.5(0.15)6.0^2 = 2.7$ J.

(*b*) The inductor has no energy at this time, since the current is zero. All the energy is in the capacitor, and that energy is $(1/2)CV^2 = 2.7$ J. Thus, $V^2 = 2.7(2)/1.22 \times 10^{-3} = 4426$, $V = 66.5$ V. The charge stored is $Q = CV = 1.22 \times 10^{-3}(66.5) = 0.081$ C. Alternatively, one could have used Energy $= (\frac{1}{2})Q^2/C = 2.7$ J, which also yields $Q = 0.081$ C.

Supplementary Problems

Problem 10.28. A coil has an inductance of 25 mH.

(*a*) If the current in the coil is 2.0 A, what is the flux in the coil?

(*b*) If there is a back EMF of 0.30 V, what is the average rate of change of the current?

Ans. (*a*) 0.050 Wb; (*b*) 12 A/s

Problem 10.29. A coil starts with a current of 5.0 A, which is changing at the rate of 0.90 A/s. This change produces a back EMF of 0.030 V.

(*a*) What is the inductance of the coil?

(*b*) What is the flux in the coil at the start?

Ans. (*a*) 33 mH; (*b*) 0.167 Wb

Problem 10.30. A length of 0.50 m of a long solenoid, with a cross-sectional area of 0.030 m^2, has an inductance of 0.080 H. How many turns/length are on the solenoid?

Ans. 2060 turns/m

Problem 10.31. A toroid has 1500 turns, a mean radius of 1.1 m, a cross-sectional area of 0.95 m^2 and carries a current of 7.0 A.

(*a*) What is the magnetic field at the mean radius?

(*b*) What is the inductance of the toroid?

Ans. (*a*) 1.91×10^{-3} T; (*b*) 0.39 H

Problem 10.32. A coil, with an inductance of 0.50 H, has a uniform magnetic field in its area of 0.40 m^2. It carries a current of 0.50 A. What is the magnetic field in its area?

Ans. 0.625 T

Problem 10.33. A circuit, consisting of one coil, has an inductance of 5.0 mH and carries a current of 1.5 A.

(*a*) What is the flux through the coil?

(*b*) If one increases the circuit to 15 coils, with the same current, what would be the flux through *each* coil? What is the flux through all 15 coils?

(*c*) What is the inductance of the 15 coils?

Ans. (*a*) 7.5×10^{-3} Wb; (*b*) 0.113 Wb, 1.69 Wb; (*c*) 1.13 H

Problem 10.34. Two coils are near each other, so that when one changes the current in the first at the rate of 5.0 A/s, there is an EMF of 2.0 mV induced in the second.

(*a*) What is the mutual inductance between the two coils?

(*b*) What flux goes through the first coil, if there is a current of 8.0 A in the second coil?

Ans. (*a*) 0.40 mH; (*b*) 3.2×10^{-3} Wb

Problem 10.35. A long wire carries a current of 5.0 A. Nearby, there is a loop of area 0.80 m^2, as in Fig. 10-7. The mutual inductance between the wire and loop is 6.0×10^{-5} H.

(*a*) What is the flux through the loop?

(*b*) What is the average field within the loop?

(*c*) What is the average distance between the wire and loop?

Ans. (*a*) 3.0×10^{-4} Wb; (*b*) 3.75×10^{-4} T; (*c*) 2.7×10^{-3} m

Problem 10.36. A long solenoid has 500 turns/m, and produces a magnetic field of 4.0×10^{-3} T inside the solenoid. The solenoid has a cross-sectional area of 0.020 m^2.

(*a*) What current is flowing in the wires?

(*b*) What is the flux through the area of one turn of the solenoid?

(*c*) If one winds 25 turns on the outside of the solenoid, what is the mutual inductance between the solenoid and the turns?

Ans. (*a*) 6.4 A; (*b*) 8.0×10^{-5} Wb; (*c*) 3.1×10^{-4} H

Problem 10.37. A toroid has 750 turns, and a secondary winding on the toroid has 25 turns. The cross-sectional area of the toroid is 0.0090 m^2, and the mean radius is 0.72 m.

(*a*) What is the mutual inductance between the toroid and the secondary winding?

(*b*) If one fills the toroid with material of magnetic permeability 75, what is the mutual inductance?

(*c*) If, in part (*b*), a current of 5.0 A in the toroid is removed in 1.0×10^{-3} s, what average voltage is induced in the secondary?

Ans. (*a*) 4.69×10^{-5} H; (*b*) 3.5×10^{-3} H; (*c*) 17.6 V

Problem 10.38. A small coil, of area 5.0×10^{-4} m^2, is on the axis of a large coil at a distance of 2.1 m from the center of the large coil, as in Fig. 10-14. The large coil has a radius of 0.60 m, and contains 2000 turns. The large coil has a current of 10 A.

(*a*) What is the magnetic field at the center of the small coil?

(*b*) If the field is uniform within the small coil, what is the flux in the small coil?

(*c*) What is the mutual inductance between the coils?

Ans. (*a*) 4.34×10^{-4} T; (*b*) 2.17×10^{-7} Wb; (*c*) 2.17×10^{-8} H

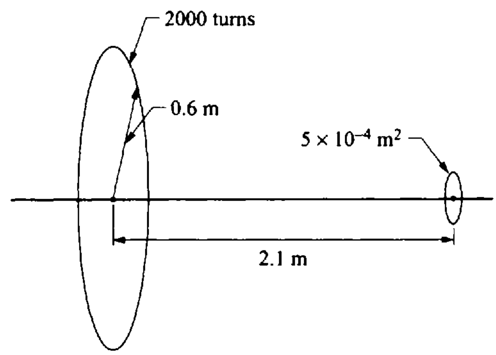

Fig. 10-14

Problem 10.39. A coil with a single turn has a self inductance of 1.5 mH. A tightly wound coil of the same radius, but with 16 turns, is placed flush against the first coil. A current of 0.60 A flows through the first coil.

(*a*) What is the flux through the area of the first coil?

(*b*) What flux goes through the area of each turn of the second coil?

(*c*) What is the mutual inductance between the coils?

Ans. (*a*) 9.0×10^{-4} Wb; (*b*) 9.0×10^{-4} Wb; (*c*) 0.024 H

Problem 10.40. A coil has a self inductance of 250 mH. How much work has to be done to cause a current of 1.1 A to flow in the coil?

Ans. 0.15 J

Problem 10.41. A coil has a self inductance of 250 mH. How much work has to be done to increase the current from 1.1 A to 2.2 A?

Ans. 0.45 J

Problem 10.42. A coil, with an inductance of 150 mH, is carrying a current of 2.1 A. The current is reduced to zero in a time of 2.0×10^{-2} s.

(*a*) How much energy was released to some external circuit?

(*b*) What average power was applied to that external circuit?

Ans. (*a*) 0.33 J; (*b*) 16.5 W

Problem 10.43. A capacitor, with capacitance 2.0×10^{-3} f, is charged to 110 V. The capacitor is then connected to an inductor, of inductance 0.20 H, and current begins to flow. The energy stored in the capacitor is transferred to the inductor.

(*a*) How much energy is initially stored in the capacitor?

(*b*) When the voltage is reduced to 50 V, what current is flowing in the inductor?

(*c*) What is the maximum current in the inductor?

Ans. (*a*) 12.1 J; (*b*) 9.8 A; (*c*) 11 A

Problem 10.44. A toroid has an inductance of 0.15 H, and a volume of 0.80 m^3. It carries a current of 0.50 A.

(*a*) How much energy is stored in the toroid?

(*b*) What is the average energy density in the toroid?

(*c*) What is the average magnetic field in the toroid?

Ans. (*a*) 1.88×10^{-2} J; (*b*) 2.34×10^{-2} J/m^3; (*c*) 2.43×10^{-4} T

Problem 10.45. An air conditioner is built to work at a voltage of 208 V. If a 110 V circuit is to be used to bring power to the air conditioner, does one need a step-up or a step-down transformer, and what ratio of turns is required?

Ans. step-up with turns ratio of 1.89:1

Problem 10.46. A step-down transformer is used in Switzerland, where the voltage is 220 V, to run an American appliance rated at 110 V, 250 W. What is the turns ratio in the transformer?

Ans. 1:2

Problem 10.47. A transformer is built from a rectangular ring, as in Fig. 10-9. The iron core is not perfect (some flux lines leave the core), and the flux in the iron at the secondary is only 75% of the flux at the primary. If the turns ratio is $N_2/N_1 = 20$, what is the voltage ratio V_2/V_1?

Ans. 15