Chapter 6

Magnetism—Effect of the Field

6.1 INTRODUCTION

In previous chapters, we learned about forces exerted by one mass on another mass (gravitational force) and by one charge on another charge (electrical force). Experimentally we find that there is also a force exerted by one *moving* charge on another *moving* charge (in addition to the electrical force). This force is the **magnetic force**. The most common occurrence of this force is when two magnets attract (or repel) each other, but this attraction (or repulsion) is due to subtle properties of the materials, which we will leave to a later chapter.

In discussing the magnetic force, we will use the concept of a **magnetic field**, for which we use the symbol **B**. The magnetic field is a vector, and is the link between the two moving charges that interact with each other. One of the charges is the *source* of the field, and this field, in turn, has the *effect* of exerting a force on the second moving charge. Thus, the magnetic field has two aspects: (1) its effect—to exert a force on a moving charge and (2) its source—the origin of the field, which can be another moving charge, or possibly there may be another means of producing the field. These two aspects are totally independent of each other and therefore we will discuss each in a separate chapter.

The unit for a magnetic field is a **tesla** (T) in our system. A more common unit which is widely used in practice is the **gauss** (G). One gauss equals 10^{-4} tesla. The strength of the magnetic field near the surface of the earth is approximately one gauss.

Note that in general a magnetic field can vary from point to point in space, and can also change from moment to moment. For the present we will assume that the magnetic field remains constant in both space (uniform magnetic field) and time.

6.2 FORCE ON A MOVING CHARGE

In this chapter the *effect* of the magnetic field, **B**, will be discussed. This means that we ask ourselves the following question. Given a magnetic field produced by some means, which is not necessarily of concern to us, what is the force, **F**, that this field, **B**, exerts on a charge, q, moving with a velocity, **v**?

Note. We have to find a vector, **F**, that results from some interaction of a scalar, q, and two vectors, **v** and **B**. This is depicted in Fig. 6-1.

We seek to know the magnitude and direction of the as yet unknown force, **F**, that the magnetic field **B** exerts on the charge q moving with velocity **v** when the angle between the vectors **v** and **B** (when

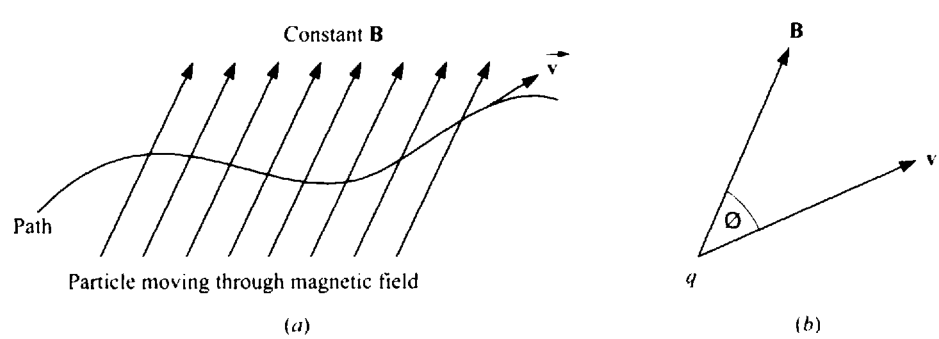


Fig. 6-1

their two tails are touching) is ϕ. We do this in two steps. First we discuss the magnitude, and then we discuss the direction of the force.

Magnitude of the Force

The formula for the magnitude of the force is:

$$|F| = |qvB \sin \phi| \tag{6.1}$$

We have used absolute value signs, since the magnitude is always positive. The sign of q does not affect the *magnitude* of the force. It will, however affect the *direction* of the force. Note that the force is zero when the angle ϕ = zero or 180°, i.e. when the velocity and the magnetic field are along the same line. Also, the largest force occurs when $\sin\phi$ is ± 1, i.e. when the velocity is perpendicular to the magnetic field (see Fig. 6-2).

Problem 6.1.

(*a*) A charge of 2×10^{-6} C is moving with a velocity of 3×10^4 m/s at an angle of 30° with a magnetic field of 0.68 T. What is the magnitude of the force exerted on the charge?

(*b*) What is the magnitude of the force if the charge were -2×10^{-6} C?

(*c*) What is the magnitude of the force if the angle ϕ were 150°?

Solution

(*a*) Substituting $q = 2 \times 10^{-6}$ C, $v = 3 \times 10^4$ m/s, $\phi = 30°$ and $\mathbf{B} = 0.68$ T into Eq. (*6.1*), we get $|F| = (2 \times 10^{-6})(3 \times 10^4)(\sin 30°)(0.68) = 0.0204$ N.

(*b*) Since only the absolute value of each variable enters, the answer is the same as for part (*a*).

(*c*) Since $|\sin 150°| = \sin 30°$, the answer is still the same.

Problem 6.2. A charge of 3×10^{-5} C is at the origin in Fig. 6.3. There is a uniform magnetic field of 0.85 T pointing in the positive x direction. Calculate the magnitude of the force exerted on the charge if it is moving with a velocity of 2×10^5 in the direction (*a*) from A to B; (*b*) from A to E; (*c*) from D to A; (*d*) from A to F; and (*e*) from A to H.

Solution

In all five cases, $qvB = (3 \times 10^{-5})(2 \times 10^5)(0.85) = 5.1$ N. The difference between each case is the value of $\sin\phi$. Thus, the solution for each case is

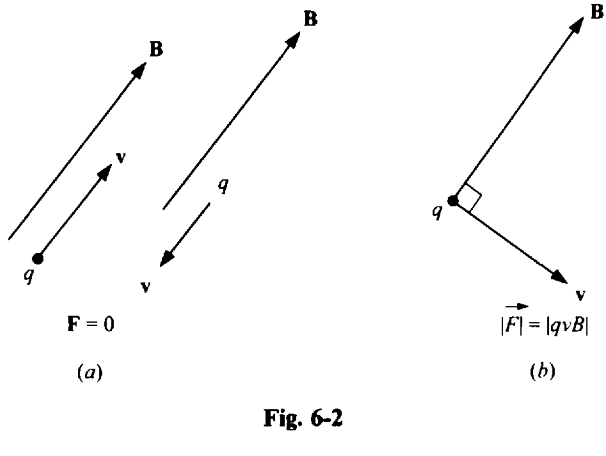


Fig. 6-2

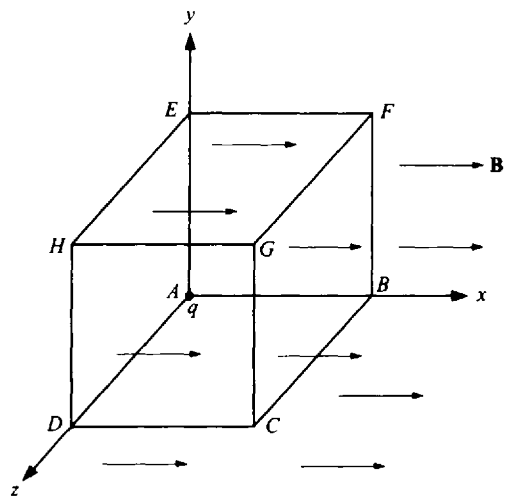


Fig. 6-3

(a) $\sin\phi = 0$ and therefore $|F| = 0$ N.

(b) $\sin\phi = 1$ and therefore $|F| = 5.1$ N.

(c) $\sin\phi = 1$ and therefore $|F| = 5.1$ N.

(d) ϕ is 45° so $\sin\phi = 0.707$ and therefore $|F| = 3.61$ N.

(e) ϕ is 90°, $\sin\phi = 1$ and therefore $|F| = 5.1$ N.

Direction of the Force

The direction of the force is perpendicular to both **v** and **B**, and it is therefore necessary to consider the problem in three dimensions. The solution is done in two steps. First one determines the line along which the force acts and then one determines the proper direction along that line.

The two vectors **v** and **B** can be considered as forming a plane. In Fig. 6-4, we draw the plane formed by a combination of vectors **v** and **B**. The direction that is perpendicular to this plane we call the normal to the plane. You can picture placing the palm of your right-hand in this plane containing

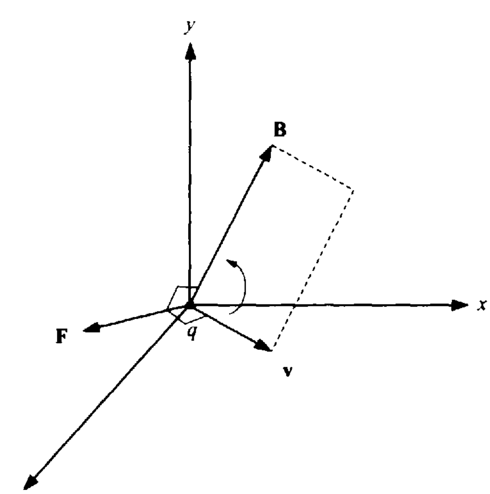


Fig. 6-4

both **v** and **B**. The normal is the direction perpendicular to your palm. On Fig. 6-4, we also draw this normal direction for this combination of **v** and **B**. This normal direction is the line along which the force vector lies. This direction is perpendicular to both **v** and **B**. We now have to choose between the two possible directions along this line. This is done by using the "**right-hand rule**". There are many different ways of applying a right-hand rule, and if you already know a particular method, you should continue to use that method. Here is one method that you can use. For the case of a positive charge, when the vectors **v** and **B** are tail to tail, curl the fingers of your right-hand in the direction from **v** to **B** with your thumb perpendicular to the other fingers. Your thumb then points in the direction of the force. If the charge is negative, then the direction is reversed.

Note. Both the magnitude and the direction of the magnetic force are completely analogous to the magnitude and direction of the vector torque discussed in Chap. 10, Section 10.3 when we replace **r** and **F** in that chapter by (q**v**) and **B**.

Problem 6.3. Determine the direction of the force in Problems 6.1(*a*), (*b*) and (*c*).

Solution

(*a*) The orientation of v and **B** is shown in Fig. 6-5 (*a*). Both **v** and **B** are in the plane of the paper. The perpendicular to the paper is the line going in and out of the paper. Curling the fingers of our right-hand from **v** to **B**, we see the perpendicular thumb points out of the paper. Thus the direction of the force is *out* of the paper. We use a dot, reminding us of the point of an arrow, to indicate that the force is out of the paper.

(*b*) The only change from (*a*) is that the sign of the charge is negative. Therefore, we reverse the direction of **F**, and it is now *into* the paper. We use a cross, reminding us of the cross hair at the back of an arrow, to indicate that the force is into the paper.

(*c*) Suppose that the directions of **v** and **B** are as shown in Fig. 6-5 (*b*). Rotating our fingers through the 150° angle from **v** to **B**, the thumb points into the paper. Thus, the direction of the force is *into* the paper.

Problem 6.4. Determine the direction of the force in Problem 6.2.

Solution

(*a*) Since the magnitude of the force is zero, there is obviously no direction needed.

(*b*) **v** is in the y direction, and **B** is in the x direction. The plane formed by these vectors is the x–y plane. The normal to this plane is the z direction (either $+z$ or $-z$). We use the right-hand rule to choose

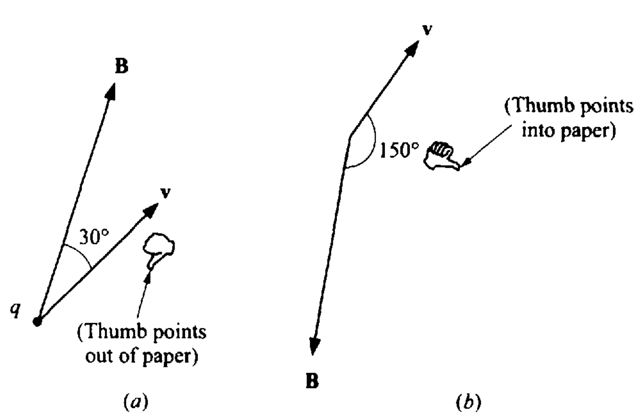


Fig. 6-5

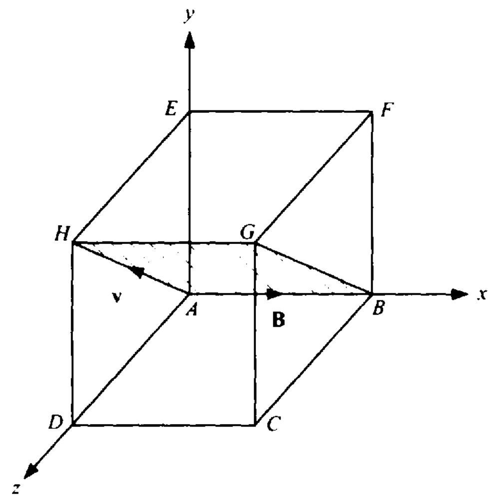


Fig. 6-6

between $+z$ and $-z$. Rotating from the positive y direction (the direction of **v**) to the positive x direction (the direction of **B**) our thumb points in the $-z$ direction. This is the direction of **F**.

(*c*) **v** is in the $-z$ direction, and **B** is in the x direction. The plane formed by these vectors is the x–z plane. The normal to this plane is the y direction (either $+y$ or $-y$). We use the right-hand rule to choose between $+y$ and $-y$. Rotating our fingers from the negative z direction (the direction of **v**) to the positive x direction (the direction of **B**) our hand now pushes in the $-y$ direction. This is the direction of **F**.

(*d*) **v** is in the x–y plane, at an angle of 45° with the positive x axis, and **B** is in the x direction. The plane formed by these vectors is the x–y plane. The normal to this plane is the z direction (either $+z$ or $-z$). We use the right-hand rule to choose between $+z$ and $-z$. Rotating from **v** to the positive x direction (the direction of **B**) our thumb points into the paper, which is the $-z$ direction. This is the direction of **F**.

(*e*) **v** is in the y–z plane, at an angle of 45° with the positive z axis, and **B** is in the x direction. The plane formed by these vectors is shown in Fig. 6-6 (plane *ABGH*). The normal to this plane is parallel to the direction *DE* (or *ED*). We use the right-hand rule to choose between *DE* and *ED*. Curling our fingers from the direction of **v** toward the positive x direction (the direction of **B**) our hand now pushes in the *DE* direction. This is the direction of **F**.

6.3 APPLICATIONS

If the magnetic force is the only force exerted on a moving charged particle, then the particle will move with constant speed. This is because the force is always perpendicular to the direction of the motion, and a force perpendicular to the velocity only changes the direction and not the magnitude of the motion. To change the magnitude of the velocity, one needs a force that is parallel to the velocity, which the magnetic force does not provide. The force, and thus the acceleration, is also perpendicular to **B**. Suppose that, in addition, the magnetic field is also perpendicular to the initial velocity. The entire motion (both **v** and **a**) are now in the plane perpendicular to **B**, with **v** and **a** each having constant magnitude. This is exactly what is needed to produce **circular motion** at constant speed. The centripetal force needed for the circular motion is supplied by the magnetic force. The magnitude of the magnetic force must equal the centripetal force required and we can therefore say that

$$qvB = mv^2/R \tag{6.2}$$

or

$$R = mv/qB \tag{6.3}$$

This is a formula for the radius of the circle traversed by the particle of mass, m, charge, q, moving with a velocity, v, in a perpendicular magnetic field, B. There are many applications of this relationship. The circular motion that a magnetic field can create is useful in many diverse areas. We will discuss some of these applications below.

Applications of Circular Motion

If one has a charged particle of unknown sign, one can use the circular motion created by a magnetic field to determine the sign of the charge. Suppose one has a charged particle moving upward, as in Fig. 6-7, in a magnetic field that is directed into the paper. The resultant circular path of the particle could be either path 1 or path 2, depending on whether the force is in the direction of $\mathbf{F}_1$ or $\mathbf{F}_2$. If the charge is positive, then the right-hand rule shows that the force is in the direction of $\mathbf{F}_1$ and the particle moves along path 1. On the other hand, if the charge is negative, the force is reversed and points in the direction of $\mathbf{F}_2$ causing the particle to move along path 2. Thus, by making the particle go in a circle in a perpendicular magnetic field, one can determine the sign of the charge.

Problem 6.5. A particle, with a charge equal to that of an electron is moving in a circle of radius 5 cm in a magnetic field of 0.2 T. What momentum does this particle have?

Solution

Using Eq. (*6.3*), $R = mv/qB$, one gets that the momentum, p, which is mv equals $p = mv = qBR$. Thus, $p = 1.60 \times 10^{-19}(0.2)(0.05) = 1.60 \times 10^{-21}$.

Problem 6.6. Two negatively charged particles, each with a charge equal to that of an electron are moving in a circle with the same velocity. The circular motion is due to a perpendicular magnetic field of 0.2 T. One of the particles is a charged atom of carbon with approximately 12 times the mass of a hydrogen atom, and the other is a charged atom of unknown mass. The radius of the circular path of the carbon atom is R_c, while that of the unknown atom is R_u.

(*a*) Show that one can get the unknown mass by measuring the ratio of the two radii.

(*b*) If $R_u = 1.33R_c$ what is the unknown atom?

Solution

(*a*) $R_u = m_u v/qB$, and $R_c = m_c v/qB$

Thus $R_u/R_c = m_u/m_c$

(*b*) $m_u = 1.33m_c = 16$ hydrogen masses, so the unknown atom is oxygen.

This problem illustrates the principle behind the operation of a mass spectrometer. In practice, a mass spectrometer typically does not have the particles moving with the same speed. Instead, each particle gains its speed by being accelerated from near rest through the same difference of potential, V. This is illustrated in Problem 6.7.

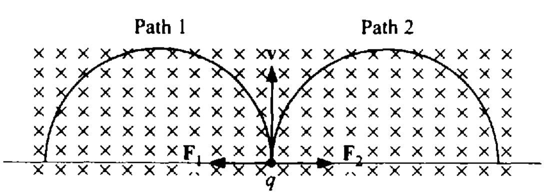


Fig. 6-7

Problem 6.7. The two charged particles in the previous problem are accelerated through a difference of potential, V, and then they travel in a perpendicular magnetic field, B.

(*a*) Derive an expression for the ratio of the two radii of their circular paths. Figure 6-8(*a*) illustrates the geometry.

(b) In this case, what would R_u/R_c be?

Solution

(*a*) At a the charges have no velocity. As they travel to point b, they lose potential energy equal to eV since b is at a higher potential and the charges are negative. This lost potential energy is converted to kinetic energy, so that at b the particles have a kinetic energy of

$$KE = (\tfrac{1}{2})mv^2 = eV, \quad \text{or} \quad v^2 = 2eV/m$$

$$v = \sqrt{2eV/m}$$

Each particle enters the magnetic field region with the velocity corresponding to its mass, and is then turned into a circular path with the appropriate radius. Thus, from Eq. (*6.3*) and with $q = e$

$$R^2 = m^2v^2/q^2B^2 = m^2(2eV/m)/e^2B^2 = 2V(m/e)/B^2$$

$$R^2B^2 = 2Vm/e \quad \text{and} \quad m = eR^2B^2/2V$$

The ratio of the masses is therefore

$$m_u/m_c = (R_u/R_c)^2.$$

(*b*) Again, by measuring the ratio of the radii, one can get the ratio of the masses. Since we are assuming that the unknown mass is oxygen, $R_u/R_c = \sqrt{4/3} = 1.15$.

There are many other uses to which one can put the ability of the magnetic field to produce circular motion. For instance, nearly all particle accelerators use magnetic fields to make particles return to an area where they are accelerated. There is often only one small region where particles are given an increase in speed, as a result of a parallel electric field, and the circular motion created by the magnetic

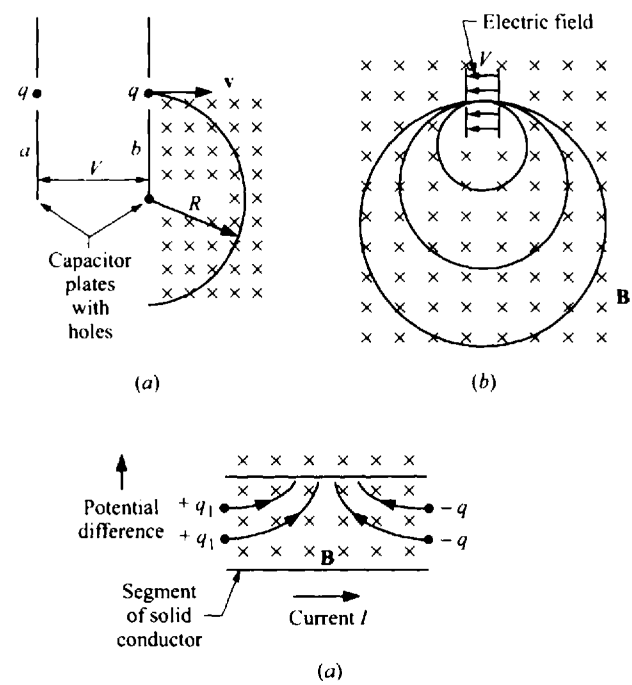


Fig. 6-8

field causes the particles to return to this region regularly and receive additional kinetic energy, as shown in Fig. 6-8(*b*).

Another application is in the "**Hall Effect**", which is used to determine the sign of the charges that produce currents in various solid conductors. Here, one deflects the moving charges in an electric current in the direction of the magnetic force. If a current is flowing to the right, this could be the result of positive charges moving to the right, or of negative charges moving to the left. For a perpendicular magnetic field, for instance a field going into the paper, both the positive and the negative charges are deflected upward, since a negative charge moving in a direction opposite to a positive charge has the same direction for the force. This is illustrated in Fig. 6-8(*c*). Thus, the magnetic field will deflect the conducting charges upward and one can detect the sign of the charge by determining whether positive or negative charge has gathered at the top. If positive charge is at the top, and negative at the bottom, then the top surface will be at a higher potential than the bottom, as we learned from a the case of a parallel plate capacitor. The opposite potential difference would result if negative charges gathered at the bottom. By this method, it has been determined that in some materials, called **semiconductors**, there are cases of positive as well as negative charge conductors.

Another interesting phenomenon occurs if the magnetic field is not perpendicular to the velocity vector. In that case one can resolve the velocity vector into one component which is parallel to the magnetic field and another component that is perpendicular to the magnetic field (see Fig. 6-9). The parallel component will be unaffected by the magnetic field since there is no force produced by B on a parallel velocity. The perpendicular component, however will be deflected into a circular path. circling the direction of the magnetic field. The resultant motion will be a spiral around the field direction (see the figure). There are many cases in which this actually happens, such as when particles in the "solar wind" meet the magnetic field in the earth's atmosphere.

Velocity Selector

In the previous problems there were cases of only a magnetic force, and some cases of both electrical and magnetic forces acting on the particles. But in the region in which the electrical force was active (when the particles were accelerated by the difference of potential), there was no magnetic force, and in the region of the magnetic force (the circular motion) there was no electrical force. By using a combination of both electric and magnetic fields, we can produce a mechanism to separate out particles of a particular velocity. This is known as a **velocity selector**, and is shown in Fig. 6-10, and described in the following problem.

Problem 6.8. The particle, of charge q, is moving with velocity, $\mathbf{v}$, in a region between two charged parallel plates a distance d apart, that produce a uniform electric field, $\mathbf{E}$. A uniform magnetic field, $\mathbf{B}$, pointing into the paper, also exists in this region.

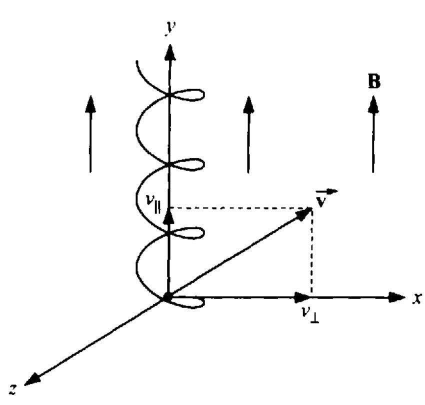


Fig. 6-9

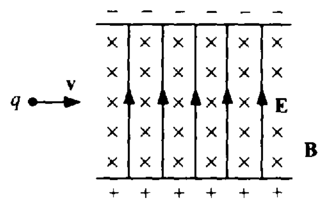


Fig. 6-10

(*a*) For what velocity, **v**, will the electrical and magnetic forces be equal and opposite, thus canceling each other's effect?

(*b*) Find the velocity when $V = 100$ volts, $d = 2.0$ cm and $B = 0.5$ T.

Solution

(*a*) The electrical force, $\mathbf{F}_E$, will equal $q\mathbf{E}$ and point in the direction of **E** (for a positive charge). The magnetic force, $\mathbf{F}_B$, will equal qvB, and point in the direction opposite to $\mathbf{F}_E$. Thus, the two will cancel if their magnitudes are equal. This is also true for a negative charge, since the direction of both forces changes. Therefore there will be no net force if $qE = qvB$, or $E = vB$.

Note. For a velocity of $v = E/B$ there is no force to deflect the particle, and it will travel in a straight line. If the particle has a bigger velocity than this, then the magnetic force will exceed the electrical force, and the particle will be deflected in the direction of the magnetic force. Similarly, if the velocity is smaller than this velocity, then the magnetic force will be less than the electrical force and the particle will be deflected in the direction of the electrical force. Thus only particles with this particular velocity will be undeflected, and they can be easily selected out from the rest. We can choose the velocity we want by varying E, simply by changing the potential difference across the two plates, which is producing the electric field.

(*b*) Remembering that the electric field produced by two parallel plates is V/d, where d is the distance between the plates, we have

$$v = E/B = V/dB = 100 \text{ V}/(2 \times 10^{-2} \text{ m})(0.5 \text{ T}) = 10^4 \text{ m/s}.$$

6.4 MAGNETIC FORCE ON A CURRENT IN A WIRE

Whenever current flows in a wire, one has charge that is moving. If a segment of the wire is in a magnetic field, then the magnetic field will exert a force on that segment of the wire. To obtain this force one has to determine how to adjust the formula for a single moving charge to accommodate a current in a wire. The answer to this is that all one has to do is to substitute IL for qv in the equation for the force. Here, I is the current flowing in the wire, and **L** is a vector whose magnitude is the length of the segment of the wire, and the direction of **L** is the direction of the current. We can see this intuitively by noting that in a small time Δt the amount of charge passing a point in the wire is $q = I\Delta t$. If this charge moves with an average velocity v, it will cover a distance $L = v\Delta t$. Thus $q/I = L/v$ or $qv = IL$. Therefore, the magnitude of the force on a segment is given by (see Fig. 6-11)

$$|F| = |ILB \sin \phi| \tag{6.4}$$

The direction of the force is calculated by the same procedure that was used for a single charge. The force is perpendicular to both **L** and **B**, and therefore normal to the plane containing those vectors. We use the right-hand rule to choose the correct direction along this normal, where the direction of the current replaces the direction of **v**.

The force calculated in this manner is the force on that segment of wire, of length L, carrying the current I. Each segment of the wire is affected separately by the magnetic field, and we can separately

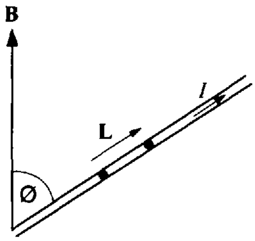


Fig. 6-11

calculate the magnitude and direction of the force on each segment. To get the total force on the wire we would then add together, vectorially, the forces on each segment. This is illustrated by the next problem.

Problem 6.9. Consider a wire *abcd*, in the shape shown in Fig. 6-12 which is in a magnetic field **B** pointing out of the paper. The current is 1.5 A, the magnetic field is 0.3 T and the lengths are $L_1 = 0.5$ m and $L_2 = L_3 = 0.8$ m.

(*a*) Calculate the force (magnitude and direction) acting on segment *ab*.

(*b*) Calculate the force (magnitude and direction) acting on segment *bc*.

(*c*) Calculate the force (magnitude and direction) acting on segment *cd*.

(*d*) Calculate the force (magnitude and direction) acting on the wire *abcd*.

Solution

(*a*) Using Eq. (6.4), the magnitude of the force is $|F_1| = IL_1B$ since $\phi = 90°$. The direction is perpendicular to $\mathbf{L}_1$ (to $+y$) and to **B** (out of the paper) and thus in the x direction, either $+x$ or $-x$. Using our right-hand rule, (we rotate our fingers from $\mathbf{L}_1$ to **B**) and our thumb then points in the $+x$ direction, which is therefore the direction of $\mathbf{F}_1$. For the magnitude we get $F_1 = (1.5\text{ A})(0.5\text{ m})(0.3\text{ T}) = 0.225$ N.

(*b*) Applying the same formula to segment *bc*, we get the magnitude of the force to be $|F_2| = IL_2B$. The direction of the force is perpendicular to $\mathbf{L}_2$ (to $+x$) and to **B**, and thus in the y direction. To choose between $\pm y$, we use our right-hand rule and find that $\mathbf{F}_2$ is in the $-y$ direction. For the magnitude we get $F_2 = (1.5\text{ A})(0.5\text{ m})(0.3\text{ T}) = 0.360$ N.

(*c*) Applying the same formula to segment *cd*, we get the magnitude of the force to be $|F_3| = IL_3B$. The direction of the force is perpendicular to $\mathbf{L}_3$ (to $-y$) and to **B**, and thus in the x direction. To choose between $\pm x$, we use our right-hand rule and find that $\mathbf{F}_3$ is in the $-x$ direction. The magnitude of F_3 is the same as F_1.

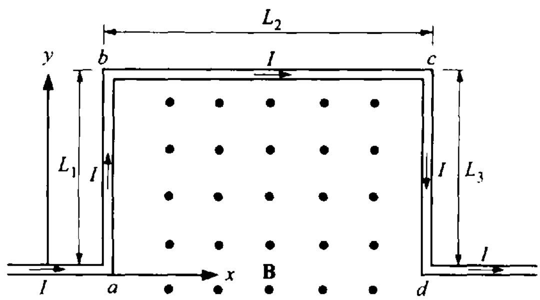


Fig. 6-12

(*d*) The force on *abcd* is the vector sum of the forces on the three segments. Since F_1 and F_3 are in opposite directions and of equal magnitude, they cancel each other when added together. Thus

$$\mathbf{F}_{\text{total}} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = \mathbf{F}_2 = 0.3 \text{ N in the } -y \text{ direction.}$$

Problem 6.10. Consider a cube, with a side of 0.5 m, as shown in Fig. 6-13. A current of 2 A is flowing along the x direction. Calculate the force (magnitude and direction) on the segment *ab* when the magnetic field of 0.3 T points in (*a*) the x direction; (*b*) the $-y$ direction. (*c*) the direction from *a* to *h*; and (*d*) the direction from *a* to *c*.

Solution

(*a*) Using Eq. (6.4), the magnitude of the force is $|F_1| = 0$ since $\phi = 0$.

(*b*) Now the angle $\phi = 90°$, and therefore $|F| = ILB = 2(0.5)(0.3) = 0.3$ N. To get the direction, we note that L and **B** are in the x–y plane and the normal to that plane is the z direction. To choose between $\pm z$, we apply the right-hand rule, curling our fingers from L to **B**. Our perpendicular thumb then faces the $-z$ direction, which is the direction of **F**.

(*c*) The angle between **L** and **B** is still 90° (not 45°) since *ah* is in the y–z plane, which is perpendicular to the direction of **L** (the x direction). Therefore $|\mathbf{F}| = 0.3$ N. Getting the direction is somewhat harder. The plane of **L** and **B** is now *abgh* whose normal is along the diagonal *ed* (or *de*). Using the right-hand rule the thumb faces the direction *ed*, which is the direction of **F**.

(*d*) The angle between **L** (*ab*) and **B** (*ac*) is now 45°. Thus $|F| = 2(0.5)(0.3) \sin 45 = 0.212$ N. The plane of **L** and **B** is now the x–z plane whose perpendicular is the y direction. The right-hand rule selects between $\pm y$. Rotating our fingers from **L** to **B**, our thumb faces in the $-y$ direction which is therefore the direction of **F**.

Problem 6.11. Consider an equilateral triangle, with a side of 0.5 m, as shown in Fig. 6-14. A current of 2 A is flowing around the triangle in the direction shown. Calculate the force (magnitude and direction) on each segment of the triangle, and on the whole triangle when a magnetic field of 0.3 T points in the direction *ab*.

Solution

Along *ab*, the force is zero, since *ab* is along the direction of **B**.

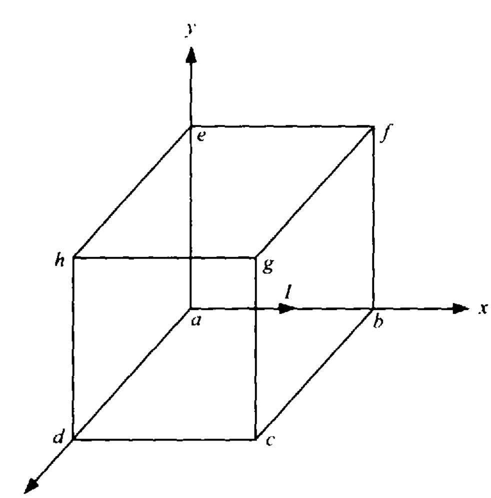


Fig. 6-13

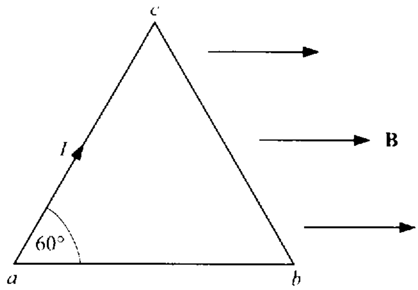


Fig. 6-14

Along *ac*, the force is $|F| = 0.5\ (2)\ (0.3)\ \sin 60^\circ = 0.26$ N. The plane containing **L** and **B** is the plane of the paper whose normal is in or out of the paper. The right-hand rule gives the correct direction as into the paper.

Again, along *cb*, the force is $|F| = 0.5\ (2)\ (0.3)\ \sin 60^\circ = 0.26$ N. The plane containing **L** and **B** is the plane of the paper whose normal is in or out of the paper. The right-hand rule gives the correct direction as out of the paper.

Adding these forces vectorially gives $\mathbf{F} = 0$.

Problem 6.12. Consider a circular metal disc, which is free to rotate about its center. The bottom of the disc is in contact with a pool of liquid mercury as seen in Fig. 6-15. A battery is connected between the center of the disc and the pool of mercury, so that a current flows vertically downward from the center to the mercury. If a magnetic field is established in the direction out of the paper, what motion, if any, will the disc undergo?

Solution

Because we have current flowing downward in the bottom of the disc, we have a case of current in a magnetic field. The magnetic force will be perpendicular to **L** (which is downward) and to **B** (which is out of the paper) and will therefore point in the horizontal direction. The right-hand rule determines that the direction is to the left. Since the disc is not free to move from its position (it is only free to rotate), the only possible motion is a rotation about its center. On the lower half of the disc there is a force to the left, and there is no force on the top of the disc. The disc will therefore rotate in the clockwise direction. This is an example of a very crude **electromagnetic motor**.

6.5 MAGNETIC TORQUE ON A CURRENT IN A LOOP

We have seen in Problem 6.11, that the magnetic field did not exert a net force on a triangle in which current was flowing around the perimeter. This is an example of current flowing around a closed loop, where the net magnetic force will always be zero. However, even with no net force it is possible

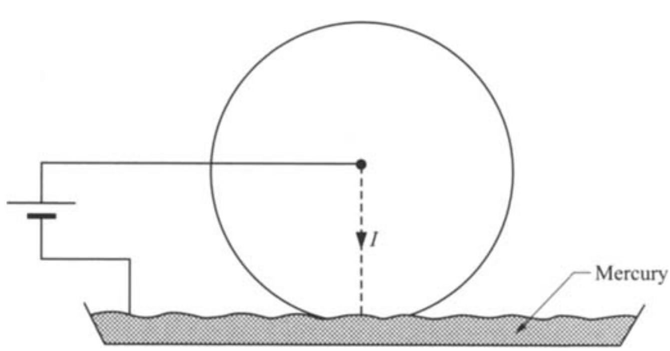


Fig. 6-15

that there will be a tendency for the coil to rotate, and this tendency is determined by the torque (or moment) which these forces exert on the coil. Let us examine the case of a rectangular coil, *PQRS*, whose sides are of lengths *a* and *b*, and which carries a current *I*. This coil is in a magnetic field which is constant throughout the area of the coil. In Fig. 6-16(*a*) the coil is pictured in three dimensions, while Fig. 6-16(*b*) shows the same coil projected on the *x*–*y* plane. The magnetic field is in the *x* direction.

Problem 6.13. Find an expression for the torque, Γ, on coil *PGRS* in Fig. 6-16. A current, *I*, flows, as shown. Discuss the rotation of the coil.

Solution

We will first calculate the forces acting on each of the four sides of the coil, in order to determine the net force (which we know should be zero) and the torque which may be exerted. On each of sides *PS* and *QR*, the magnitude of the force is $IbB \sin \phi$ [Fig. 6-16(*b*)]. The direction of the two forces are opposite to each other, since the current flows in the opposite direction for the two sides. For one of the sides the force is in the $+z$ direction [out of the paper in Fig. 6-16(*b*)], and for the other side the force is in the opposite, or $-z$ direction. The line of action of these two forces is clearly the same and they exert no net torque on the coil. Thus any net torque will have to come from sides *PQ* and *SR*. The force exerted on each of these sides is $IbB \sin 90°$ and the directions are opposite to each other. The net force will therefore be zero, as we expected. However, the line of action of these two forces will not be identical and there will usually be a net torque. Let us calculate the direction of the forces and their line of action, and from this information we will then be able to calculate the torque.

On side *PQ* the force will be perpendicular to *PQ* (the *z* direction) and to **B** (the *x* direction). The force is therefore in the $\pm y$ direction. Since the current in the coil is flowing in the direction *PQRS*, then using the right-hand rule gives a force in the $-y$ direction. For this same current direction, the force on side *SR* will be in the $+y$ direction. This is depicted in Fig. 6-17. The line of action for the force on *PQ* is the *y* axis, while the line of action for the force on side *SR* is the line parallel to the *y* axis, but at a distance of $a \sin \phi$ from that axis. If we take the torque about the origin, only the force on *SR* will contribute and the torque will be $\Gamma = F\, a \sin \phi = IbB\, a \sin \phi = IAB \sin \phi$, where $A = ab =$ the area of the coil. (Actually, the torque will be the same about any axis because the two forces producing the torque are equal in magnitude and opposite in direction, thus forming a couple; see Ibid., 9.2, p. 234). This torque will try to rotate the coil about the *z* axis in the counter-clockwise direction, until the plane of the coil is parallel to the *y* axis. At this point, the angle ϕ is zero, and the torque is zero. Thus the coil will try to line up with its plane perpendicular to the magnetic field. If the current in the coil had been in the opposite direction, from *Q* to *P*, then the direction of all the forces would have been reversed, and the torque would have been in the direction to rotate the coil clockwise around the *y* axis. Again, when the plane of the coil is perpendicular to the magnetic field the torque will be zero.

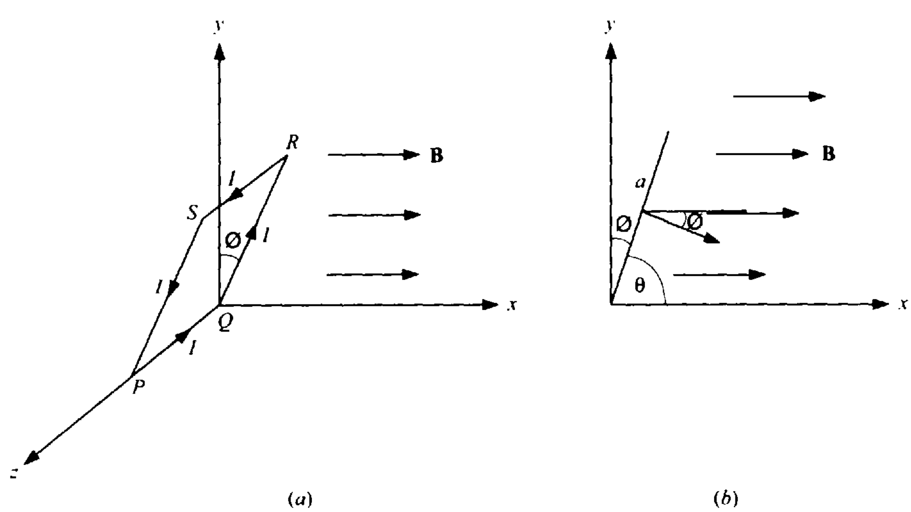


Fig. 6-16

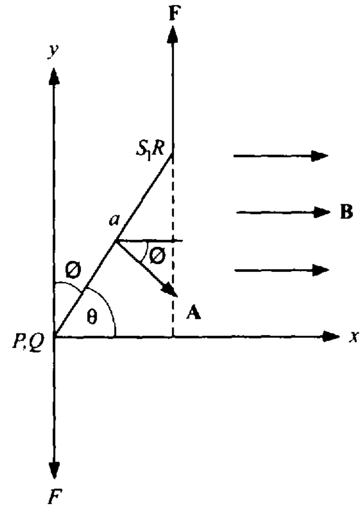


Fig. 6-17

It is useful to define a vector area for the coil, **A**, whose magnitude is $A = ab$, and whose direction is perpendicular to the plane of the coil. The $\pm$direction of **A** is determined by the right-hand rule. Curl the fingers of your right-hand around the coil in the direction of the current. Your thumb then points in the positive **A** direction. Thus ϕ is the angle between **A** and **B**, as can be seen in Fig. 6-17. We see that in general the torque is given by

$$\Gamma = IAB \sin \phi \tag{6.5}$$

where Γ tends to rotate the coil in the same direction as rotating the vector **A** through ϕ to **B**. When **A** is parallel to **B**, $\phi = 0$ and the torque is zero.

We define a new vector, **M**, the magnetic dipole moment of the coil, whose magnitude is IA and whose direction is the same as **A** (with the convention we defined earlier). If the coil consists of several turns, then each turn has a magnetic moment IA, and the entire coil has a magnetic moment NIA, where N is the number of turns in the coil. The torque will turn the coil in the direction of making **M** point in the direction of **B**.

For the case shown in Fig. 6-16, where the current flows from P to Q, the torque will rotate the coil in the counter-clockwise direction, until the plane of the coil is parallel to the y axis. At that point, the torque is zero. If the coil rotates past the y axis, then the torque will again try to align **M** with **B**, and the rotation will now be clockwise. Thus the torque will always be rotating the coil back to the equilibrium position, and we see that the coil is in *stable* equilibrium, when **M** is parallel to **B**. If the current were in the opposite direction, then **M** would point in the opposite direction (see Fig. 6-18). If the coil then starts in position (1) in the figure the torque would be clockwise, trying to rotate the coil further away from the y axis. If the coil starts on the other side of the y axis (position 2 in the figure), the torque would be counter-clockwise, again rotating the coil away from the y axis. Of course, when the coil is precisely lined up with the y axis ($\phi = 180°$), the torque is zero, and the coil is in equilibrium, but the equilibrium is unstable because any move away from the y axis will result in the coil continuing to rotate even more, rather than returning to the equilibrium position. After the coil has rotated 180°, the coil will have its vector **M** pointing in the direction of **B**, and the coil will be in stable equilibrium.

Although this result was derived for the special case of a rectangle, the result is valid for any coil shape, with the moment of the coil equaling $M = NIA$, and the torque on the coil equaling $MB \sin \phi$, with the usual counter-clockwise, clockwise conventions.

It should be clear that this phenomenon of a torque on a coil can be used to build a motor, which will continuously rotate in the magnetic field. Such motors are built by constructing a coil from many

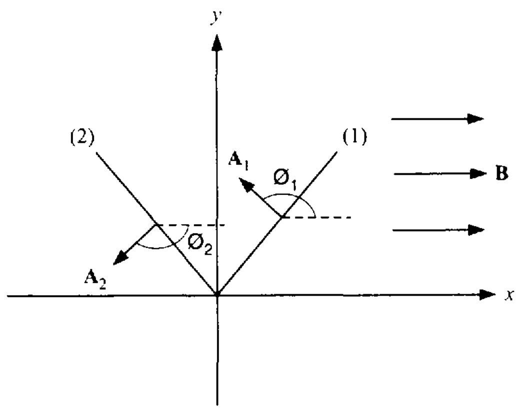


Fig. 6-18

turns (to increase **M** and thereby, the torque), and suspending the coil on an axis in a constant magnetic field. The direction of the current in the coil is chosen to make the coil rotate in one particular direction, for instance clockwise. When the coil passes the y axis the direction of the torque would normally reverse, making the coil turn counter-clockwise. In order to prevent this from happening, we arrange to have the current direction reverse as the coil passes through the y axis, thus maintaining a clockwise torque. This is accomplished by the split in the rings where the current enters from the source of EMF (see Fig. 6-19).

Problem 6.14. Consider a circular ring carrying a current of 2 A. The plane of the ring is at an angle of 60° to the yz plane, as shown in Fig. 6-20. The ring has a radius of 1.5 m, and is in a uniform magnetic field of 0.3 T pointing in the positive x direction. What torque is exerted on the coil?

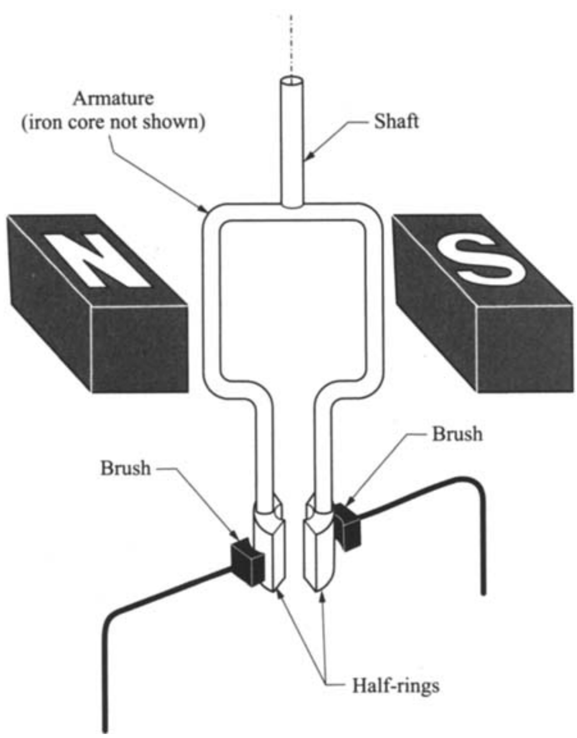


Fig. 6-19

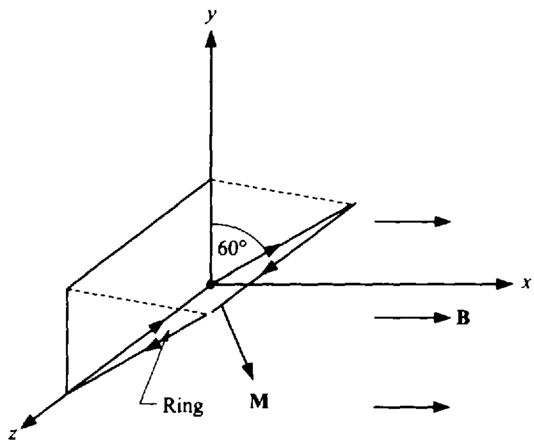


Fig. 6-20

Solution

The magnitude of the torque is $IAB \sin \phi = 1.5\ (\pi r^2)\ (0.3)\ \sin 60°$ where 60° is the angle between **M** and **B**. Thus the torque will equal 2.75 N · m. For the direction of current shown in the figure, the vector **M** points below the xz plane. Since the torque tries to align **M** with **B**, it will try to rotate the plane upward toward the yz plane.

Another example of a torque exerted by a magnetic field is a charged particle which is spinning. Consider the case of a charged sphere spinning on its axis (see Fig. 6-21). Every part of the sphere is moving in a circle about the axis, and we therefore have charges going in concentric circles which make a current. This is like many different coils, all with planes perpendicular to the axis of rotation, or with area vectors parallel to the axis. For positive charge, the current is in the same direction as the velocity of the charge, and for the rotation in the figure, the area vector is vertically upward. For a negative charge, the current is opposite to the velocity, and the area vector would be vertically downward. The magnetic moment vector is in the same direction as the area vector, upward for positive charge and

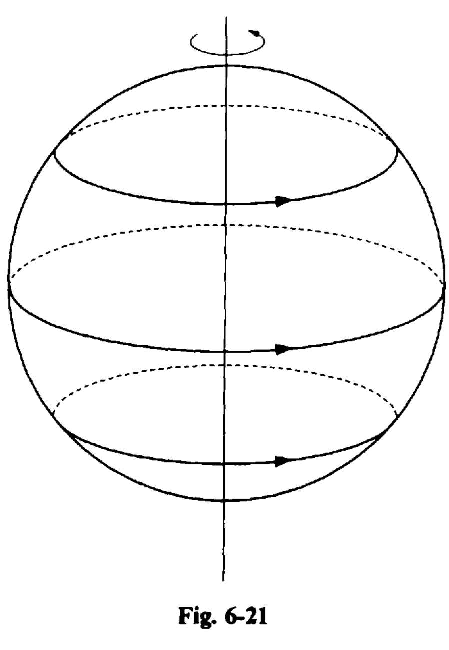


Fig. 6-21

downward for negative charge. In a magnetic field there will be a torque on this spinning sphere which tries to align the moment vector in the direction of **B**. Since spinning particles are like tiny spinning spheres, a charged spinning particle tries to rotate so that its spin axis is along the magnetic field. The direction of rotation about this magnetic field will be interchanged for positive and for negative particles. This is part of the meaning when one talks about electrons with spin "up" or spin "down". In both cases the spin axis aligns with a magnetic field, either parallel or anti parallel to **B**.

The last example of magnetic moments in a magnetic field that we will discuss is that of a compass needle in a magnetic field. We will find out later that a bar magnet consists of many charged particles producing a circulating current about the axis of the magnet. The particles produce this current either because they are spinning in unison about parallel axes to the axis of the magnet, or because they are moving in orbit-like paths circulating the magnet axis. Thus a magnet is actually similar to a coil with a magnetic moment parallel to its axis. Magnets are often described by "poles" at each end. The direction of the moment of the magnet is from the south to the north pole of the magnet. The north pole is the end that tries to align itself facing north, when the magnet is free to rotate about a vertical axis through its center. Such a magnet is called a **compass**. This alignment is due to the magnet being affected by the Earth's magnetic field. In any magnetic field, such as the intrinsic field of the earth, the magnet rotates, with the north pole of the magnet lining up parallel to the magnetic field. Since the Earth's magnetic field points approximately due north, this use of the magnetic needle (compass) has been an ancient method of determining the northerly direction.

Problems for Review and Mind Stretching

Problem 6.15. An electron is in an upward, vertical magnetic field of 0.8 T. What horizontal velocity must the electron have (magnitude and direction) for the magnetic force to be 1.6×10^{-13} N to the east?

Solution

The magnitude of the force is $|F| = |qvB \sin \phi|$. Since **B** is vertical and **v** is horizontal, $\phi = 90^\circ$ and $\sin \phi = 1$. Thus,

$$1.6 \times 10^{-13} = (1.6 \times 10^{-19})v(0.8)(1) \qquad (i)$$

Therefore,

$$v = 1.25 \times 10^{6} \text{ m/s} \qquad (ii)$$

Since **F** is perpendicular to **v**, and they both are horizontal, **v** must be in the north–south direction. Suppose **v** is north. Then rotating **v** upward toward **B** would give east as the direction of **F** for a positive charge (right-hand rule). However, an electron has a negative charge, so the force on an electron moving north is to the west, which is not the direction we seek. Thus, the velocity must be south.

Problem 6.16. A particle with a charge of 2×10^{-9} C is moving horizontally toward the east at point a, as shown in Fig. 6-22. The particle has a mass of 5×10^{-15} kg and is moving with a velocity of 4×10^{4} m/s. We want to make this charge move in a circle through point b, which is 1.0 m south of a. What magnetic field (magnitude and direction) is required?

Solution

Since the circle is in the horizontal plane, the magnetic field must be vertical (either in or out of the paper). At point a, the (positively charged) particle is moving to the east, and the centripetal force needed to make the particle move in the desired circle must be to the south. Using the right-hand rule, we find that if the field is out of the paper, the force would be to the south, while if the field is into the paper the force is to the north. Thus, the direction of the field must be out of the paper.

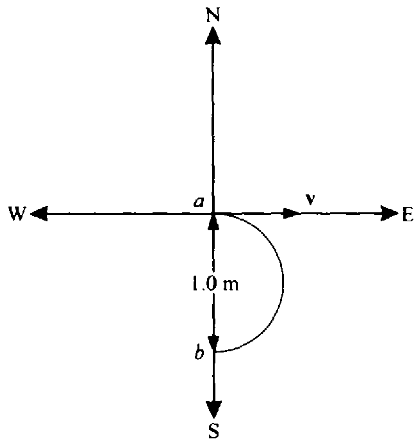


Fig. 6-22

The radius of the circle must be half of the distance between a and b, or

$$R = 0.5 \text{ m} \qquad (i)$$

Using $$R = mv/qB \qquad (ii)$$

$$\text{B} = mv/qR = (5 \times 10^{-15})(4 \times 10^4)/(2 \times 10^{-9})(0.5) = 0.2 \text{ T} \qquad (iii)$$

Problem 6.17. A metal rod of length 0.025 m, is free to roll along a railing as in Fig. 6-23. There is a uniform magnetic field of 0.03 T in the entire region, pointing into the plane of the railing, as shown. A current of 20 A is flowing through the railing and rod, as a result of the battery and resistor shown in the figure.

(*a*) What force (magnitude and direction) is exerted on the rod?

(*b*) If the polarity of the battery is changed, what change, if any, occurs to the force?

Solution

(*a*) The current is flowing from a to b in the rod. The magnitude of the force is:

$$|F| = ILB \sin \phi = (20)(0.025)(0.3) \sin 90° = 0.15 \text{ N} \qquad (i)$$

The direction of the force is perpendicular to L (which points from a to b) and to **B** (which points into the paper), and is therefore along the direction of the railing. The right-hand rule (curl the fingers of the right-hand from **L** to **B** and then **F** is in the direction of the thumb) gives the direction of **F** to the right.

(*b*) Changing the polarity of the battery reverses the direction of the current (and therefore **L**). The force is then reversed and is to the left.

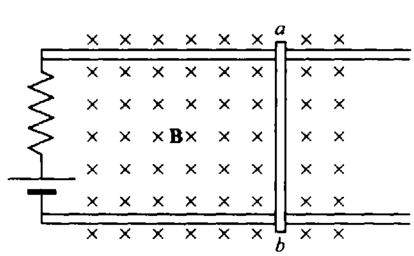


Fig. 6-23

Problem 6.18. A coil with 100 turns is free to rotate about an axis AA' as in Fig. 6-24. There is a magnetic field of 0.05 T in the plane of the coil, as shown in the figure. The coil has an area of 0.07 m^2, and a current, I, of 2×10^{-3} A is made to flow in the coil in the direction shown.

(*a*) Calculate the torque produced by the magnetic field on the coil while in this position.

(*b*) Calculate the torque produced by the magnetic field on the coil if the coil rotates through an angle θ from this initial position.

Solution

(*a*) The torque produced by the magnetic field is given by

$$|\Gamma| = MB \sin \phi, \text{ where } \phi \text{ is the angle between } \mathbf{M} \text{ and } \mathbf{B} \quad (i)$$

$$M = NIA = (100)(2 \times 10^{-3})(0.07) = 0.014 \quad (ii)$$

and the direction is perpendicular to the plane of the coil. Using our convention (curl the fingers of the right-hand in the direction of I, and the thumb points in the direction of **M**), we get that **M** is into the paper. Thus

$$\Gamma_B = (0.014)(0.05)(1) = 7 \times 10^{-4} \text{ N} \cdot \text{m} \quad (iii)$$

The torque tends to rotate the coil in the direction you get by rotating **M** to **B**.

(*b*) If the plane of the coil rotates by θ then the angle ϕ becomes $(90 \pm \theta)$. The torque, Γ_B' is now

$$\Gamma_B' = (0.014)(0.05) \sin (90 \pm \theta) = 7 \times 10^{-4} \cos \theta \quad (iv)$$

Problem 6.19. In Problem (6.18) a wire along axis AA' can produce a restoring torque on the coil, given by $|\Gamma_R| = 7 \times 10^{-4}\, \theta$, where θ is the angle, measured in radians, through which the coil rotates from the original position. The coil will be in equilibrium if the restoring torque is equal to the magnetic torque in magnitude and opposite in direction.

(*a*) Calculate the restoring torque produced by the wire on the coil at angles of 5°, 10° 15°, 20°, 25°, and 30°.

(*b*) Calculate the current needed in the wire so that the magnetic torque equals, in magnitude, the restoring torque at each of those angles.

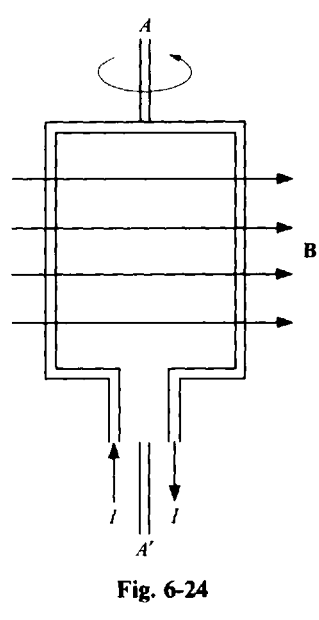


Fig. 6-24

(c) Plot a graph of the angle of equilibrium vs. current, using the data calculated in (a) and (b).

Solution

(a) $$|\Gamma_R| = 7 \times 10^{-4}\ \theta, \text{ where the angle } \theta \text{ is in radians} \qquad (i)$$

Thus:

θ (degree)	θ (radians)	Γ_R
5	0.087	0.61×10^{-4}
10	0.175	1.22×10^{-4}
15	0.262	1.83×10^{-4}
20	0.349	2.44×10^{-4}
25	0.436	3.05×10^{-4}
30	0.524	3.67×10^{-4}

(b) From the previous problem,

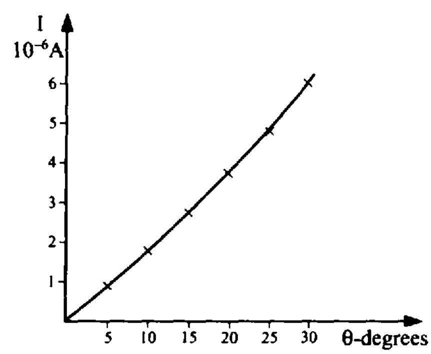
$$|\Gamma_B| = MB \sin \phi = NIA \sin (90° \pm \theta) = 7I \cos \theta \qquad (ii)$$

If $$|\Gamma_B| = |\Gamma_R|,\ 7I \cos \theta = 7 \times 10^{-4}\ \theta \qquad (iii)$$

$$I = 10^{-5}\ \theta/\cos \theta \qquad (iv)$$

Thus

θ (degree)	θ (radians)	I
5	0.087	0.87×10^{-6}
10	0.175	1.78×10^{-6}
15	0.262	2.71×10^{-6}
20	0.349	3.71×10^{-6}
25	0.436	4.81×10^{-6}
30	0.524	6.05×10^{-6}



Supplementary Problems

Problem 6.20. A magnetic field of 0.3 T is in the x-direction. Calculate the force (magnitude and direction) on a charge of 3×10^{-5} C moving with a velocity of 3×10^{6} m/s in the direction shown in Fig. 6-25.

Ans. (a) 27 N, into paper; (b) 0; (c) 13.5 N, into paper; (d) 13.5 N, into paper

Problem 6.21. A magnetic field of 0.3 T is out of the paper. Calculate the force (magnitude and direction) on a charge of 3×10^{-5} C moving with a velocity of 3×10^6 m/s in the direction shown in Fig. 6-25.

Ans. (*a*) 27 N, in $+x$ direction; (*b*) 27 N, in $-y$ direction; (*c*) 27 N, 60° below $+x$ axis; (*d*) 27 N, 60° above $+x$ axis

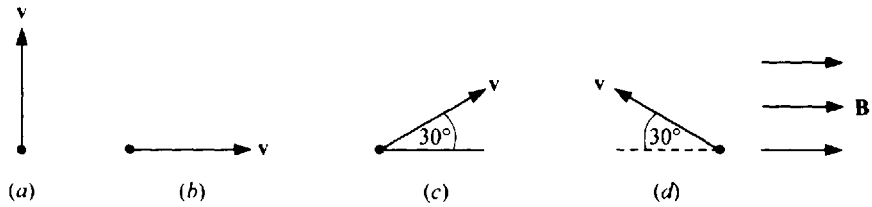


Fig. 6-25

Problem 6.22. A particle with charge -1.6×10^{-19} C is moving horizontally in the air above the earth with a speed of 7×10^4 m/s. It has a mass of 1.67×10^{-27} kg. What magnetic field (magnitude and direction) is needed so that the magnetic force cancels out the gravitational force of the earth?

Ans. 1.5×10^{-12} T, horizontal and perpendicular to **v**. (This illustrates that magnetic forces, for normal values of B, are very much larger than gravitational forces.)

Problem 6.23. A vertical magnetic field of 0.3 T causes a charged particle, moving with a velocity of 3×10^5 m/s, to move in a circle of radius 0.01 m. What is the charge to mass ratio (q/m) of this particle?

Ans. 10^8 C/kg

Problem 6.24. A magnetic field, coming out of the paper, causes a charged particle to move around a circle in a clockwise direction. Is the particle positively or negatively charged?

Ans. positively charged

Problem 6.25. A magnetic field, of 0.6 T, going into the paper, causes a charged particle to move in a horizontal circle. The particle has mass 1.67×10^{-27} kg and charge 1.6×10^{-19} C.

(*a*) How long does it take for the particle to go once around the circle?

(*b*) How many times per second does the particle go around the circle?

Ans. (*a*) 1.33×10^{-7} s; (*b*) 7.4×10^6 Hz

Problem 6.26. Two isotopes have masses of 9.87×10^{-26} kg and 9.97×10^{-26} kg, and each have a charge of 1.6×10^{-19} C. They each move in a circle with a velocity of 5×10^6 m/s in a magnetic field of 0.5 T. After going through a semi-circle, they strike a screen perpendicular to their velocity. How far apart are they on this screen?

Ans. 0.125 m

Problem 6.27. Particles move through a velocity selector, in which the fields are $E = 10^4$ V/m and $B = 0.5$ T. Outside of the velocity selector, there is only the magnetic field of 0.5 T. The particles have a charge of 1.6×10^{-19} C, and a mass of 6.7×10^{-27} kg.

(*a*) What is the velocity of those particles that leave the velocity selector?

(*b*) What is the radius of the circle in which the particles move after they leave the velocity selector?

(*c*) If the electric field is doubled, what is the radius of the circle in which the particles now move?

Ans. (*a*) 2×10^4 m/s; (*b*) 1.68×10^{-3} m; (*c*) 3.35×10^{-3} m

Problem 6.28. One wants to build a velocity selector to select particles with a speed of 6×10^6 m/s. A magnetic field of 0.3 T is available.

(*a*) What electric field is needed?

(*b*) If the electric field is produced by parallel plates, spaced 2 mm apart, what voltage must be applied?

(*c*) If one wants to select particles with half of this velocity, what voltage is needed?

Ans. (*a*) 1.8×10^6 V/m; (*b*) 3.6×10^3 V; (*c*) 1.8×10^3 V

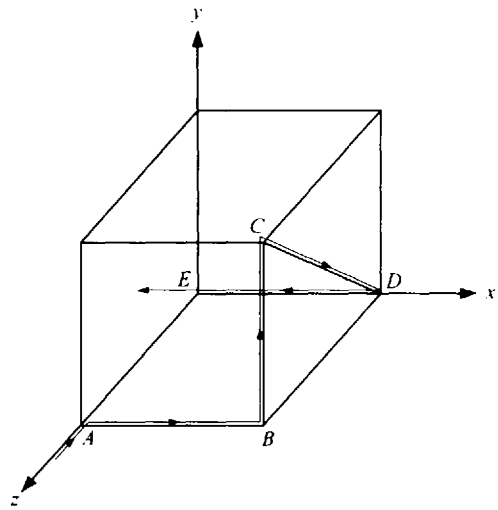


Fig. 6-26

Problem 6.29. Consider a cube, with a side of 0.5 m, as shown in Fig. 6-26. A magnetic field of 0.3 T points in the y direction. A current of 2 A flows along the direction $ABCDE$. Calculate the force (magnitude and direction) on (*a*) segment AB; (*b*) segment BC; (*c*) segment CD; (*d*) segment DE; and (*e*) the entire path $ABCDE$.

Ans. (*a*) 0.3 N, $+z$ direction; (*b*) 0; (*c*) 0.3 N, $+x$ direction; (*d*) 0.3 N, $-z$ direction; (*e*) 0.3 N, $+x$ direction

Problem 6.30. Consider a cube, with a side of 0.5 m, as shown in Fig. 6-26. A magnetic field of 0.3 T points in the $-x$ direction. A current of 2 A flows along the direction $ABCDE$. Calculate the force (magnitude and direction) on (*a*) segment AB; (*b*) segment BC; (*c*) segment CD; (*d*) segment DE; and (*e*) the entire path $ABCDE$.

Ans. (*a*) 0; (*b*) 0.3 N, $+z$ direction; (*c*) 0.42 N, in y-z plane 45° above the $-z$ direction; (*d*) 0;
(*e*) $F_x = 0.3$ N, $F_y = -0.3$ N, $F_z = 0$

Problem 6.31. Consider a cube, with a side of 0.5 m, as shown in Fig. 6-26. A magnetic field of 0.3 T points in the $-z$ direction. A current of 2 A flows along the direction $ABCDE$. Calculate the force (magnitude and direction) on (*a*) segment AB; (*b*) segment BC; (*c*) segment CD; (*d*) segment DE; and (*e*) the entire path $ABCDE$.

Ans. (*a*) 0.3 N, $+y$ direction; (*b*) 0.3 N, $-x$ direction; (*c*) 0.3 N, $+x$ direction; (*d*) 0.3 N, $-y$ direction; (*e*) 0

Problem 6.32. A square plate of length 0.5 m, with a mass of 0.03 kg, is hinged and free to rotate about the z axis. Current flows along three edges in the direction shown in Fig. 6-27(*a*), and there is a magnetic field of 0.6 T in the $-x$ direction. For this current, the plate is in equilibrium at an angle, θ, of 30°.

(*a*) What is the direction of the net magnetic force on the plate?

(*b*) By taking torques about the z axis, determine the current flowing along the edges, assuming that the center of gravity of the plate is at its center.

Ans. (*a*) $+y$ direction; (*b*) 0.5 A

Problem 6.33. A square plate of length 0.5 m, with a mass of 0.03 kg, is hinged and free to rotate about the z axis. Current flows along three edges in the direction shown in Fig. 6-27(*a*), and there is a magnetic field of 0.6 T in the $+y$ direction. For this current, the plate is in equilibrium at an angle, θ, of 30°.

(*a*) What is the direction of the net magnetic force on the plate?

(*b*) By taking torques about the z axis, determine the current flowing along the edges, assuming that the center of gravity of the plate is at its center.

Ans. (*a*) $+x$ direction; (*b*) 0.25 A

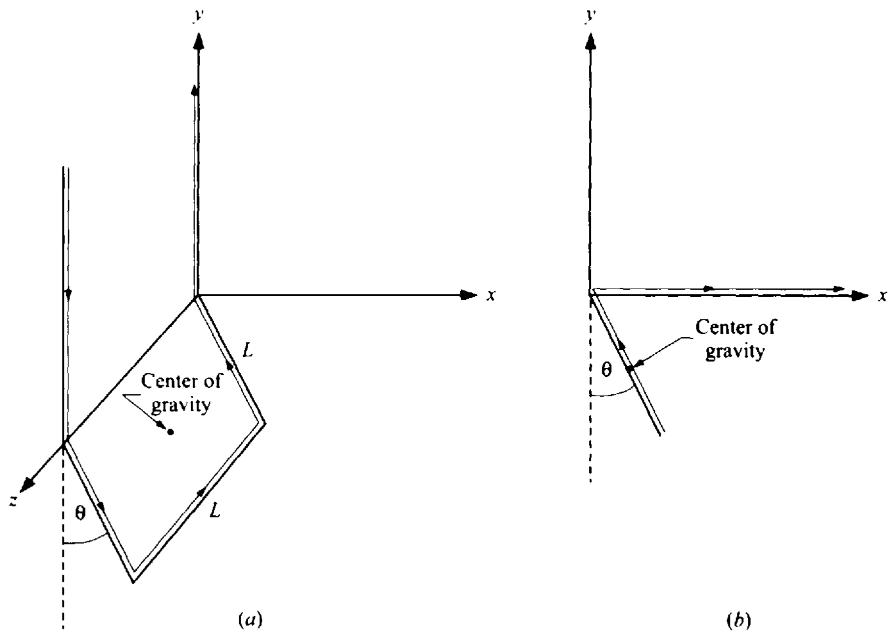


Fig. 6-27

Problem 6.34. An electron is orbiting about a proton with a speed of 3×10^7 m/s in a circle of radius 1.5×10^{-10} m (see Fig. 6-28).

(*a*) What current is moving in the circle? Is it clockwise or counter-clockwise?

(*b*) What is the magnetic moment due to this current?

(*c*) Is the magnetic moment in or out of the paper?

Ans. (*a*) 0.0051 A, clockwise; (*b*) 3.6×10^{-22} A · m²; (*c*) into

Problem 6.35. A current of 2 A flows along the edges of the rectangle *ABCD* in Fig. 6-29. The sides of the rectangle are 0.06 m and 0.10 m, respectively. What torque is exerted on the rectangle by a magnetic field of 1.1 T, if the magnetic field points (*a*) in the *x* direction?; (*b*) in the *y* direction?; (*c*) in the *z* direction?; and (*d*) in the direction from *D* to *B*?

Ans. (*a*) 0; (*b*) 0.0132 N · m, $+z$ direction; (*c*) 0.0132 N · m, $-y$ direction; (*d*) 0.0132 N · m, direction from *A* to *C*

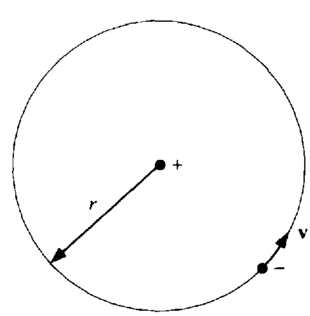


Fig. 6-28

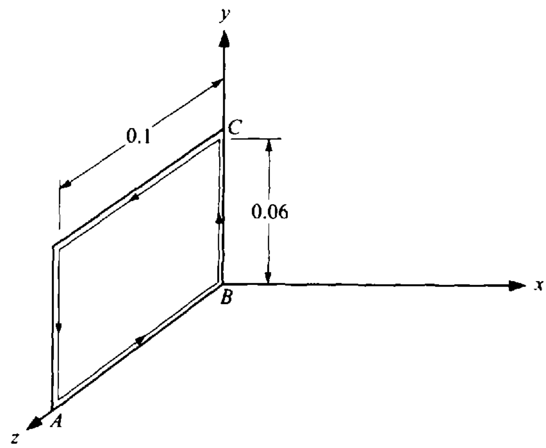


Fig. 6-29

Problem 6.36. A rectangular gate, of sides 2 m × 1.5 m, carries 1500 turns of wire with a current of 0.2 A along its edge. One wants the gate to swing open, with a torque of 90 N · m. What magnetic field is needed?

Ans. 0.1 T

Problem 6.37. A circular coil, of 2000 turns, and area 0.15 m^2, carries a current of 0.3 A. It is in the earth's magnetic field of 1.6×10^{-5} T, which we will assume is directed due north [see Fig. 6-30(*a*)].

(*a*) If the coil is in stable equilibrium in the *x*–*y* plane, does the current flow clockwise or counter-clockwise in the coil?

(*b*) If one turns the coil so that the plane of the coil makes an angle, $\theta = 30°$ with the *x*-axis, as in Fig. 6-30(*b*), what torque is exerted on the coil?

Ans. (*a*) clockwise; (*b*) 1.87×10^{-7} N · m

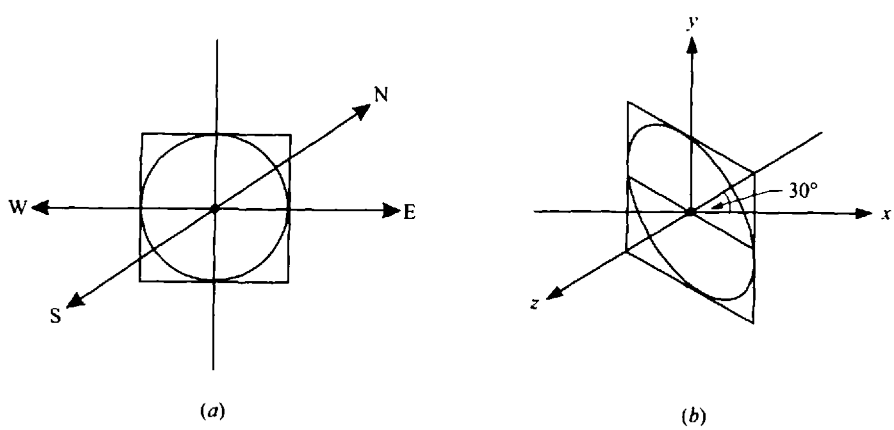


Fig. 6-30