# Chapter 2

# Sound

## 2.1 MATHEMATICAL ADDENDUM—EXPONENTIAL AND LOGARITHMIC FUNCTIONS

While exponentials and logarithms are not major mathematical tools in a second semester physics course, there are enough references to them to warrant a brief mathematical review here. We start with the **exponential function**:

$$y = A^x \tag{2.1}$$

where $A$ is an arbitrary number.

### *Integer Powers of a Number*

When the exponent, $x$, is a positive integer we have the usual powers of $A$: $A^1 = A$; $A^2 = A \cdot A$; $A^3 = A \cdot A \cdot A$, etc. By convention, $A^0 = 1$. For negative integers we define $A^{-1} = 1/A$; $A^{-2} = 1/A^2$ and in general: $A^{-n} = 1/A^n = (1/A)^n$. Clearly, by examination of a few simple examples, we have for any two non-negative integers: $A^n \cdot A^m = A^{(n+m)}$; $A^n \cdot A^{-m} = A^{n-m}$. These two can be combined into the single statement:

$$A^n \cdot A^m = A^{(n+m)} \tag{2.2}$$

where now $n$ and $m$ are any positive, negative or zero integers.

**Problem 2.1.** Show that for any two non-negative integers, $n$ and $m$:

(*a*) $(A^n)^m = (A^m)^n = A^{(n \cdot m)}$

(*b*) $A^n \cdot A^{-m} = A^{-n} \cdot A^m = A^{-(n \cdot m)}$

**Solution**

(*a*) This can be shown by generalizable example. Consider the case $n = 2$ and $m = 3$. Then: $(A^2)^3 = A^2 \cdot A^2 \cdot A^2 = A^6$; Similarly, $(A^3)^2 = A^3 \cdot A^3 = A^6$. Since $n \cdot m = 6$ we have our result. This reasoning works for all positive integers, $n$ and $m$. If $n$ or $m$ is zero, the result follows from the definition.

(*b*) Again by example, consider $n = 2$ and $m = 3$. Then: $(A^2)^{-3} = (1/A^2) \cdot (1/A^2) \cdot (1/A^2) = 1/A^6 = A^{-6}$, and so on. Again, the reasoning works for all positive integers and zero.

The results of Problem 2.1 can be summarized as:

$$(A^n)^m = A^{(n \cdot m)} \tag{2.3}$$

for any positive, negative or zero integers, $n$ and $m$.

### *Fractional Powers of a Number*

We now turn to fractional powers. By definition, $A^{1/2} = \sqrt{A}$; $A^{1/3} = \sqrt[3]{A}$ and, in general, $A^{1/n}$ = the $n$th root of $A$. This means that: $(A^{1/n})^n = A$. As for integers, it is understood that: $A^{-1/n} = 1/A^{1/n} = (1/A)^{1/n}$. The $n$th roots are defined for all positive numbers $A$, but for $n$ even this is not true for negative $A$ (e.g., no number times itself equals a negative number, so square roots, $n = 2$, are not defined). We will assume that $A$ is positive when dealing with fractional powers. We now ask the question what do we mean by: $A^{n/m}$, where $n$ is any integer and $m$ is a non-zero integer. An obvious definition is:

$$A^{n/m} = (A^n)^{1/m} \tag{2.4}$$

For this to make sense, we must have that if $n'/m' = n/m$, $A^{n'/m'} = A^{n/m}$, or, $(A^{n'})^{1/m'}$ $(A^n)^{1/m}$. This is true, and we illustrate it for the simple example $n' = 2n$ and $m' = 2m$. We must show that $(A^{2n})^{1/2m} = (A^n)^{1/m}$. Let $B = A^n$, and $C = A^{2n} = B^2$. Since $C = B^2$, the $2m$th root of $C$ is, indeed, the $m$th root of $B$, and we have our result. For Eq. (*2.4*) to be a useful definition, it must also be shown that $(A^{1/m})^n = (A^n)^{1/m}$.

**Problem 2.2.** Show that $(A^{1/m})^n = (A^n)^{1/m} = A^{n/m}$ for any integer $n$, and non zero integer $m$.

**Solution**

Let $A^{1/m} = B$. We then have to show that:

$$B^n = (A^n)^{1/m} \tag{i}$$

Noting from the definition that $B^m = A$, we substitute $B^m$ for $A$ in the right side of Eq. (*i*), to get:

$$(A^n)^{1/m} = [(B^m)^n]^{1/m} = (B^{m \cdot n})^{1/m} = B^{(m \cdot n/m)} = B^n \tag{ii}$$

which is just the result, Eq. (*i*), that we needed to prove.

It can be demonstrated that Eqs. (*2.2*) and (*2.3*) are valid for any two positive or negative fractional powers, $a$ and $b$:

$$A^a \cdot A^b = A^{(a+b)} \tag{2.5a}$$

$$(A^a)^b = A^{a \cdot b} \tag{2.5b}$$

where we have generalized the definition of negative powers to include fractions:

$$1/A^a = A^{-a} \tag{2.5c}$$

### *General Powers of a Number and their Properties*

Powers of $A$ can be defined not only for all proper and improper fractions as we have done, but more generally for all real numbers, including the myriad of numbers such as $\sqrt{2}$ and $\pi$, which are infinite non-repeating decimals, and correspond to points on a line (such as the $x$ axis of a graph), but cannot be expressed as a fraction. For all such powers, Eqs. (*2.5*) hold. Returning, to our exponential function, Eq. (*2.1*): $y = A^x$, we see that for positive $A$, $x$ can be any number on the real line, from $-\infty$ to $\infty$, and that $y$ takes on positive values which depend on $A$ and $x$. There are two values of $A$ that are most often used in dealing with powers. One is $A = 10$, which is particularly useful since for historical reasons numbers are most often expressed in "base 10 ", i.e., using the **decimal** system. (As most students now know from computer science courses, one can express numbers in the **binary** (base 2) system, and in fact any positive integer can be used as the base for the integer system). For the powers of 10 our exponential function becomes: $y = 10^x$. Every time $x$ increases by a unit ($x \to x + 1$), $y$ increases by a multiplicative factor of $10^1 = 10$. It is this rapid increase in $y$ with increasing $x$ that characterizes an exponential function. Of course, if $x$ is negative, every decrease in $x$ by one unit causes $y$ to decrease by a factor of 10, so that exponential decreases are drastic as well. Figure 2-1(*a*) and (*b*) shows graphs of the exponential function $y = 10^x$.

### *The Logarithmic Function*

As $x$ increases continuously from $-\infty$ to $\infty$, $y$ is always positive and increases continuously from 0 to $\infty$, passing through $y = 1$ when $x = 0$. Thus, for any positive $y$ there is a unique number $x$ for which

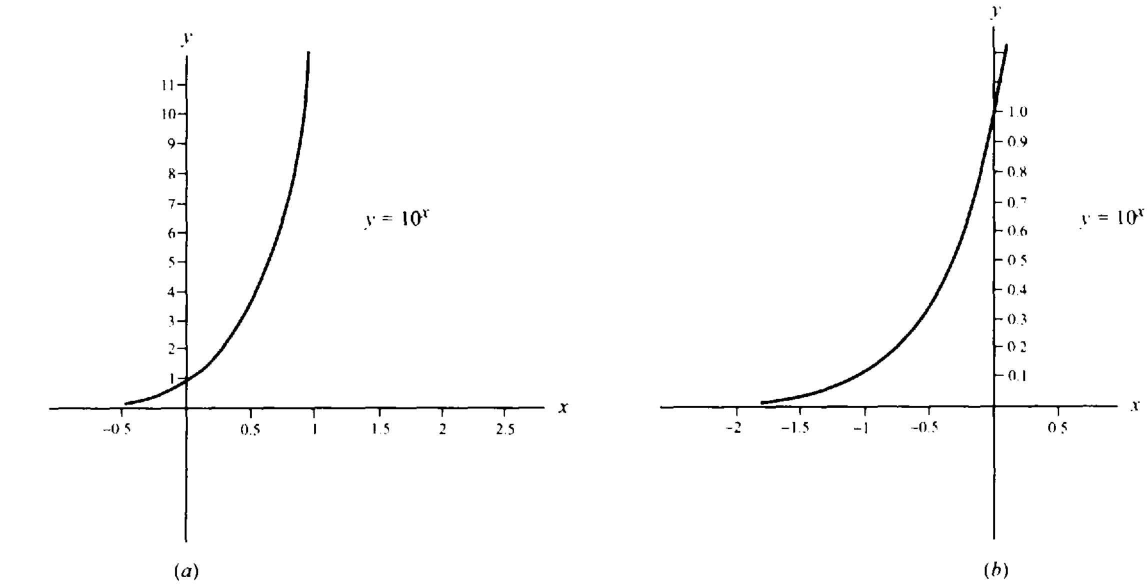



**Fig. 2-1**

$y = 10^x$ holds. The **logarithmic function** "to the base 10" is then defined as: $\log_{10} y = x$. As such, the base 10 logarithm is the inverse function to $y = 10^x$:

$$y = 10^x \Leftrightarrow \log_{10}(y) = x \tag{2.6}$$

For example: $\log_{10}(1) = \log_{10}(10^0) = 0$; $\log_{10}(10) = \log_{10}(10^1) = 1$; $\log_{10}(100) = \log_{10}(10^2) = 2$; $\log_{10}(1000) = \log_{10}(10^3) = 3$. Often it is understood that "log" stands for $\log_{10}$, and the subscript is omitted. The logarithmic function can be shown to obey the following rules:

$$\log(A \cdot B) = \log(A) + \log(B) \tag{2.7a}$$

$$\log(A^z) = z \cdot \log(A) \tag{2.7b}$$

for all positive $A$ and $B$, and all $z$. It follows from Eqs. (*2.7a*) and (*2.7b*) that:

$$\log(A/B) = \log(A) - \log(B) \tag{2.7c}$$

**Problem 2.3.**

(*a*) Show that for any two positive numbers, $A$ and $B$, $\log(A \cdot B) = \log(A) + \log(B)$.

(*b*) Show that $\log(A^z) = z \cdot \log(A)$ for all positive $A$ and any $z$.

(*c*) Show that $\log(A/B) = \log(A) - \log(B)$

(*d*) Using the results of parts (*a*) and (*b*), find the following in terms of log (2) and/or log (3): log (8), log (18), log (27), log (80), $\log(\frac{1}{2})$, $\log(\frac{1}{8})$, $\log(\frac{3}{8})$.

**Solution**

(*a*) Since $A$ and $B$ are positive we know we can find real numbers $a$ and $b$ such that: $A = 10^a$ and $B = 10^b$. Then, $\log(A) = a$, and $\log(B) = b$. Next, we have: $\log(A \cdot B) = \log(10^a \cdot 10^b) = \log(10^{(a+b)}) = a + b = \log(A) + \log(B)$, which is the desired result.

(*b*) Again, let $A = 10^a$, so that $a = \log(A)$. Then, $A^z = (10^a)^z = 10^{a \cdot z} \Rightarrow \log(A^z) = \log(10^{a \cdot z}) = a \cdot z = z \log(A)$, which is the desired result.

(c) $\log(A/B) = \log(A \cdot B^{-1}) = \log(A) + \log(B^{-1}) = \log(A) - 1 \cdot \log(B) = \log(A) - \log(B)$.

(d) $\log(8) = \log(2^3) = 3 \log 2$;
$\log(18) = \log(2 \cdot 9) = \log(2) + \log(3^2) = \log 2 + 2 \log 3$;
$\log(27) = \log(3^3) = 3 \log 3$;
$\log(80) = \log(8) + \log(10) = \log 8 + 1$.
$\log(\frac{1}{2}) = \log(2^{-1}) = -1 \cdot \log 2 = -\log 2$,
$\log(\frac{1}{8}) = \log(2^{-3}) = -3 \cdot \log 2$
$\log(\frac{3}{8}) = \log 3 - \log(8) = \log 3 - 3 \log 2$.

Figure 2-2 shows a plot of $x = \log(y)$. Note that $x$ is positive for $y > 1$ and negative for $y < 1$, properties that follow directly from the behavior of the exponential function, $y = 10^x$. Notice that the logarithmic function compresses huge variations into smaller increments: $\log(10{,}000) = 4$, $\log(100{,}000) = 5$, $\log(1{,}000{,}000) = 6$, and so on. This is the principle behind the logarithmic plot in which, for example, the horizontal axis for the independent variable is equi-spaced, as usual, while the vertical plot is on a "log scale" where equi-spaced intervals correspond to equal multiplicative factors, such as powers of 10. Figure 2.3 shows a log scale plot of $y = 10^x$. Note that on a log scale the vertical axis has no zero or negative values, so the origin is just an arbitrarily chosen positive number, e.g, $y = 0.001$ in the case of Fig. 2-3. Equal upward spacings correspond to a multiplicative factor (10 in our case) while equal downward spacings correspond to decreases by the same factor.

### *Natural Exponential and Logarithm*

There is a number with special mathematical properties, called the **naperian base**, e, that is particularly useful for exponentials. The number e, like $\sqrt{2}$ and $\pi$, is a real number that cannot be expressed as a fraction. Its approximate value is: e = 2.7183. The exponential function using e is: $y = e^x$, sometimes called the "natural" exponential function, and is often written as: $y = \exp(x)$. The inverse function, called the **natural logarithm**, is $x = \log_e(y)$. $\log_e$ is often given the shorthand notation "ln" so that $\log_e(y)$ is expressed as: $\ln(y)$. The counterpart of Eq. (2.6) is then:

$$y = e^x \equiv \exp(x) \Leftrightarrow x = \ln(y) \tag{2.8}$$

The basic rules for exponentials, Eqs. (2.5), hold for exp (x), as do the logarithmic rules, Eqs. (2.7), with "log" replaced by "ln" in those expressions.

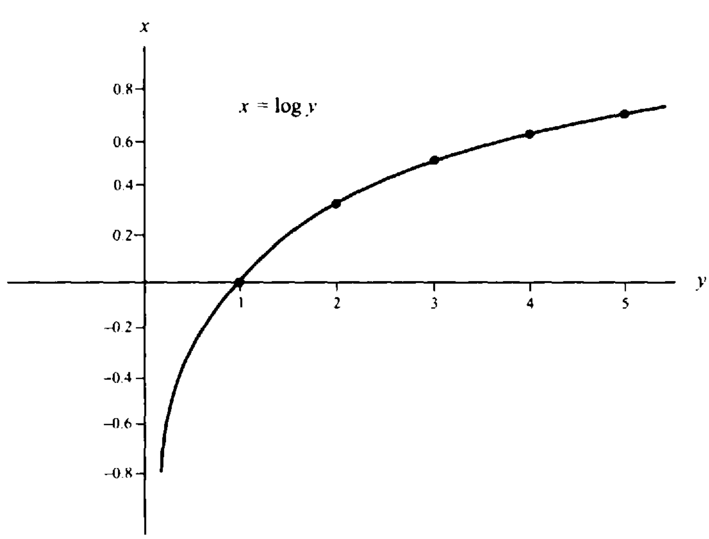



Fig. 2-2

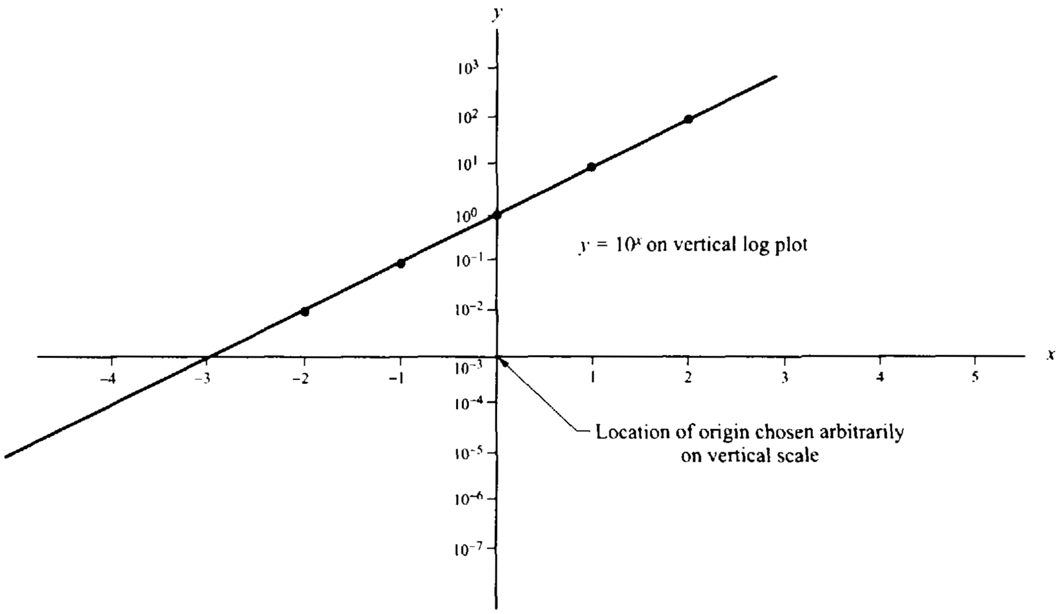



**Fig. 2-3**

**Problem 2.4.**

(*a*) Find the values of exp $(x)$ for $x = 0, 1, 2, -1, \frac{1}{2}, \frac{3}{2}$ (keep values to four place accuracy).

(*b*) Find the values of ln $(z)$ for $z = 1, 10, 10^2, \frac{1}{2}$.

**Solution**

(*a*) These results can be obtained from our approximate value of e given above, or more easily using the "$e^x$" function on a good calculator. $\exp(0) = e^0 = 1$(exact); $\exp(1) = e^1 = e = 2.718$; $\exp(2) = e^2 = 7.389$; $\exp(-1) = 1/e = 0.3679$; $\exp(\frac{1}{2}) = \sqrt{e} = 1.649$; $\exp(\frac{3}{2}) = \exp(1) \cdot \exp(\frac{1}{2}) = (2.718)(1.649) = 4.482$.

(*b*) $\ln(1) = 0$; from ln tables or calculator: $\ln(10) = 2.303$; $\ln(100) = 2 \ln(10) = 4.605$; $\ln(\frac{1}{2}) = -\ln(2) = -0.6931$.

**Problem 2.5.**

(*a*) Show that the natural and decimal logarithms are related by:

$$\ln(A) = \ln(10) \cdot \log(A) \quad (i)$$

(*b*) Use the results of part (*a*) to find ln (8) and ln (18) in terms of log 2 and log 3.

**Solution**

(*a*) Let $A = 10^x$, and $10 = e^a$. Then: $A = (e^a)^x$. Using the rules for logarithms:

$$\ln(A) = \ln[(e^a)^x] = x \cdot \ln(e^a) = x \cdot a \quad (ii)$$

We also have, however:

$$\log(A) = \log(10^x) = x \quad (iii)$$

and

$$\ln(10) = \ln(e^a) = a \quad (iv)$$

Substituting (*iii*) and (*iv*) into the right side of (*ii*) we get our result: $\ln(A) = \ln(10) \cdot \log(A)$.

(*b*) $\ln(8) = 3 \cdot \ln(2) = 3 \cdot \ln(10) \cdot \log 2$. Using the value of $\ln(10)$ from Problem 2.4 we have: $\ln(8) = 6.908 \cdot \log 2$. Similarly, $\ln(27) = 3 \cdot \ln(3) = 3 \cdot \ln(10) \cdot \log(3) = 6.908 \cdot \log(3)$.

## 2.2 PROPAGATION OF SOUND—VELOCITY, WAVE-FRONTS, REFLECTION, REFRACTION, DIFFRACTION AND INTERFERENCE

***Sound Velocity in Air***

In Chap. 1 we discussed the propagation of waves in different media. In the case of fluids we saw that the general formula for the velocity of propagation, $v_p$, of longitudinal sound waves, was:

$$v_p = (B/\rho)^{1/2} \qquad (2.9)$$

where $B$ is the **bulk modulus** and $\rho$ the density of the fluid. The bulk modulus is given by (see, e.g. Beginning Physics I, Chap. 11, Sec. 3)

$$B = \text{stress/strain} = -\Delta P/(\Delta V/V) \qquad (2.10)$$

where $\Delta P$ is the increase in hydrostatic pressure on the fluid and $\Delta V/V$ is the consequent fractional change in volume. Since the volume decreases as the pressure increases, the minus sign assures that $B$ is positive. To calculate the bulk modulus for air we can reasonably assume that the ideal gas law holds:

$$PV = nRT \qquad (2.11)$$

Eq. (*2.11*) gives us a relationship between $P$ and $V$, and would allow us to calculate the actual value of $B$ if we knew that $T$ was constant during the compression (or rarefaction) of gas. We know from the study of heat and thermodynamics that a compression of a gas is generally accompanied by a temperature rise unless there is adequate time for heat to flow from the compressed gas to the surroundings so that a common temperature with the surroundings is maintained during the compression period, and the process is isothermal. In the case of longitudinal waves, the compressions and rarefactions at a given location are typically very rapid, so that there is no time for a significant amount of heat to flow to or from the surrounding air as the local air goes through its accordion-like paces. Indeed, under such conditions the process is adiabatic rather than isothermal. Thus, instead of Eq. (*2.11*), we can use the relationship between $P$ and $V$ for an ideal gas undergoing an adiabatic process (see, e.g., Beginning Physics I, Problem 18.9):

$$PV^\gamma = \text{constant} \qquad (2.12)$$

where $\gamma$ is the ratio of the molar heat capacity at constant pressure to that at constant volume: $\gamma = c_{\text{mol, p}}/c_{\text{mol, v}}$. Eq. (*2.12*) implies that a change in $P$ must be accompanied by a corresponding change in $V$, so we should have a relationship between $\Delta P$ and $\Delta V$ and hence an expression for $B$. Using the calculus it can be shown that for small $\Delta P$:

$$\Delta P/\Delta V = -\gamma P/V \qquad (2.13)$$

Substituting into Eq. (*2.10*) we get:

$$B_{\text{adiabatic}} = \gamma P \qquad (2.14)$$

From this, and the ideal gas law, it can be shown that:

$$v_p = [\gamma RT/M]^{1/2} \qquad (2.15)$$

where $M$ is the molecular mass of the gas. We derive this result in the following problem.

**Problem 2.6.**

(*a*) Using Eqs. (*2.9*), (*2.11*) and (*2.14*), find an expression for the speed of sound in an ideal gas in terms of the temperature, $T$, and pressure $P$, of the gas.

**Solution**

Substituting Eq. (*2.14*) into Eq. (*2.9*), we get:

$$v_p = [\gamma P/\rho]^{1/2} \qquad (i)$$

The ideal gas law, Eq. (*2.11*), can be re-expressed in terms of the density, $\rho$, and the molecular mass, $M$, by recalling that: $n = m/M$, where $n$ = number of moles in our gas, $m$ = mass of the gas. The molecular mass, $M$, represents the mass per mole of the gas (see, e.g., Beg. Phys. I, Problem 16.7).

Then:

$$PV = nRT = mRT/M \qquad (ii)$$

Dividing both sides by $V$, and noting that $\rho = m/V$, we get:

$$P = \rho RT/M \qquad (iii)$$

Substituting (*iii*) into (*i*) we get: $v_p = [\gamma RT/M]^{1/2}$, which is the desired result, Eq. (*2.15*).

**Problem 2.7.** Recall that $\gamma$ for monatomic and diatomic gases are approximately $5/3 = 1.67$, and $7/5 = 1.40$, respectively. Also, the gas constant $R = 8.31$ J/mol · K, and atmospheric pressure is $P_A = 1.013 \cdot 10^5$ Pa.

(*a*) Calculate the velocities of sound in hydrogen and helium at $P = P_A$ and $T = 300$ K (27 °C).

(*b*) Calculate the velocity of sound in air at $P_A$ at: $T = 273$ K, $T = 300$ K, and $T = 373$ K. Assume $M_{air} = 29.0$ kg/kmol.

**Solution**

(*a*) Using Eq. (*2.15*), and noting that helium is monatomic, with molecular mass 4.0 kg/km, we have:

$$v_p = [1.67(8310\ \text{J/kmol} \cdot \text{K})(300\ \text{K})/(4.0\ \text{kg/kmol})]^{1/2} = 1020\ \text{m/s}.$$

Similarly, for hydrogen, which is diatomic and has molecular mass 2.0 kg/kmol, we have:

$$v_p = [1.40(8310\ \text{J/kmol} \cdot \text{K})(300\ \text{K})/(2.0\ \text{kg/kmol})]^{1/2} = 1321\ \text{m/s}.$$

(*b*) Here, the dominant gases are oxygen and nitrogen, which are diatomic, so:

$$v_p = [1.40(8310\ \text{J/kmol} \cdot \text{K})T/(29.0\ \text{kg/kmol})]^{1/2} = 20T^{1/2}\ \text{m/s}.$$

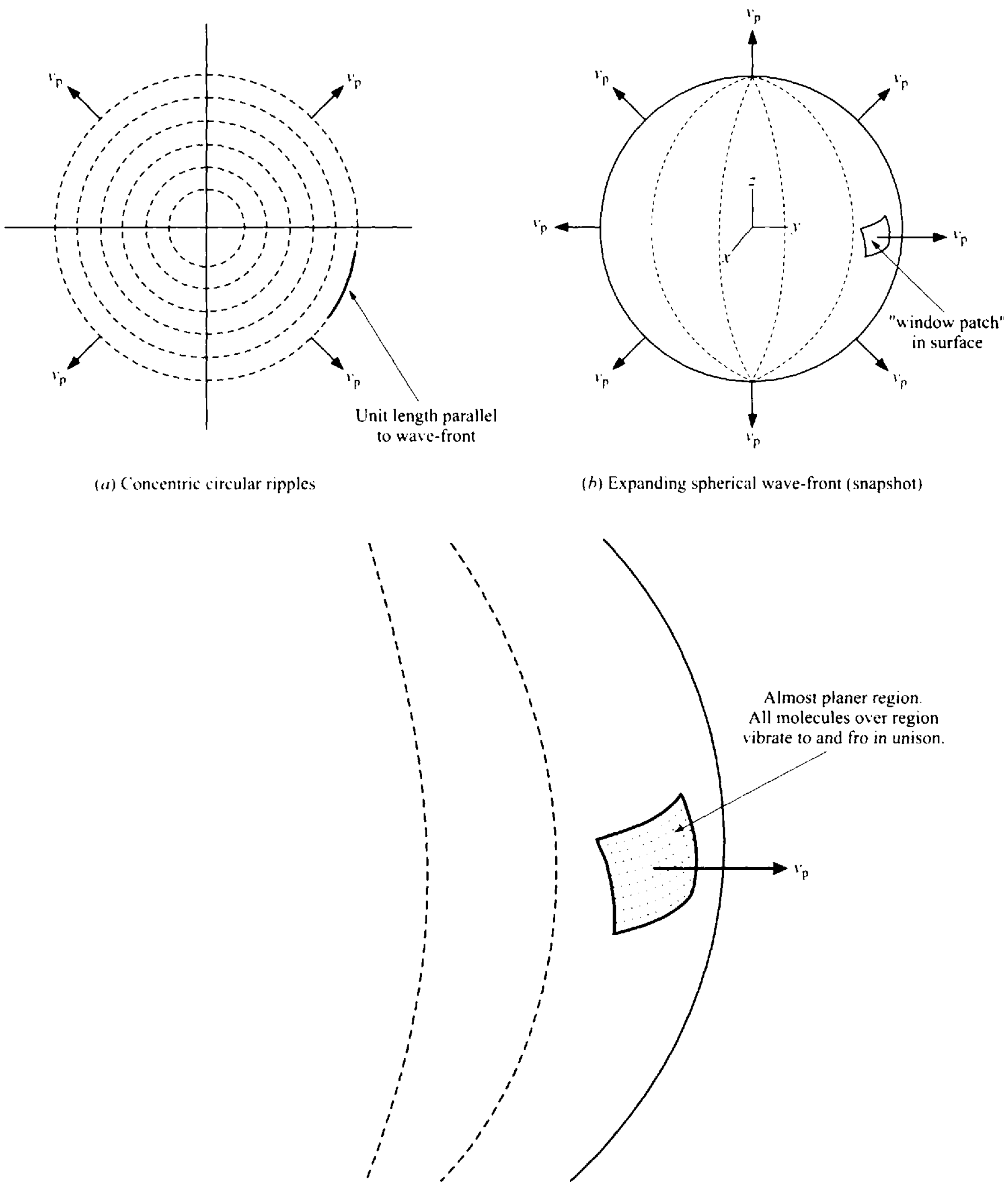
Substituting our temperatures, we get:

$$T = 273\ \text{K} \Rightarrow v_p = 330\ \text{m/s}; \qquad T = 300\ \text{K} \Rightarrow v_p = 346\ \text{m/s}; \qquad T = 373\ \text{K} \Rightarrow v_p = 386\ \text{m/s}.$$

It is interesting to note that the formula for the speed of sound in a gas, Eq. (*2.15*), is very similar to the equation for the root-mean-square velocity of the gas molecules themselves (see, e.g., Beg. Phys. I, Problem 16.12): $v_{rms} = (3RT/M)^{1/2}$. Both the velocity of sound and the mean velocity of the molecules decrease with molecular mass and increase with temperature, and $v_p$ is slightly less than $v_{rms}$ for the same gas and temperature.

### *Waves in Two and Three Dimensions*

In Chap. 1 all of the waves we considered were constrained to propagate in one dimension, such as transverse waves in a cord or sound waves in a rail or tube. Waves in bulk material such as air, tend to spread out in all available directions. A two-dimensional analogue of this is the ripple effect when a stone is tossed into the still water of a pond. The disturbance of the water surface at the point of contact




(*a*) Concentric circular ripples

(*b*) Expanding spherical wave-front (snapshot)

(*c*) Magnification of small "window patch" on large spherical wave-front

**Fig. 2-4**

sends circular ripples moving out at the appropriate wave propagation speed for this system. The fact that the ripples are circular tells us that the disturbance is traveling at equal speed in all directions. This demonstrates that the speed and direction of the disturbance over the water surface has nothing to do with the speed and direction of the stone. Once a disturbance in any material is created, the wave propagation of the disturbance is *characteristic of the material through which the wave moves.* If we draw an imaginary line through the crest (or trough) of one of the ripples at a given instant of time, we are looking at the same phase of the disturbance at all different locations on the water surface. Such a line is called a **wave-front**. If we plot this wave-front at many different instants of time we get a clear picture of how the disturbance moves through the water surface. For our water surface the motion of the wave-front mimics our own observation of the motion of the ripple. If a given point in our pond were disturbed with a constant frequency vibrator, the wave would consist of a continuous train of circular ripples and corresponding wave-fronts, with crests and troughs spaced equally from one another. Figure 2-4(*a*) shows a pictorial display of the wave-fronts, with the origin as the location of the vibrator.

This analysis can be extended to sound waves in three dimensions. Consider a medium such as air at rest; a disturbance (such as caused by a snap of the fingers) or a continuous simple harmonic vibration (such as caused by a vibrating tuning fork) will have wave-fronts that travel in all directions with equal speed, just as the ripples in the water. In the case of the longitudinal sound waves in the still air at constant temperature, the disturbance propagates in three dimensions with constant velocity, so the wave-fronts now take the form of spherical shells—at least until they hit some boundary. A representation of the wave-front of a spherical wave in three dimensions is shown in Fig. 2-4(*b*) where, again, the origin is at the source of the disturbance.

> ***Note.*** The direction of propagation of the wave at any location is perpendicular to the wave front at that location.

### *Energy and Power in Waves in Two and Three Dimensions*

In general, it is complicated and beyond the scope of this book to quantitatively describe wave motion in two or three dimensions. Nonetheless, there are a number of characteristics that can be described fairly easily. For example, for our water ripples, the energy of the wave in any small wave-front region, and the associated power transmitted through a unit length parallel to the wave-front, see Fig. 2-4(*a*), are being diluted as the circular wave-front expands to larger circumference. Since the circumference of a ripple increases in proportion to its growing radius $R$, the power per unit wave-front length must decrease as $1/R$.

A similar analysis can be made for our SHM sound waves in three dimensions. The energy and power of the wave, per unit area perpendicular to the direction of propagation of the wave [see, e.g., the small "window" in Fig. 2-4(*b*)] now fall off as $1/R^2$. The power per unit area perpendicular to the direction of propagation is called the intensity, $I$, and is given by:

$$I = P/A \tag{2.16}$$

where $P$ is the power transmitted through a "window" concentric to the wave-front and of cross-sectional area $A$, as shown schematically in Fig. 2-4(*c*).

**Problem 2.8.** A spherical sound wave emanates from a small whistle suspended from a ceiling of a very large room, emitting a single frequency simple harmonic wave.

(*a*) If the power generated by the whistle is 0.0020 W, find the intensity of the spherical wave 1.0 m, 2.0 m, and 3.0 m from the source. [*Hint*: Recall that the surface area of a sphere of radius $r$ is $4\pi r^2$].

(*b*) Find the power passing through an imaginary circular window of area 12.0 $cm^2$, which is facing (parallel to) the wave fronts and at a distance of 3.0 m from the source.

**Solution**

(*a*) All the power must pass through any imaginary concentric spherical shell, and by symmetry will flow out with equal intensity in all directions. For a spherical shell of radius $r$, the intensity would thus be: $I = P/A = P/(4\pi r^2)$. Substituting the value of $P$ and the various $r$ values into this relationship, we get:

$$r = 1 \text{ m}, I = 0.159 \text{ mW/m}^2; \quad r = 2 \text{ m}, I = 0.0398 \text{ mW/m}^2; \quad r = 3 \text{ m}, I = 0.0177 \text{ mW/m}^2.$$

(*b*) Since $I$ represents power per unit area passing perpendicular to the imaginary window, we must have:

$$P_A = IA = (0.0177 \cdot 10^{-3} \text{ W})(12.0 \cdot 10^{-4} \text{ m}^2) = 0.212 \text{ mW}.$$

**Problem 2.9.**

(*a*) Assuming a simple harmonic disturbance in the water of a pond, how would you expect the amplitude of any given ripple to change with radius $R$ as the ripple expands out? Ignore thermal losses.

(*b*) Repeat for the sound wave in Problem 2.8; If the wave amplitude were 0.20 mm at $R = 1.0$ m, what is the amplitude at $R = 3.0$ m?

**Solution**

(*a*) We recall from Chap. 1 that the power of a wave of a given frequency and velocity of propagation is proportional to the square of the amplitude. Since the power falls off as $1/R$ for the case of our circular ripples, the amplitude of the wave decreases as $1/\sqrt{R}$.

(*b*) In this case the power falls off as $1/R^2$, so the wave amplitude falls off as $1/R$. If the amplitude were 0.20 mm at $R = 1.0$ m, then it would be 1/3rd that amount, or 0.0667 mm, at $R = 3.0$ m.

### *Plane Waves*

Another interesting result for our water ripples and our spherical sound waves is that at large distances from the source, a small portion of the circular (or spherical) wave-front looks almost like a straight line (or flat plane) at right angles to the direction of motion of the wave. For a spherical sound wave whose source is a long way off, the wave-front appears to be a planar surface perpendicular to the direction of motion of the wave, as long as we are observing a portion of the wave-front whose dimensions are small compared with the distance to the source. Thus, for example, the imaginary "window" shown in Fig. 2-4(*c*) is almost planar if the dimensions are small compared to the distance from the source of the wave. A wave moving through space in which the wave-front is planar is called a **plane wave**, and is characterized by the fact that every point on the planar wave-front is in phase at the same time. Thus, the air molecules are all vibrating in and out along the direction of motion of the wave (longitudinal) in lock-step at all points on the wave-front in the window of Fig. 2-4(*c*).

Since all the points on a plane wave act in unison, the wave equation for such a wave is exactly the same as for our longitudinal waves in a long tube. Indeed, if $x$ is along the direction of wave propagation, then the wave-front is parallel to the $(y, z)$ plane. Under those circumstances our SHM sound wave is described by:

$$d_{y,z}(x, t) = A \sin(2\pi t/T - 2\pi x/\lambda) \tag{2.17}$$

where $d_{y,z}$ represents the displacement of air molecules at any point $(y, z)$ on the wave-front, and at a distance $x$ (measured from some convenient point) along the direction of wave propagation, at any time $t$. $A$, $T$ and $\lambda$ are the amplitude, period and wavelength of the wave, and all three are constant for all $y$ and $z$ in our plane wave region. Thus $d_{y,z}$ does not depend on $y$ or $z$ in our plane wave region. We note that in reality the amplitude $A$ does decrease with increasing $x$ but only slightly if we limit ourselves to changes in $x$ that are small compared to the distance from the source of the wave-front.

**Problem 2.10.** Consider the spherical wave in Problem 20.8(*b*).

(*a*) Could the portion of the wave passing through the imaginary window be considered a plane wave?

(*b*) When the portion of the wave-front passing through the imaginary window, which is 3.0 m from the source, moves an additional 4.0 cm, how much larger is the area it will occupy?

(*c*) What will be the intensity of the wave of Problem 20.8(*b*) when it moves an additional 4.0 cm, as in part (*b*), above?

**Solution**

(*a*) The imaginary window is like a circular patch of radius less than 2 cm on a spherical surface of radius 3.0 m. It therefore is, indeed, almost flat and a plane wave would be an excellent approximation to the part of the spherical wave passing through it.

(*b*) The new area would correspond to the equivalent window on a concentric spherical shell of radius 3.04 m. Since the areas go as the square of the radius, the ratio of the window areas would be (letting $a_1$ and $a_2$ represent the areas at 3.00 m and 3.04 m, respectively):

$$a_2/a_1 = r_2{}^2/r_1{}^2 = (3.04/3.00)^2 = 1.027 \Rightarrow a_2 = 1.027a_1 = 12.3\ \text{cm}^2, \qquad \text{or a 2.7\% increase.}$$

(*c*) Since the portion of the wave passing through the first window is precisely the portion of the wave passing through the second window, the new intensity will just be: $I_2 = P_{A1}/A_2$, where $P_{A1}$, the power through the first window, was already calculated in Problem 2.8(*b*). Substituting in numbers we get:

$$I_2 = (0.212 \cdot 10^{-3}\ \text{W})/(12.3 \cdot 10^{-4}\ \text{m}^2) = 0.172\ \text{W/m}^2.$$

*Note.* This is only slightly less than $I_1 = 0.177\ \text{W/m}^2$, as calculated in Problem 2.8(*a*). Indeed, $I_2$ could have been calculated by noting that if the area has gone up by 2.7% the intensity must go down by 2.7% so: $I_2 = I_1/1.027 = 0.172\ \text{W/m}^2$.

**Problem 2.11.**

(*a*) Find an expression for the intensity of a sinusoidal planar sound wave traveling through air in terms of the density of air $\rho$, the angular frequency $\omega$, the amplitude $A$, and the velocity of propagation $v_p$, of the wave.

(*b*) A sinusoidal planar sound wave travels through air at atmospheric pressure and a temperature of $T = 300$ K. The intensity of the wave is $5.0 \cdot 10^{-3}\ \text{W/m}^2$. Find the amplitude of the wave if the frequency is 2000 Hz. (Assume the mean molecular mass of air is M = 29 kg/kmol and $\gamma = 1.40$.)

**Solution**

(*a*) From Eq. (*1.12*) and Problem 1.16 we have for the power $P$ of the wave passing through a cross-sectional area $C_A$ perpendicular to the direction of propagation:

$$P = \tfrac{1}{2}\rho C_A \omega^2 A^2 v_p \qquad (i)$$

From the definition, $I = P/C_A$, and dividing we get:

$$I = \tfrac{1}{2}\rho\omega^2 A^2 v_p \qquad (ii)$$

(*b*) Recalling (Problem 2.6) that gas pressure is $p = \rho RT/M$, we have for air at atmospheric pressure:

$$1.013 \cdot 10^5\ \text{Pa} = \rho(8314\ \text{J/kmol})(300\ \text{K})/(28.8\ \text{kg/kmol}), \qquad \text{and} \qquad \rho = 1.17\ \text{kg/m}^3.$$

Similarly (Problem 2.6), $v_p = [\gamma p/\rho]^{1/2} = [1.40(1.013 \cdot 10^5\ \text{Pa})/(1.17\ \text{kg/m}^3)]^{1/2} = 348$ m/s. Substituting into (*ii*) above, we get:

$$5 \cdot 10^{-3}\ \text{W/m}^2 = \tfrac{1}{2}(1.17\ \text{kg/m}^3)(6.28)^2(2000\ \text{Hz})^2(348\ \text{m/s})A^2 \Rightarrow A = 3.94 \cdot 10^{-7}\ \text{m}.$$

By examining a region of space where a sound wave can be approximated by a plane wave (or the corresponding two-dimensional region on the surface of a lake where a ripple wave can be approximated by a "straight line" wave-front) one can gain interesting insight into many wave phenomena. These wave phenomena are very similar to those associated with light waves, which we will be studying later on. They include: reflection, refraction, interference and diffraction. These are examined in the following sections.

### *Reflection and Refraction of Sound*

When sound wave-fronts hit a barrier, such as the floor or a wall for the case of the whistle in Problem 2.9, or the side of a mountain or canyon wall for the case of a person making a noise in the great outdoors, part of the wave reflects and part is transmitted into the barrier. The part that is transmitted can penetrate deeply into the barrier material, or it can quickly lose amplitude with wave energy converting to thermal energy (absorption). The rate of absorption depends on such factors as the composition of the barrier, its elasticity and the frequency of the wave. The part of the wave that is reflected has diminished amplitude but the same frequency and velocity as the original wave, and hence the same wavelength. The echo we hear in a canyon is a consequence of such a reflection, and the time elapsed between emission of a sound and the echo we hear can be used to roughly measure the speed of sound in the air if the distances are known, or the distances if the speed of sound is known.

**Problem 2.12.**

(*a*) A man standing 3360 ft from a high cliff hits a tree stump with an axe, and hears the faint echo 6.4 s later. What is the velocity of sound in the air that day?

(*b*) A child standing with his parents somewhere between the two walls of a wide canyon shouts "hello". They hear two loud echoes, which one parent times with a stop watch. The first echo arrived after an interval of 1.2 s, while the second arrived 1.8 s later. How wide is the canyon? Assume the same speed of sound as in part (*a*).

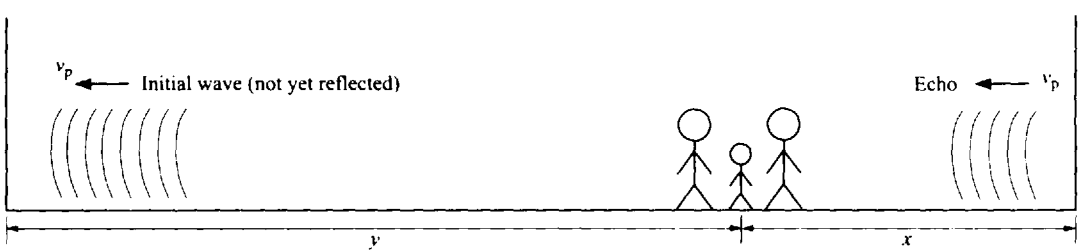
**Solution**

(*a*) The sound created by the axe hitting the stump first travels the distance, $x$, to the cliff, where it is reflected and makes the return trip of the same distance to the man. We must then have that: $2x = v_p t$, where $t$ is the elapsed time for the echo and $v_p$ is the velocity of sound in air. Substituting the given values for $x$ and $t$ in the equation, we have:

$$v_p = 2(3360 \text{ ft})/(6.4 \text{ s}) = 1050 \text{ ft/s}.$$

(*b*) The situation is shown schematically in Fig. 2-5. The family is clearly not midway between the two walls because the echoes took different times. Letting $x$ and $y$ be the respective distances to the near and far walls, we have:

$$2x = v_p t_1 = (1050 \text{ ft/s})(1.2 \text{ s}) = 1260 \text{ ft} \Rightarrow x = 630 \text{ ft}.$$




A short time after a child shouts

**Fig. 2-5**

Similarly, we have:

$$2y = v_p t_2 = (1050 \text{ ft/s})(1.8 \text{ s} + 1.2 \text{ s}) = 3150 \text{ ft} \Rightarrow y = 1575 \text{ ft}.$$

Then the distance between the walls is

$$d = x + y = 2205 \text{ ft}.$$

The reflection of sound is of great importance in modern high frequency detection devices. Sonar is used by submarines to find and map out objects at various distances from the sub. The time between emission and detection of reflected sound pulses is measured, and from a knowledge of the speed of sound the distance to the object is determined. Ultrasound is used in medical imaging by detecting changes in tissue density in the body through examination of reflected and transmitted waves.

In our discussion of the velocity of sound in air, we concluded that the velocity is temperature dependent, as shown in Eq. (*2.15*). In general, the layers of air above the ground are not at a constant temperature. Depending on circumstances, e.g. time of day or night, specific atmospheric conditions, etc., the layers of air near the earth's surface can be either colder or warmer than the layers above. Consider the portion of a sound wave emitted from the horn of a ship at sea. Part of the wave initially travels parallel to the surface of the sea, and to an observer at some distance from the ship the wave-fronts can be approximated by those of a plane wave traveling from the ship to the observer. Indeed, if the air is at a uniform temperature, a cross section of the wave-fronts in some local region would look something like that in Fig. 2.6(*a*). Suppose the wave passes a region where the temperature is higher at sea level and drops with increasing altitude. Eq. (*2.15*) indicates that the propagation velocity would be highest at sea level and decreasing upward. Then the bottom of a given wave-front would move faster thàn a point higher up and the wave-front would start to bend as shown in Fig. 2-6(*b*). Since the direction of propagation of a wave is perpendicular to the wave-front, the wave velocity would start to develop an upward component and would therefore not carry as far along the sea surface. On the other hand, if the layers of air at the water surface were colder than those above, the speed of the wave-front near the surface would be less than that above, and we would get the effect shown in Fig. 2-6(*c*). Here the net effect is that more of the wave-front from higher levels is pushed down toward the surface, ensuring that a substantial amount of wave energy would travel along the surface, and thus be audible a long way off. In general, when a wave travels through a medium of varying densities (for example, layers of air at different temperatures) the velocity of different parts of the wave-front are different, and the direction of propagation of the wave changes as a consequence. This is called **refraction**, and will be discussed in greater detail in our discussion of light waves.

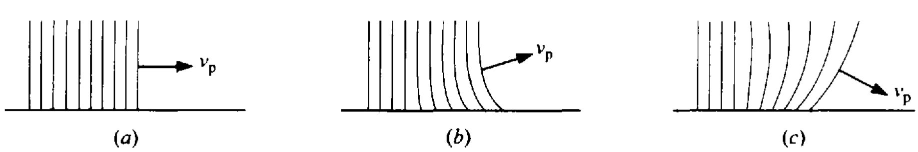



**Fig. 2-6**

### *Interference and Diffraction*

We can now examine interference and diffraction of sound waves. Interference was already encountered in Chap. 1, and was used to demonstrate the formation of standing waves in a cord or a tube. **Interference** is the effect of having more than one wave passing a given point, and the possibility that the two waves will reinforce or weaken each other as a consequence of the phase difference between the waves. For the case of continuous waves, such as sinusoidal traveling waves, the ability of two waves to completely cancel each other out at a given point over an extended time period (e.g. a node for standing waves) requires three things to be true:

1. The frequencies of the two waves are the same.
2. The amplitudes of the two waves are the same.
3. The wave vibrations are in the same direction in space.

The most important of these requirements is the first. If the frequencies are not identical (or almost so) then at any point in space the relative phases of the two waves would rapidly be changing due to the different frequencies and the positive or negative interference would average to zero over even short time intervals. Our second requirement is less important because even if the amplitudes are somewhat different and one does not get complete cancellation, the interference effect might still be quite significant. The third requirement does not come up in the context of sound waves in a tube, since the vibrations of all waves are constrained to be in the same direction. This is no longer true for waves traveling through space. Nonetheless, if the direction of vibrations of two waves passing a point make a relatively small angle with each other, one could still get substantial cancellation of the two waves—assuming of course that requirements 1 and 2 is satisfied.

## 2.3 HUMAN PERCEPTION OF SOUND

### *Intensity Scale of Sound Waves*

The human ear responds to the intensity of the sound waves hitting it with the perception of loudness. While the sense of loudness is a physiological and psychological response of human beings and varies somewhat from person to person (and therefore is not exactly the same as sound intensity), it is true that the human ear can perceive an exceptionally broad range of sound intensities. To describe that range it is useful to create a logarithmic scale called the **decibel scale** (db), which gives a quantitative measure to "loudness", which we label $n$, and define as:

$$n = 10 \log (I/I_0) \tag{2.18}$$

where $I$ is the intensity of sound and $I_0$ is a fixed reference intensity taken to approximate the lowest level of sound audible to a human being: $I_0 = 1.0 \cdot 10^{-12}$ W/m$^2$. It turns out that an intensity of about $I = 1.0$ W/m$^2$ represents the highest intensity to which the ear can respond without feeling pain. Substituting this intensity into Eq. (*2.18*) we obtain $n = 10 \log (1.0/1.0 \cdot 10^{-12}) = 10 \log (10^{12}) = 120$ db, so that the threshold of pain is 120 db. Note that because of the logarithmic scale each factor of 10 increase in intensity corresponds to an addition of 10 db. Thus a thousand-fold increase in noise corresponds to a thirty decibel increase in loudness level.

**Problem 2.13.** A powerful firecracker is tossed in the air and explodes 5 m from a person walking nearby. The peak sound power generated by the explosion is 16 W.

(*a*) What is the intensity of sound that enters the persons ear?

(*b*) To how many decibels does this correspond?

(*c*) At what distance $r$ from the explosion would the person's ear have to be if the sound was at the threshold of pain?

**Solution**

(*a*) We assume the energy disperses in a spherically symmetric shell away from the burst site, so

$$I = P/4\pi r^2 \Rightarrow I = (16 \text{ W})/[12.56 \cdot (5.0 \text{ m})^2] = 5.09 \cdot 10^{-2} \text{ W/m}^2.$$

(*b*) $n = 10 \log [(5.09 \cdot 10^{-2} \text{ W/m}^2)/(1.0 \cdot 10^{-12} \text{ W/m}^2)] = 107$ db.

(c) Here $n = 120$, so: $12 = \log (I/I_0) \rightarrow I = I_0 \cdot 10^{12} = 1.0\ \text{W/m}^2$. Then: $16\ \text{W} = (1.0\ \text{W/m}^2)(12.56)r^2$, or $r = 1.13$ m.

**Problem 2.14.** A symphonic passage produces a sound level at a person's ear in the auditorium of 60 db while a person speaking in the next row produces a sound level of 40 db at the same ear. What is the ratio of the intensities of the two sounds?

**Solution**

$$n_1 - n_2 = 10 \log (I_1/I_0) - 10 \log (I_2/I_0) = 10 \log (I_1/I_2) \Rightarrow 20\ \text{db} = 10 \log (I_1/I_2), \qquad \text{or: } I_1/I_2 = 10^2.$$

While human perception of loudness will approximately follow the decibel scale it does not exactly do so. Indeed, loudness perception is dependent on a variety of factors specific both to individuals in a species and the species as a whole. An important factor for human (and other) species is the frequency of the sound. The human ear is most sensitive to frequencies in the range 1000–6000 Hz and people with the most acute hearing are able to detect sounds at $I_0$ intensity, or 0 db, at those frequencies. As the frequency drops below 1000 Hz this threshold of hearing rapidly rises to higher and higher intensities requiring about 30 db at 100 Hz and 100 db at 20 Hz—about the lowest frequency that human beings can hear. For frequencies higher than 6000 Hz the threshold intensity rises relatively slowly (about 20 db) as the frequency reaches toward 12,000–15,000 Hz, and then rises more rapidly to about 100 db at 20,000 Hz—about the highest frequency that human beings can hear. Thus, a 30 decibel sound at 40 Hz will be inaudible, while the same level sound at 1000 Hz will sound quite loud. As it turns out the threshold of pain is about 120 decibels at all frequencies from 20 to 20,000 Hz. It should be noted that few people can hear the full frequency range from 20 to 20,000 Hz, and most cannot hear even intermediate frequencies at the lowest threshold intensities.

### *Reverberation Time*

When a sound is emitted in a closed environment such as a room or auditorium it takes a certain amount of time for the intensity of the sound to dissipate. This is because the sound reflects off the walls and the people and objects in the room, and dies down only as a consequence of the absorption of some of the energy by each object at each reflection. The reverberation time is defined as the time it takes for the intensity of a given steady sound to drop 60 db (or six orders of magnitude in intensity) from the time the sound source is shut off. Reverberation times are important because if they are too long successive sounds run into one another and can make it difficult to make out speech (too much echo). For music the quality of the performance is negatively impacted if the reverberation time is too long or too short, the latter case corresponding to a thin or dry effect. Reverberation times depend on the total acoustic energy pervading the room, the surface areas of the absorbing materials and their absorption coefficients. The absorption coefficient of a surface is defined as the fraction of sound energy that is absorbed at each reflection. Thus, an open window has an absorption coefficient of 1 since all the energy passes out of it and none reflects back in. Heavy curtains have a coefficient of about 0.5, and acoustic ceiling tiles have a coefficient of about 0.6. Wood, glass, plaster, brick, cement, etc. have coefficients that range from 0.02 to 0.05. A formula that gives good estimates of the **reverberation time** was developed by Sabine, a leading acoustic architect, and is given by:

$$t_r = 0.16V/A \tag{2.19}$$

where $t_r$ is the reverberation time (s), $V$ is the volume of the room ($\text{m}^3$) and $A$ is called the absorbing power of the room. The absorbing power $A$ is just the sum of the products of the areas of all the absorbing surfaces ($\text{m}^2$) and their respective absorption coefficients.

**Problem 2.15.**

(*a*) Find the reverberation time for an empty auditorium 15 m wide by 20 m long by 10 m high. Assume that the ceilings are acoustic tile, the side walls are covered with heavy drapes, and that the floor and the back and front walls are concrete. Assume the following absorption coefficients: ceiling tiles, 0.6; drapes, 0.5; concrete, 0.02.

(*b*) How would the answer to part (*a*) change if the auditorium were filled with 50 people, each with an absorbing power of 0.4. Assume no change in absorption of the floor.

**Solution**

(*a*) We use Sabine's formula, Eq. (*2.19*). The volume of the room is $15 \times 20 \times 10 = 3000\ \text{m}^3$, so $V = 3000$. To get the absorbing power we multiply absorption coefficients by areas:

$$A = 0.6(300) + 0.5(200 + 200) + 0.02(300 + 150 + 150) = 392.$$

Then: $$t_r = 0.16(3000)/392 = 1.22\ \text{s}.$$

(*b*) The only change from part (*a*) is that the absorbing power $A$ is increased by the contribution of the people: $A = 392 + 0.4(50) = 412$. Then: $t_r = 0.16(3000)/412 = 1.17$ s.

**Problem 2.16.** If it were desirable to raise the reverberation time of the auditorium in part (*b*) of Problem 2.15 to 1.70 s, How many $\text{m}^2$ of drapes would need to be removed if the walls behind them were concrete?

**Solution**

For $t_r$ to be 1.70 s, we must have an absorbing power $A$ given by:

$$1.70 = 0.16(3000)/A \Rightarrow A = 282.$$

If $x$ is the number of $\text{m}^2$ of drape that need to be removed, exposing a like amount of concrete, we have, recalling the absorbing power of Problem 2.15(*b*):

$$412 - 0.5x + 0.02x = 282 \Rightarrow 130 = 0.48x \Rightarrow x = 271.$$

Thus, 271 $\text{m}^2$ of the original 400 $\text{m}^2$ of drapes must be removed.

### *Quality and Pitch*

In addition to loudness, humans can distinguish other sound factors related to frequencies and combinations of frequencies of sound. When a note on a musical instrument is played, the fundamental is typically accompanied by various overtones (harmonics, i.e., integer multiples of the fundamental) with differing intensity relative to that of the fundamental. The intensities of the harmonics will vary from instrument to instrument. The sound of harmonics is pleasing to the ear, and while the note is identified by the listener with the fundamental frequency, the same note from different instruments will sound differently as a consequence of the different harmonic content. The time evolution of the note also contributes to the different sounds. These different sound recognitions by the human ear are called the *quality* of the note. The **pitch** of a note is the human perception of the note as "high" or "low" and is closely related to the frequency but is not identical to it. The pitch involves human subjective sense of the sound. While a higher frequency will be perceived as a higher pitch, the same frequency will be perceived as having slightly different pitches when the intensity is changed: higher intensity yields lower pitch. Another difference between frequency and pitch is the perception of simultaneous multiple frequencies. As noted above, when the human ear hears a fundamental and harmonics it perceives the pitch as that of the fundamental.

With regard to musical notes, it is found that certain combinations of notes have particularly pleasing sounds. The frequency of such notes are found to be close whole number ratio to each other. In particular two notes an octave apart are in the ratio of 1 to 2, and such notes are labeled with the same letter. Thus, middle C on the piano, which corresponds to 264 Hz and the C an octave above (C') is 528 Hz while the C an octave below is 132 Hz. Similarly, the notes C, E, G form what is called a major triad in "the key of C", having frequencies in the ratio of 4 to 5 to 6. Their actual frequencies are then: C = 264 Hz, E = 330 Hz, G = 396 Hz. Similarly, F, A, C' and G, B, D' form major triads in the keys of F and G, with actual frequencies: F = 352 Hz, A = 440 Hz, C' = 528 Hz, G = 396 Hz, B = 495 Hz, D' = 594 Hz. All the main piano notes of the C octave can be determined from these triads, recalling that $C = \frac{1}{2}C'$ and $D = \frac{1}{2}D'$. The octave then has seven notes: C, D, E, F, G, A, B. If one starts with D and tries to make a major triad in "the key of D" new notes would be necessary. In general practice five new notes are added to the piano octave in part to address this problem: C#, D#, F#, G#, and A#. The new scale is then: C, C#, D, D#, E, F, F#, G, G#, A, A#, B. In the **diatonic scale** in the key of C, the original seven notes have the same frequencies as given above. A quick check shows that for the original seven, any two adjacent notes are either in the ratio of 9/8 or 10/9 or 16/15. The intervals between adjacent notes that have either of the first two ratios are called whole-note intervals, while those with the last ratio are called half-note intervals. The new notes are placed between those that have the 9/8 or 10/9 ratios, so that all adjacent notes are approximately half-note intervals. Even with the added notes, if one tried to have the major triad in all keys many more notes would be necessary. To avoid this the **equally tempered scale** was created, in which all the twelve notes of the octave are tuned so that the ratio of any two adjacent notes are the same. Since there are twelve notes in the octave the adjacent notes must be in the ratio of the twelfth root of 2, $(2)^{1/12} = 1.05946$, so that C' = 2C, D' = 2D, etc. By agreement the note A is taken as 440 Hz, and all the other notes are then determined. In this scale the notes have slightly different frequencies than in the diatonic scale. The advantage is that for this choice every note has a major triad, while the disadvantage is that the ratios are not exactly 4 to 5 to 6 for any key. In the key of C for example, the new frequencies are: C = 261.6 Hz, E = 329.6 Hz, G = 392.0 Hz, so the ratios are 3.97 to 5 to 5.95. Since the ear finds it more pleasing to have the ratios: 4 to 5 to 6, a piano tuned in the diatonic scale will sound better than the even tempered scale in the keys of C, F and G, but would sound worse in some other keys such as D, E, and A, etc.

## 2.4. OTHER SOUND WAVE PHENOMENA

### *Beats*

In discussing interference of waves we noted that it was necessary to have the same frequency if one was to have interference effects observable. Nonetheless, if we have two frequencies that differ only by a few Hz we can indeed detect "interference" effects that oscillate in time slowly enough to be easily detectable. Consider two sound waves of equal amplitude $A$, and slightly different frequencies, $f_1$ and $f_2$, traveling along the $x$ axis. At a given point in space the actual disturbance of the air molecules from their equilibrium positions can be expressed as: $x = A\cos(2\pi f_1 t) + A\cos(2\pi f_2 t + \phi)$, where $\phi$ is the relative phase of the two waves at some arbitrary instant of time, $t_0$. Since the frequencies are different this relative phase is of no significance since the relative phases of the waves continually change as time goes on. We therefore simplify the mathematics by setting $\phi = 0$. Then: $x = A[\cos 2\pi f_1 t) + \cos 2\pi f_2 t)]$. Using the trigonometric identity: $\cos\theta_1 + \cos\theta_2 = 2\cdot\cos[(\theta_1 - \theta_2)/2]\cdot\cos[(\theta_1 + \theta_2)/2]$, we get:

$$x = 2A\cdot\cos[2\pi t(f_1 - f_2)/2]\cdot\cos[2\pi t(f_1 + f_2)/2] \qquad (2.20)$$

We let $f = (f_1 + f_2)/2$, and $\Delta f = (f_1 - f_2)$. Then $f$ is the average of the two frequencies and is midway between them, while $\Delta f$ is the difference of the two frequencies. Since the frequencies are very close, the last cosine term on the right of Eq. (*2.20*) approximates the oscillation of either original wave, while the other cosine term represents a very slow oscillation at frequency $\Delta f/2$. For example, if $f_1 = 440$ Hz (middle A on a properly tuned keyboard) and $f_2 = 437$ Hz (e.g., middle A on an out of tune keyboard), $\Delta f/2$ would equal 1.5 Hz. Then, in Eq. (*2.20*) the expression: $2A\cdot\cos[2\pi t(f_1 - f_2)/2]$ can be thought of as a slowly varying amplitude for the "average" oscillation at $f = 441.5$ Hz. This variable amplitude reaches two maximal values: $2A$ and $(-2A)$ in each complete cycle. Each will correspond to a maximal loudness in the sound, called a **beat**. Since there are two such beats in each cycle of the $\Delta f/2$ Hz oscillation, the number of beats per second is: $\Delta f = (f_1 - f_2)$ Hz. In other words, the number of beats per second is just the difference of the two frequencies. Note that we have been assuming that $f_1 > f_2$ so that $\Delta f$ is positive. However, it doesn't matter which is larger since $\cos(\theta) = \cos(-\theta)$. Thus, in our analysis we can more generally use $\Delta f = |f_1 - f_2|$. Because beats are most clearly audible as the frequencies are closest they are an excellent vehicle for tuning an instrument against a known standard frequency such as that of a tuning fork.

**Problem 2.17.**

(*a*) A piano tuner is testing middle A on the piano against a standard tuning fork with the exact frequency of 440 Hz. She hears four beats per second, and starts to decrease the tension in the piano cord. The beats increase to five per second. What is the frequency of the cord before and after her adjustment?

(*b*) What must the piano tuner do next to correctly tune the piano?

**Solution**

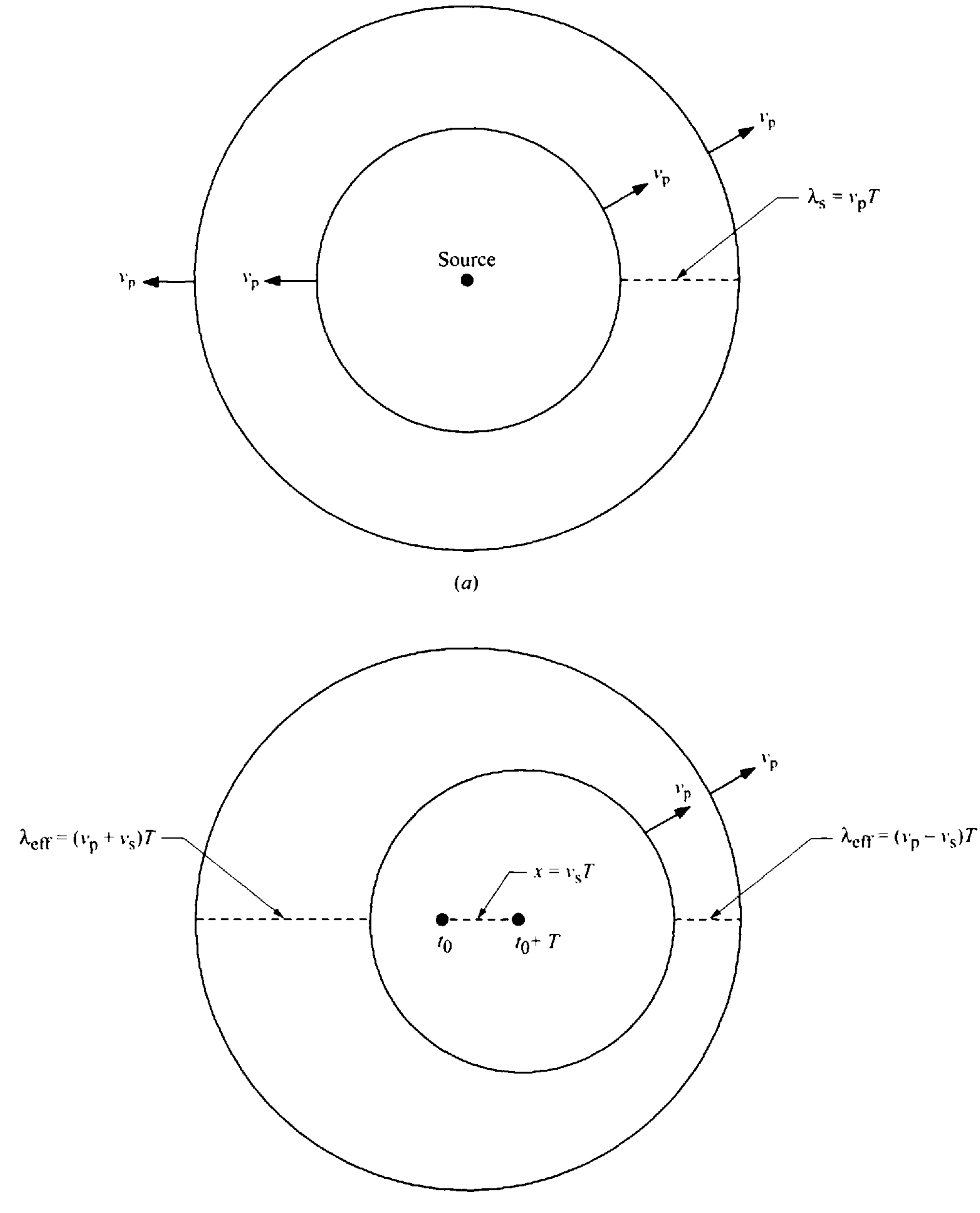
(*a*) The original frequency of the piano cord differed from the 440 Hz by 4 Hz, and was therefore either 444 Hz or 436 Hz. To choose between these two we note that reducing tension in the cord drops the frequency. Since by decreasing tension the number of beats increased, she must have started with the A cord at lower frequency than the tuning fork and it got lower still. It therefore was at 436 Hz to start, and dropped to 435 Hz after the adjustment.

(*b*) She has to increase the tension slowly and listen to a decrease in the number of beats against the tuning fork. When the beats are no longer audible the cord is properly tuned.

### *The Doppler Shift*

We now turn to a phenomenon we all recognize from every day life. When an ambulance or police car with its siren screaming approaches you it appears to have one pitch, but when it passes by and moves away from you the pitch seems to drop noticeably. Clearly the mechanical siren did not change, so what did? This change in pitch is an example of what is called the **Doppler shift** and is caused by motion of the source of a sound wave through the air (as in the example of the siren) or by the motion of the listener through the air. In the case of the source moving, the wave-fronts of successive crests of the sound wave are bunched up in the direction of motion through the air, while they are more separated in the direction opposite to the motion. In either case the crests all move through the air with the characteristic propagation velocity of the medium, $v_p$. Thus, if the source is moving toward (away from) the listener, the listener would detect shorter (longer) wavelengths or higher (lower) frequencies.

The moving source situation is depicted in Fig. 2-7. We consider a source emitting a sinusoidal wave with frequency $f_s = 1/T$. If the source is stationary (relative to the air) as in Fig. 2-7(*a*), two successive positive crests are emitted a time $T$ (one period) apart, say at times $t_0$ and $t_0 + T$. The successive crests move off as spherical wave-fronts traveling at velocity $v_p$, and at some later time would appear as depicted in the figure. The distance between the crests in any radial direction, including to the right and left in the figure, is just the wavelength, which is given as $\lambda_s = v_p T = v_p/f_s$. If, on the other hand, the source were moving to the right with some velocity $v_s$ (typically much smaller than $v_p$) relative to the air, then the second crest emitted at time $t_0 + T$ will be emitted from a location to the right of the one emitted at time $t_0$. The distance between the two points of emission would be $x = v_s T$, as shown in Fig. 2-7(*b*). Once emitted, both wave fronts travel relative to the medium with the characteristic velocity $v_p$, and again spread out as spherical shells, but the shells are of course no longer concentric. It is easy to see that the distance between the crests now depends on the direction in which one is interested. If a listener is off to the right (source traveling directly toward listener) the effective wavelength (crest to crest distance) will be: $\lambda_{eff} = v_p T - v_s T$ since the second crest is closer to the first by the distance $x$ the




(a)

(b)

**Fig. 2-7**

source moved before emitting the second crest. Similarly, if the listener were off to the left (source moving away from listener) the effective wavelength would be $\lambda_{eff} = v_p T + v_s T$, since the second crest was emitted a distance $x$ to the right of the first crest. Combining these two cases, and redefining $v_s$ so that it is positive (negative) when moving away from (toward) the listener, we have:

$$\lambda_{eff} = (v_p + v_s)T = (v_p + v_s)/f_s \qquad (2.21)$$

where $f_s$ is the stationary source frequency and $v_s$ is negative for the source moving toward the listener. It should be noted that as long as the source keeps moving at constant velocity the previous discussion will hold for all succeeding crests and the forward and backward traveling waves will indeed have wavelength $\lambda_{eff}$. Since the waves still travel with velocity $v_p$ the effective frequency is just: $f_{eff} = v_p/\lambda_{eff}$, or $\lambda_{eff} = v_p/f_{eff}$. Substituting for $\lambda_{eff}$ in our formula and shifting terms around we get finally:

$$f_{eff} = [v_p/(v_p + v_s)]f_s \tag{2.22}$$

where $f_{eff}$ is the frequency heard by a stationary listener facing along the line of motion of the source, $f_s$ is the stationary source frequency and $v_s$ is positive (negative) for source moving away from (toward) the listener.

**Problem 2.18.** A fire engine has a siren with a frequency of 1000 Hz. The engine is hurtling down the street at 25 m/s in the direction of a pedestrian standing on the curb. Assume the speed of sound in air is 350 m/s, and that there is no wind blowing.

(*a*) What is the frequency of the siren heard by the pedestrian as the engine approaches?

(*b*) What is the frequency heard by the same pedestrian once the engine has passed by?

**Solution**

(*a*) Using Eq. (*2.22*) and noting that $v_p = 350$ m/s, $v_s = -25$ m/s, $f_s = 1000$ Hz, we get:

$$f_{eff} = [(350 \text{ m/s})/(350 \text{ m/s} - 25 \text{ m/s})](1000 \text{ Hz}) = 1077 \text{ Hz}.$$

(*b*) We again use Eq. (*2.22*), the only difference being that now $v_s = 25$ m/s. Then:

$$f_{eff} = [350/375] \cdot 2000 = 933 \text{ Hz}.$$

We now consider the case where the listener is moving at some speed, $v_L$, relative to the air, toward or away from the source. If the listener moves toward the source the apparent speed with which the crests pass the listener is no longer $v_p$ but $v_p + v_L$. If the listener moves away from the source the corresponding velocity would be $v_p - v_L$. We redefine $v_L$ to be positive or negative for the listener moving toward or away from the source, respectively, so that we can always express the speed of the crests past the listener as: $v_p + v_L$. The wavelength is the distance between successive crests and is not affected by the motion of the listener. The wavelength is either $\lambda_s$ (for a stationary source) or $\lambda_{eff}$ [as given by Eq. (*2.21*)] for a moving source. Considering the more general case of a moving source, the frequency heard by the listener would be: $f_L = (v_p + v_L)/\lambda_{eff}$. Substituting from Eq. (*2.21*) for $\lambda_{eff}$, we get:

$$f_L = [(v_p + v_L)/(v_p + v_s)]f_s \tag{2.23}$$

where $v_L$ is positive (negative) for the listener moving toward (away from) the source, and $v_s$ is positive (negative) for the source moving away from (toward) the listener. The special case of the source not moving is obtained by setting $v_s = 0$ in Eq. (*2.23*). Similarly, the special case of the listener not moving is obtained by setting $v_L = 0$, reproducing Eq. (*2.22*). The use of Eq. (*2.23*) is illustrated in the following problems.

**Problem 2.19.** Consider the case of Problem 2.18, except that now the listener is driving a car initially moving toward the fire engine with a speed of 15 m/s.

(*a*) Find the frequency heard by driver before passing the fire engine.

(*b*) Find the frequency heard by the driver after passing the fire engine.

**Solution**

(*a*) We use Eq. (*2.23*) with $v_s = -25$ m/s (toward listener) and $v_L = 15$ m/s (toward source), and again $f_s = 1000$ Hz and $v_p = 350$ m/s. Then,

$$f_L = [(350 \text{ m/s} + 15 \text{ m/s})/(350 \text{ m/s} - 25 \text{ m/s})](1000 \text{ Hz}) = 1123 \text{ Hz}.$$

(b) Here the only difference in Eq. (2.23) is that both $v_s$ and $v_L$ change signs (away from listener and away from source, respectively):

$$f_L = [(350 - 15)/(350 + 25)](1000 \text{ Hz}) = 893 \text{ Hz}.$$

**Problem 2.20.** Suppose in Problem 2.19 the automobile were moving at the same speed of 15 m/s but this time in the same direction as the fire engine. All else being the same:

(a) Find the frequency heard by the driver before the fire engine overtakes the automobile.

(b) Find the frequency heard by the driver after the fire engine overtakes the automobile.

(c) Suppose after the fire engine passes the automobile, the automobile speeds up to match the speed of the engine. What would be the frequency heard by the driver?

**Solution**

(a) Here the fire engine is traveling toward the listener who is traveling at a slower speed in the same direction, so, $v_s$ is negative (toward listener) while $v_L$ is negative (away from source), so we have from Eq. (2.23):

$$f_L = [(350 \text{ m/s} - 15 \text{ m/s})/(350 \text{ m/s} - 25 \text{ m/s})](1000 \text{ Hz}) = 1031 \text{ Hz}.$$

(b) Here the fire engine has passed the listener who is now following the fire engine at the slower speed of the automobile. Now $v_s$ is positive (away from listener), while $v_L$ is also positive (toward source), so we get:

$$f_L = [(350 + 15)/(350 + 25)](1000 \text{ Hz}) = 973 \text{ Hz}.$$

(c) We again apply Eq. (2.23), with the same sign conventions for $v_s$ and $v_L$ as in part (b). The only difference is that $v_l$ is now 25 m/s. Then:

$$f_L = [(350 + 25)/(350 + 25)](1000 \text{ Hz}) = 1000 \text{ Hz},$$

the actual frequency of the source.

Note that the answer to Problem 2.20(c) is a general result: If the source and listener are both moving in the same direction with the same speed, the listener hears the actual frequency of the source. The Doppler shift occurs in any medium in which waves travel and a comparable phenomenon occurs with light waves, although the formulas for light are somewhat different.

### *Shock Waves*

In the Doppler shift we assumed that the velocity of the source (or listener) is less than the velocity of propagation of the wave through the medium. There are circumstances where that is not the case, such as the travel of a **supersonic** (faster than the speed of sound) jet aircraft (SST). When supersonic motion occurs a compressional wave, due to the object cutting through the air, is emitted by the traveling body and forms what is called a shock wave. The shock wave moves at a specific angle relative to the direction of motion of the object through the air, and can sometimes be of sufficient intensity to cause a loud booming sound, as in the case of an SST. To understand this phenomenon we consider an object moving to the right at supersonic speed $v$ through the air. As the object passes any point the disturbance of the air at that point expands out in a spherical ripple. Since the object travels faster than sound it is always beyond the shell of any previous ripple. This is shown in Fig. 2-8. The object is shown at its location at time $t_3$ while the ripples from earlier times $t_1$, and $t_2$ are also shown. We draw a tangent line to the emitted ripples to get the wave-front of the shock wave, which makes an angle $\theta$ with the direction of motion. If $R$ is the radius of the ripple starting at time $t_1$, after a time $(t_3 - t_1)$ has elapsed, and $x$ is the distance the object has moved in that time interval, we can see from the figure that

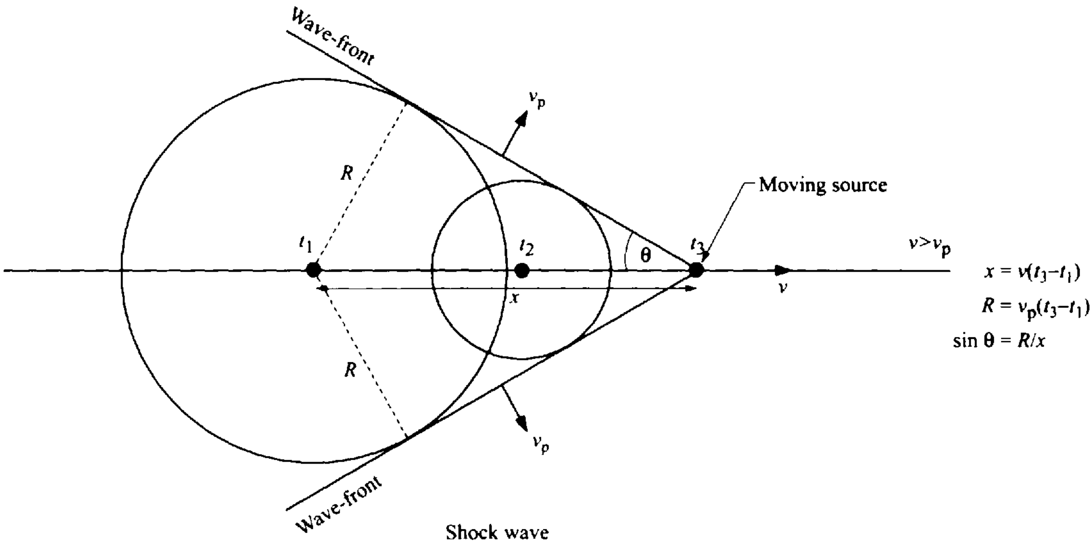



**Fig. 2-8**

$R/x$ is the ratio of the opposite side to the hypotenuse of a right triangle with angle $\theta$ as shown. Then:

$$R/x = v_p/v = \sin\theta \tag{2.24}$$

The direction of propagation of the shock wave is perpendicular to the wave-front and makes an angle $(90° - \theta)$ to the direction of motion of the object. Shock waves accompany speeding bullets as well as SSTs, and an example in a medium other than air is the bow wave of a speed boat in water.

**Problem 2.21.**

(*a*) A supersonic airliner creates a shock wave whose wave-front makes an angle of 65° with its direction of motion through the air. Find the speed of the airliner, assuming the speed of sound in air is $v_p = 350$ m/s.

(*b*) A bullet moves through the air at 2000 m/s. Find the angle the shock wave makes with the direction of motion of the bullet.

**Solution**

(*a*) We have $v_p/v = \sin\theta \Rightarrow$

$$v = v_p/\sin\theta = (350 \text{ m/s})/\sin(65°) = 386 \text{ m/s (or 869 mph)}$$

(*b*) $v_p/v = \sin\theta \Rightarrow \sin\theta = 350/2000 = 0.175 \Rightarrow \theta = 10.1°$.

# Problems for Review and Mind Stretching

**Problem 2.22.** Find the expression for the velocity of propagation of a sound wave [replacing Eq. (*2.15*)] if the compression were isothermal rather than adiabatic.

**Solution**

From the ideal gas law, $PV$ is constant for constant $T$, so: $(P + \Delta P)(V + \Delta V) = PV$, which results in: $PV + P\Delta V + V\Delta P + \Delta P\Delta V = PV$. Canceling like terms on both sides and noting that for infinitesimal changes in $P$ and $V$ the term $\Delta P \Delta V$ is negligible compared to the others, we get: $P\Delta V = -V\Delta P$ or: $\Delta P/\Delta V = -P/V$. Then, from Eq. (*2.10*) the bulk modulus becomes: $B = P$. Comparing to Eq. (*2.14*) and recalling Eq. (*2.9*), Eq. (*2.15*) changes to: $v_p = [RT/M]^{1/2}$.

**Problem 2.23.** In the equally tempered scale

(*a*) find the frequencies of the following notes: middle G, B, D, and D′ (one octave above D) and

(*b*) What are the ratios in the major triad G to B to D′? [express as $x:5:y$]

**Solution**

(*a*) Recalling that middle A = 440 Hz, we have: $G = A/[(2)^{1/12}]^2 = 440/(1.059{,}46)^2 = 392$ Hz; $B = A \cdot [(2)^{1/12}]^2 = 493.9$ Hz; $D = G/[(2)^{1/12}]^5 = 392/1.334{,}84 = 293.7$ Hz: $D' = 2D = 587.4$.

(*b*) $x/5 = G/B = 392/493.9 \Rightarrow x = 3.97$; $5/y = B/D' = 493.9/587.4 = 0.7937 \Rightarrow y = 5.95$. Thus, G to B to D = 3.97 to 5 to 4.95.

*Note.* This is the same as any other major triad ratio in the equally tempered scale since all the adjacent notes are related by the same multiplicative factor.

**Problem 2.24.** Two pianos are tuned to different scales. One is the equally tempered scale and the other is the diatonic scale in the key of C.

(*a*) What is the beat frequency when the note G is struck on both pianos?

(*b*) What is the beat frequency when the note G′ is struck on both pianos?

**Solution**

(*a*) From the text, G on the diatonic scale is 396 Hz, while from Problem 2.23, G on the tempered scale is 392 Hz. Since the beat frequency is just the frequency difference between the two notes, we have: beat frequency = 4 Hz.

(*b*) Since in either scale G′ = 2G for an octave shift, the difference in frequencies is also doubled, so the beat frequency is now 8 Hz.

**Problem 2.25.** Consider Problem 2.19, with the fire engine traveling down the street at 25 m/s toward the car which is traveling toward the engine at 15 m/s. As before, the engine's 1000 Hz siren is blaring. Suppose a wind of 10 m/s were blowing in the direction from the engine to the car, all else being the same.

(*a*) What is the frequency of the siren heard by the listener in the car as the engine approaches?

(*b*) What is the frequency of the siren heard by the listener once the engine has passed by?

(*c*) How would part (*a*) change if the wind velocity was 30 m/s (gale force wind).

**Solution**

(*a*) We still can use Eq. (*2.23*) if we recall that the velocities $v_s$ and $v_L$ in that equation represent the velocities of source and listener, respectively, relative to the medium in which the sound travels, namely the air. In Problem 2.19 the air was assumed to be still so the velocities relative to the ground and those relative to the air were the same. This is no longer the case when the mass of air is moving at 10 m/s from fire engine toward car. In our present case we must consider the velocities as seen from the frame of reference at rest relative to the air. In that frame of reference the fire engines velocity relative to the air is still pointed toward the listener, so, with our sign convention: $v_s = -(25\text{ m/s} - 10\text{ m/s}) = -15$ m/s. The listener's velocity relative to the air is still toward the source, so: $v_L = (15\text{ m/s} + 10\text{ m/s}) = 25$ m/s. $v_p$, the velocity of propagation in sound waves in air is, of course unchanged, so

$v_p = 350$ m/s. Substituting into Eq. (2.23) we get:
$f_L = [(350\text{ m/s} + 25\text{ m/s})/(350\text{ m/s} - 15\text{ m/s})](1000\text{ Hz}) = 1119$ Hz.

(*b*) The only difference in Eq. (2.23) is that both $v_s$ and $v_L$ change signs (away from listener, away from source), so:

$$f_L = [(350 - 25)/(350 + 15)](1000\text{ Hz}) = 890\text{ Hz}.$$

(*c*) The only difference is that the fire engine would appear to be going "backward" relative to the air, so $v_s$ would be positive. This automatically comes out of the equation: $v_s = -(25\text{ m/s} - 30\text{ m/s}) = +5$ m/s. $v_L$ is still toward the engine and so: $v_L = (15\text{ m/s} + 30\text{ m/s}) = 45$ m/s. Then: $f_L = [(350 + 45)/(350 + 5)](1000\text{ Hz}) = 1113$ Hz.

**Problem 2.26.** A jeep travels in a canyon at a speed of 10 m/s perpendicular to the parallel cliff walls that form the canyon boundary. The jeep blows its 200 Hz horn as it passes the midpoint between the cliffs. An observer at rest on the canyon floor has an instrument which joins the two echoes into a single wave signal and re-emits their combined sound. What is the beat frequency heard from the instrument?

**Solution**

The two cliffs detect the same frequencies that a listener at rest near each cliff would hear. When the sound reflects off these cliffs it reflects the same frequency that hit the cliffs. For cliff 1, in front of the jeep, we have a Doppler shift in frequency given by Eq. (2.23) with $v_L = 0$ and $v_s = -10$ m/s. Then, the reflected frequency for an observer at rest in the canyon is: $f_{1L} = (350\text{ m/s})/(350\text{ m/s} - 10\text{ m/s})(200\text{ Hz}) = 206$ Hz. For the other cliff, behind the jeep we again use Eq. (2.23) with $v_L = 0$, but now $v_s = 10$ m/s, so the reflected frequency is: $f_{2L} = 350/(350 + 10)(200\text{ Hz}) = 194$ Hz. The beat frequency heard is therefore $206 - 194 = 12$ Hz.

## Supplementary Problems

**Problem 2.27.** Using the fact that $(6.5)^{1/2} = 2.5495$ and $(6.5)^3 = 274.625$:

(*a*) Find the value of $(6.5)^{1/4}$.

(*b*) Find the value of $(6.5)^{3.5}$.

(*c*) Find the value of $(6.5)^{3/2}$.

(*d*) Find the value of $(6.5)^{3.25}$.

*Ans.* (*a*) 1.5967; (*b*) 700.16; (*c*) 16.572; (*d*) 438.49

**Problem 2.28.** Reduce the following expressions to most simplified terms:

(*a*) $\log [x^3 \cdot y^{-1/2}/z^n]$; (*b*) $\ln [x^{y+z}/\exp (yz)]$.

*Ans.* (*a*) $3 \log x - (\frac{1}{2})\log y - n \log z$; (*b*) $y \ln x + z \ln x - yz$

**Problem 2.29.** Suppose one defines a logarithmic function to an arbitrary base $a$: $\log_a (x)$, where $a$ is a positive number.

(*a*) Find an expression for $\log_a (x)$ in terms of $\log (x)$ [*Hint*: See Problem 2.5].

(*b*) Find the value of $\log_{100} (2)$.

*Ans.* (*a*) $\log_a (x) = \log_a (10) \cdot \log (x)$; (*b*) $(\frac{1}{2}) \log (2)$

**Problem 2.30.** The velocity of sound in $CO_2$ at 300 K is found to be 270 m/s. Find the ratio of specific heats, $\gamma$.

*Ans.* 1.29

**Problem 2.31.** A spherical sinusoidal sound wave has an intensity of $I = 0.0850\ \text{W/m}^2$ at a distance of $r = 2.0$ m from the source.

(*a*) Find the intensity of the wave at $r = 3.0$ m and $r = 4.0$ m.

(*b*) Find the total power transmitted by the spherical wave at $r = 2.0$ m, $r = 3.0$ m and $r = 4.0$ m.

*Ans.* (*a*) 0.0378 $\text{W/m}^2$, 0.0213 $\text{W/m}^2$; (*b*) 4.27 W for all three cases.

**Problem 2.32.** If the amplitude of the wave in Problem 2.31 is $A = 0.450$ mm at $r = 1.0$ m, find the amplitude at $r = 2.0$ m, 3.0 m, and 4.0 m.

*Ans.* 0.225 mm, 0.150 mm, 0.113 mm

**Problem 2.33.** Redo Problem 2.32 for the case of circular sinusoidal ripples in on the surface of a pond.

*Ans.* 0.318 mm, 0.260 mm, 0.225 mm

**Problem 2.34.** A sinusoidal plane-wave traveling through the air in the $x$ direction has an intensity $I = 0.0700$ $\text{W/m}^2$ and an amplitude $A = 0.0330$ mm. The density of air is 1.17 $\text{kg/m}^3$ and the velocity of propagation is $v_p = 350$ m/s.

(*a*) Find the frequency of the wave [*Hint*: See Problem 2.11].

(*b*) Find the energy passing through a 3.0 cm by 4.0 cm rectangular window parallel to the $(y \cdot z)$ plane in a 15 s time interval.

*Ans.* (*a*) 89.2 Hz; (*b*) $1.26 \cdot 10^{-3}$ J

**Problem 2.35.**

(*a*) A woman faces a cliff and wishes to know how far away it is. She calls out and hears her echo 4.0 s later. If the speed of sound is 350 m/s how far is she from the cliff?

(*b*) A surface ship uses sonar waves (high frequency sound waves) emitted below the water line, to locate submarines or other submerged objects. Testing the sonar using an object at a known distance of 4000 m, the time interval between emission of the signal and its return is found to be 5.52 s. What is the speed of sound in seawater?

*Ans.* (*a*) 700 m; (*b*) 1450 m/s

**Problem 2.36.** A person at a distance of 1200 m from an explosion hears an 80 db report. How close would the person have been to the explosion if the report were just at the threshold of pain?

*Ans.* 12.0 m.

**Problem 2.37.** A point source emits three distinct frequencies of 100 Hz, 1000 Hz and 10,000 Hz, each with the same power level. A student whose threshold of hearing is 0 db at 1000 Hz can just barely make out the 1000 Hz signal at a distance of 60 m from the source, but cannot hear the other tones at that distance. As the student moves closer to the source she first detects the 10,000 Hz signal at 2.0 m from the source, and first detects the 100 Hz signal at 25 cm from the source.

(*a*) What is the student's threshold of hearing at 10,000 Hz?

(*b*) What is the student's threshold of hearing at 100 Hz?

*Ans.* (*a*) 29.5 db; (*b*) 47.6 db

**Problem 2.38.** A classroom has dimensions $h = 5.0$ m, $w = 10$ m, $l = 7.0$ m, and the reverberation time of the classroom when empty is 1.80 s.

(*a*) Find the absorbing power of the room.

(*b*) If a class of 20 students with 1 professor comes into the room, find the new reverberation time of the room. [Assume each person contributes an additional absorbing power of 0.4.]

*Ans.* (*a*) 31.1; (*b*) 1.42 s

**Problem 2.39.** For the classroom of Problem 2.38(*a*), assume the absorption coefficient for the walls is 0.05 and the absorbing power of the floor and furniture is 8.0. What is the absorption coefficient of the ceiling?

*Ans.* 0.21

**Problem 2.40.** A specially constructed room has ceiling, floor and one pair of opposite walls acting as near-perfect sound reflectors (absorption coefficient zero). The remaining two opposite walls, of areas 30 $m^2$ each and separated by a distance of 6.0 m, have absorption coefficients of 0.50. Assume that a single pulse of sound is emitted from one of these walls toward the other at 80 db, and assume that the pulse reflects back and forth between the two walls as a plane-wave.

(*a*) Do a direct calculation of the reverberation time for this case (i.e., the time for the sound level to drop to 20 db). Assume the speed of sound is 350 m/s.

(*b*) What is the result one gets from Sabine's formula [Eq. (*2.19*)]?

(*c*) Explain the discrepancy?

*Ans.* (*a*) 0.34 s; (*b*) 0.96 s; (*c*) plane-wave model assumes shortest possible time between absorptions; realistically, much of the sound energy will bounce one or more times from the perfect reflecting surfaces for each reflection from the absorbing surfaces, so a longer time is involved

**Problem 2.41.**

(*a*) Show that in the diatonic scale in the key of C, the seven notes C, D, E, F, G, A, B indeed satisfy the condition that the ratios of frequencies of adjacent notes are either 9/8 or 10/9 or 16/15.

(*b*) If the same seven notes are tuned to the equally tempered scale, what are the deviations of the frequencies above (or below) the diatonic scale values, in Hz and in percent of frequency?

*Ans.* (*a*) D/C = 9/8, E/D = 10/9, F/E = 16/15; G/F = 9/8, A/G = 10/9, B/A = 9/8, C′/B = 16/15; (*b*) C: (2.4 or 0.9%), D: (3.3 or 1.1%), E: (0.4 or 0.12%), F: (2.8 or 0.8%), G: (4.0 or 1.0%), A: 0, B: (1.1 or 0.2%)

**Problem 2.42.** Using the results of Problem 2.41(*b*):

(*a*) How many beats would one hear if D on the diatonic and even tempered scales were played simultaneously?

(*b*) If one wished to tune the even tempered D note to the diatonic scale, would one increase or decrease the tension in the piano string?

*Ans.* (*a*) 3.3; (*b*) increase

**Problem 2.43.** When tuning fork A is struck at the same time as tuning fork B the beat frequency is 3 Hz. When tuning fork B is struck at the same time as tuning fork C the beat frequency is 5 Hz.

(*a*) When tuning fork A is struck at the same time as tuning fork C what beat frequency is expected?

(*b*) If tuning fork C has the lowest of the three frequencies, at 300 Hz, what are the possible frequencies of tuning forks A and B?

*Ans.* (*a*) 8 Hz or 2 Hz; (*b*) B is 305 Hz and A is either 308 Hz or 302 Hz.

**Problem 2.44.** An ambulance with siren blowing travels at 20 m/s toward a stationary observer who hears a frequency of 1272.7 Hz. Assume the speed of sound in air is 350 m/s.

(*a*) What is the actual frequency of the siren?

(*b*) What is the frequency heard by the observer once the ambulance has passed?

*Ans.* (*a*) 1200 Hz; (*b*) 1135 Hz

**Problem 2.45.** An observer travels toward a stationary whistle of frequency 1200 Hz at a speed of 20 m/s. Assume the speed of sound in air is 350 m/s.

(*a*) What is the frequency heard by the observer?

(*b*) What is the frequency heard by the observer after passing the whistle?

*Ans.* (*a*) 1268.6 m/s; (*b*) 1131 Hz

**Problem 2.46.** An observer travels north at 10 m/s and sees an ambulance traveling south at 10 m/s with siren blowing. The actual frequency of the siren is 1200 Hz. Assume the speed of sound in air is 350 m/s.

(*a*) What is the frequency heard by the observer?

(*b*) What is the frequency heard by the observer after passing the ambulance?

*Ans.* (*a*) 1270.6 Hz; (*b*) 1133 Hz

**Problem 2.47.** An automobile travels along a road parallel to railroad tracks at 60 ft/s. A train coming from behind is traveling at 90 ft/s and blows its whistle. After the train passes the automobile it blows its whistle again. A passenger in the car notes that the drop in frequency in the sound of the whistle from before passing the automobile to after is 80 Hz. Assume the speed of sound in air is 1100 ft/s.

(*a*) Find the actual frequency of the whistle.

(*b*) What are the frequencies heard by the passenger before and after the train passes?

*Ans.* (*a*) 1457 Hz; (*b*) 1500 Hz (before), 1420 Hz (after)

**Problem 2.48.** A supersonic airplane is heading due south and shock waves are observed to propagate in the directions 70° east of south and 70° west of south. The speed of sound in air is 350 m/s.

(*a*) What is the angle made by the shock wave-fronts with the direction of travel of the airplane?

(*b*) What is the speed of the airplane?

*Ans.* (*a*) 20°; (*b*) 1023 m/s

**Problem 2.49.** A bullet travels at five times the speed of sound.

(*a*) What is the angle that the shock wave-front makes with the direction of travel of the bullet?

(*b*) If the angle were half that value how many times faster than sound is the speed of the bullet?

*Ans.* (*a*) 11.5°; (*b*) ten