Chapter 17

Transfer of Heat

17.1 CONDUCTION

In Chap. 15 we saw that heat is the nonmechanical transfer of energy. In examples we gave of objects at different temperatures being brought into contact, heat was transferred via the direct contact of more energetic, "hotter," layers of molecules with less energetic, "cooler," layers of molecules. This direct transfer of thermal energy from one layer of molecules to the next is called **heat conduction**. By definition, heat conduction is a nonequilibrium process, since it involves temperature differences between objects or between parts of a single object. The rate at which heat is conducted through an object is found to obey definite laws. Consider a bar of a uniform material of length L and cross-sectional area A, as shown in Fig. 17-1. Assume the two end faces are maintained at temperatures T_1 and T_2, respectively, where T_1 is the higher temperature. We assume that the long sides of the bar are well insulated so that heat cannot leak out the sides. We also assume a "steady-state" situation where the heat conducted through any given cross section of the bar will be the same as at any other cross section so that there is no accumulation of thermal energy at any location in the bar. The amount of heat transferred per unit time, $H = \Delta Q/\Delta t$, across a given cross section of the bar is directly proportional to the temperature difference $T_1 - T_2$ and to the area A and is inversely proportional to the length L:

$$H = \frac{kA(T_1 - T_2)}{L} \tag{17.1}$$

where k is a proportionality constant that is different for each material and is called the **coefficient of thermal conductivity** (or **conductivity**, for short). Since the SI unit of H is J/s = W, the SI unit for k is W/(m · K). In Table 17.1, we give the conductivities of a number of substances at room temperature.

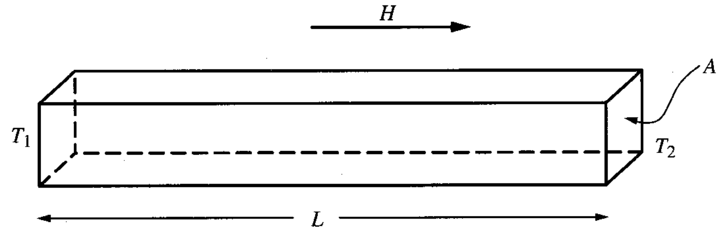

Fig. 17-1

Note that metals generally have larger conductivities than other solids and are therefore called good heat "conductors." The other materials listed clearly don't conduct heat well; they are called insulators. Stagnant liquids conduct heat to some extent, whereas stagnant gases are generally good insulators.

Problem 17.1. In constructing a calorimeter, one wants the container to be made of a substance that will allow the temperature of the container and its contents to quickly come to equilibrium. On the other hand, one doesn't want heat to be gained or lost to the surroundings during a calorimeter experiment. How would you design such a calorimeter?

Table 17.1. Coefficients of Thermal Conductivity

Substance	k, W/(m · K)
Metals:	
Aluminum	205
Brass	109
Copper	385
Silver	406
Steel	50
Nonmetallic solids:	
Brick	0.7
Concrete	0.9
Cork	0.04
Glass	0.80
Rockwool	0.04
Styrofoam	0.01
Wood	0.10

Solution

The container should be made of metal so that heat rapidly flows through it, helping to bring it and its contents to a common temperature. To avoid heat losses, the container should be surrounded (including a cover) with insulating material, such as Styrofoam, which assures that heat leaks, in or out, occur very slowly.

Problem 17.2.

(*a*) Assume that the bar in Fig. 17-1 is made of copper and that $A = 100\ \text{cm}^2$ and $L = 60$ cm. Find H, the heat flowing per unit time, if $T_1 = 600$ K and $T_2 = 300$ K.

(*b*) How would the answer change if A was doubled, L was tripled, and the temperatures stayed the same?

Solution

(*a*) From Eq. (17.1) and Table 17.1 we get

$$H = \frac{[385\ \text{W}/(\text{m}\cdot\text{K})]\,(100\times10^{-4}\ \text{m}^2)\,(600\ \text{K} - 300\ \text{K})}{0.60\ \text{m}} = 1925\ \text{W}$$

(*b*) If A doubles, so does H, and if L triples, H drops to one-third its original value, so combining these two effects, we see that H changes to two-thirds its original value: $H = 2/3\ (1925\ \text{W}) = 1283$ W.

Problem 17.3. For the situation of Problem 17.2(*a*), find the temperature T' of the bar 20 cm from the left end.

Solution

Consider a cross section of the bar 20 cm from the left end. In a steady-state situation, T' remains constant as long as the end-face temperatures T_1 and T_2 are held fixed. We apply (*17.1*) to the part of the bar between the left end and the 20-cm position.

$$1925\ \text{W} = \frac{[385\ \text{W(m}\cdot\text{K)}]\,(100\times10^{-4}\ \text{m}^2)\,(600\ \text{K} - T')}{0.20\ \text{m}}$$

While we could solve this equation for T', it is more informative to note that, since the values of k, A, and H are the same as in Problem 17.2(a), we must have

$$\frac{T_1 - T_2}{0.60\ \text{m}} = \frac{T_1 - T'}{0.20\ \text{m}} \qquad \text{or} \qquad T_1 - T' = \tfrac{1}{3}(T_1 - T_2) = 100\ \text{K}$$

Thus 600 K $-$ T' = 100 K, so T' = 500 K.

Note. More generally, from (17.1) the steady-state temperature drops linearly (from T_1 to T_2) with distance along the rod.

Problem 17.4. Two slabs α and β, of equal cross section A = 80 cm^2 but made of different materials, are in close contact, as shown in Fig. 17-2. The left side of α and the right side of β are kept at temperatures T_1 = 600 K and T_2 = 300 K, respectively. The lengths of the slabs are L_α = 20 cm and L_β = 30 cm. If slab α is made of steel and slab β is made of copper, find (a) the temperature at the interface between the two slabs, (b) the rate at which heat is transferred across the slabs.

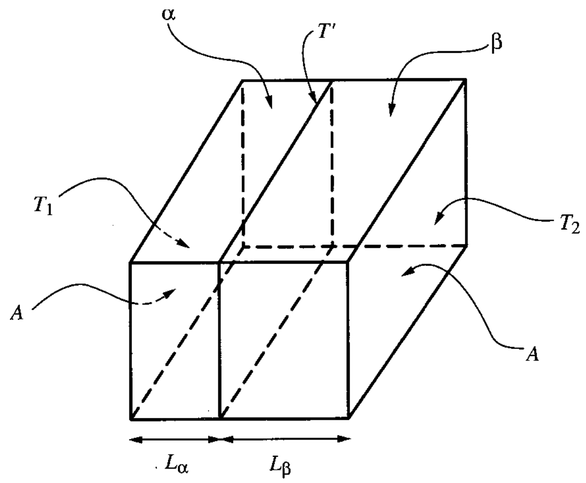

Fig. 17-2

Solution

(a) In steady state the heat flow H must be the same in each slab. Thus

$$\frac{k_\alpha(T_1 - T')A}{L_\alpha} = \frac{k_\beta(T' - T_2)A}{L_\beta} \tag{i}$$

Substituting in values from Table 17.1 and the data from the problem, we get

$$\frac{[50\ \text{W/(m}\cdot\text{K)}]\,(600\ \text{K} - T')}{0.20\ \text{m}} = \frac{[385\ \text{W/(m}\cdot\text{K)}]\,(T' - 300\ \text{K})}{0.30\ \text{m}}$$

Solving for T', we get T' = 349 K.

(*b*) Now that we have T' we can obtain H by using (*17.1*), $H = kA(T_1 - T_2)/L$, for either slab. For slab α, we have

$$\frac{[50\ \mathrm{W/(m \cdot K)}]\,(80 \times 10^{-4}\ \mathrm{m^2})\,(600\ \mathrm{K} - 349\ \mathrm{K})}{0.20\ \mathrm{m}} = 502\ \mathrm{W}$$

[As a check we can apply (*17.1*) to slab β.]

Problem 17.5. Equation (*17.1*) can be reexpressed as

$$\frac{H}{A} = \frac{T_1 - T_2}{L/k}$$

If we set $T_1 - T_2 \equiv \Delta T$, and define $L/k \equiv R$, we have $H/A = \Delta T/R$. The quantity R is called the **R-factor** of the slab.

(*a*) Find the SI units of R.

(*b*) Show that for a wall made up of multiple layers with R-factors $R_1, R_2, R_3, \ldots$, the rate of heat transfer is given by

$$\frac{H}{A} = \frac{\Delta T}{R} \qquad (i)$$

where ΔT is the total temperature difference between the two sides of the wall, and $R = R_1 + R_2 + R_3 + \cdots$.

Solution

(*a*) The units of R are those of $L/k = \mathrm{m/[W/(m \cdot K)]} = \mathrm{m^2 \cdot K/W}$.

(*b*) Consider a wall made up of three layers for definiteness. If ΔT_1, ΔT_2, and ΔT_3 represent the temperature differences across the respective layers, then

$$\frac{H}{A} = \frac{\Delta T_1}{R_1} \qquad \frac{H}{A} = \frac{\Delta T_2}{R_2} \qquad \frac{H}{A} = \frac{\Delta T_3}{R_3} \qquad (ii)$$

Since $\Delta T = \Delta T_1 + \Delta T_2 + \Delta T_3$, we get

$$\Delta T = \frac{H}{A} R_1 + \frac{H}{A} R_2 + \frac{H}{A} R_3 = \frac{H}{A}(R_1 + R_2 + R_3)$$

Setting $R \equiv R_1 + R_2 + R_3$, we get $\Delta T = (H/A)R$, which implies that $H/A = \Delta T/R$. It is easy to generalize to any number of layers.

Problem 17.6.

(*a*) Find the R-factor for each of the slabs in Problem 17.4.

(*b*) Find the R-factor for the combination of both slabs.

(*c*) Redo (*b*) of Problem 17.4 using the above result.

Solution

(*a*) For slab α,

$$R_\alpha = \frac{L_\alpha}{k_\alpha} = \frac{0.20\ \mathrm{m}}{50\ \mathrm{W/(K \cdot m)}} = 4.0 \times 10^{-3}\ \mathrm{m^2 \cdot K/W}$$

Similarly, $$R_\beta = \frac{L_\beta}{k_\beta} = \frac{0.30\ \text{m}}{385\ \text{W/(K}\cdot\text{m)}} = 7.79 \times 10^{-4}\ \text{m}^2\cdot\text{K/W}$$

(*b*) $R = R_\alpha + R_\beta = 4.78 \times 10^{-3}\ \text{m}^2\cdot\text{K/W}$.

(*c*) $$H = A\frac{\Delta T}{R} = \frac{(80 \times 10^{-4}\ \text{m}^2)(300\ \text{K})}{4.78 \times 10^{-3}\ \text{m}^2\cdot\text{K/W}} = 502\ \text{W}$$

as before.

Problem 17.7. A wall of a house of area 25 m^2, is made up of three layers: 5.0 cm of wood, 4.0 cm of rockwool, and 2.0 cm of gypsum board [$k = 0.06$ W/(m · K)].

(*a*) Find the *R*-value of the wall.

(*b*) Find the rate of heat loss through the wall when the temperature difference between the inside and outside surfaces are 80 K.

Solution

(*a*) $$R = R_1 + R_2 + R_3 = \frac{0.050\ \text{m}}{0.10\ \text{W/(m}\cdot\text{K)}} + \frac{0.040\ \text{m}}{0.04\ \text{W/(m}\cdot\text{K)}} + \frac{0.020\ \text{m}}{0.06\ \text{W/(m}\cdot\text{K)}} = 1.83\ \text{m}^2\cdot\text{K/W}$$

(*b*) $$H = A\frac{\Delta T}{R} = \frac{(25\ \text{m}^2)(80\ \text{K})}{1.83\ \text{m}^2\cdot\text{K/W}} = 1093\ \text{W}$$

Problem 17.8. *R*-values in the United States are often given in "common" units: $R = \text{ft}^2\cdot{}^\circ\text{F/(Btu/h)}$.

(*a*) The *R*-value for a concrete wall is $R = 1.4\ \text{ft}^2\cdot{}^\circ\text{F/(Btu/h)}$. If the wall has an area of $A = 120\ \text{ft}^2$, and the temperature of the outside (colder surface) of the wall is $T_1 = 20^\circ\text{F}$, find the temperature of the inside surface of the wall T_2 if the rate of heat transfer across the wall is $H =$ 12,000 Btu/h.

(*b*) How many watts does *H* correspond to?

(*c*) Find the conversion from common to SI units for *R*.

Solution

(*a*) $H = A\,\Delta T/R \Rightarrow 12{,}000\ \text{Btu/h} = (120\ \text{ft}^2)\Delta T/(1.4\ \text{ft}^2\cdot{}^\circ\text{F}\cdot\text{h/Btu}) \Rightarrow \Delta T = 140^\circ\text{F} = T_2 - T_1 = T_2 - 20^\circ\text{F} \Rightarrow T_2 = 160^\circ\text{F}$.

(*b*) Since there are 252 cal/Btu and 4.184 J/cal, we get

$$H = (12{,}000\ \text{Btu/h})(252\ \text{cal/Btu})(4.184\ \text{J/cal}) = 1.265 \times 10^7\ \text{J/h}$$

Converting hours to seconds we get

$$H = \frac{1.265 \times 10^7\ \text{J/h}}{3600\ \text{s/h}} = 3515\ \text{J/s} = 3515\ \text{W}$$

(*c*) $$\text{ft}^2\cdot{}^\circ\text{F}\cdot\text{h/Btu} = \frac{(0.305\ \text{m})^2\left(\frac{5}{9}{}^\circ\text{C}\right)(3600\ \text{s})}{1054\ \text{J}} = 0.176\ \text{m}^2\cdot\text{K/W}$$

17.2 CONVECTION

Convection is a mechanism for the transfer of thermal energy that applies to fluids (liquids and gases). Unlike, conduction, where there is no macroscopic migration of molecules, in convection the thermal energy is transferred by the motion of material from one place to another. In convection the molecules of a gas or liquid pass a hot surface and gain thermal energy. The hot gas or liquid then travels to another location where the thermal energy is deposited in a cooler environment. Convection is the mechanism for heat transfer in the hot-air and hot-water home heating systems. If the circulation of the fluid is aided by a fan or pump, it is called *forced* convection. If the circulation is a consequence of the natural difference in density of the fluid (caused by a temperature difference) at different locations, it is called *natural* convection. An example of natural convection is the rising of the hot air near a steam radiator and its circulation through the room. Here, the hot fluid is less dense than the surrounding cooler fluid and so rises by buoyancy.

The rate at which a convective fluid removes heat from a hot surface and transfers it to the bulk of the fluid depends on the geometry of the surface and involves much more complicated expressions than does conduction. Nonetheless, to a good approximation the rate H of convective heat flow is proportional to the area A of the contact surface and to the temperature difference ΔT between the surface and the bulk of the fluid away from the surface. Thus,

$$H = hA\,\Delta T \tag{17.2}$$

where h, the **coefficient of convection**, depends on the fluid, the geometry, and a variety of other factors (including a slight dependence on ΔT). By *(17.2)* h has the same units as k. The following listing of hot surfaces and their corresponding h values applies to natural convection of air at atmospheric pressure.

1. Horizontal plate with air passing the top surface:
$$h = 2.49\,(\Delta T)^{1/4}\ \mathrm{W/(m^2\cdot K)}$$
2. Horizontal plate with air passing the bottom surface:
$$h = 1.31\,(\Delta T)^{1/4}\ \mathrm{W/(m^2\cdot K)}$$
3. Vertical plate:
$$h = 1.77\,(\Delta T)^{1/4}\ \mathrm{W/(m^2\cdot K)}$$
4. Horizontal or vertical pipe of diameter D:
$$h = 4.19\left(\frac{\Delta T}{D}\right)^{1/4}\ \mathrm{W/(m^2\cdot K)}$$

In the formulas, ΔT and D are treated as pure numbers, with ΔT the numerical value in K or °C and D the numerical value in cm.

Problem 17.9. A bathroom is heated by a floor-to-ceiling steam pipe that is 10 cm in diameter. The ceiling height is $L = 3.0$ m and the temperature of the bulk of air in the room is 22°C. If the pipe surface is at 90°C, what is the rate of convective heat transfer?

Solution

Use *(17.2)* and the fourth formula above.

$$A = \pi DL = 0.942\ \mathrm{m^2} \qquad \Delta T = 90\,^\circ\mathrm{C} - 22\,^\circ\mathrm{C} = 68\ \mathrm{K}$$

$$h = 4.19\left(\tfrac{68}{10}\right)^{1/4} = 6.77\ \mathrm{W/(m^2\cdot K)} \qquad H = [6.77\ \mathrm{W/(m^2\cdot K)}]\,(0.942\ \mathrm{m^2})\,(68\ \mathrm{K}) = 434\ \mathrm{W}$$

Problem 17.10. The air outside a 1.50-m^2 glass window is at 8.0°C, and the air in the room is at 20°C. If the glass has thickness $L = 0.5$ cm, find the heat flow through the window and the temperatures of the inside and outside surfaces of the glass.

Solution

The problem is difficult because three heat-transfer processes are involved: the convection of air outside the window (H_1), the convection of air inside the room (H_2), and the conduction through the window (H_3). In steady state,

$$H_1 = H_2 = H_3 = H$$

and we can solve (*17.1*) and (*17.2*) simultaneously for the temperatures T_1 and T_2 of the inner and outer glass surfaces. Once these are known we can evaluate H.

Rather than do all that algebra, we make a simple approximation. A quick check shows that for conduction through the window

$$\frac{k}{L} = \frac{0.8\ \mathrm{W/m\cdot K}}{0.0050\ \mathrm{m}} = 160\ \mathrm{W/m^2\cdot K}$$

This is much larger than the h values for convection. Consequently (*17.1*) and (*17.2*) and the equality of the H values imply that the temperature difference across the glass must be much smaller than those between the glass surfaces and the inside and outside air. For purposes of calculating the convection rates we may therefore assume that the difference between temperatures T_1 and T_2 is negligible, and the glass is at a common temperature T'. This temperature must be midway between the temperatures of the inside and outside air if we are to have $H_1 = H_2$, so we take $T' = T_1 = T_2 = 14$°C. Applying (*17.2*) to the outside convection, with $\Delta T = 20$°C $-$ 14°C = 6.0°C, we have

$$H = hA\,\Delta T = [1.77(6.0)^{1/4}\ \mathrm{W/(m^2\cdot K)}]\,(1.5\ \mathrm{m^2})\,(6.0\ \mathrm{K}) = 24.9\ \mathrm{W}$$

To check our approximate solution let us solve (*17.1*) for $T_1 - T_2$:

$$24.9\ \mathrm{W} = \frac{[0.8\ \mathrm{W/(m\cdot K)}]\,(1.5\ \mathrm{m^2})\,(T_1 - T_2)}{0.005\ \mathrm{m}}$$

or $T_1 - T_2 = 0.104$ K, which is indeed very small.

Problem 17.11. Forced air flows over a heat exchanger in a room heater, with a convective heat-transfer coefficient $h = 150$ (Btu/h)/(ft$^2\cdot$°F). The surface temperature of the heat exchanger is held at 200°F while the air in the room is maintained at 72°F. Find the surface area of the heat exchanger if 20,000 Btu/h is delivered by the heater.

Solution

Here we are using English units, with power expressed in Btu/h. From the convection formula $H = hA\,\Delta T$, we have

$$20{,}000\ \mathrm{Btu/h} = \left[150\ \mathrm{Btu/}\left(\mathrm{h\cdot ft^2\cdot {}^\circ F}\right)\right]A(200\,^\circ\mathrm{F} - 72\,^\circ\mathrm{F}) \qquad \text{or} \qquad A = 1.04\ \mathrm{ft^2}$$

17.3 RADIATION

The third, and last, form of thermal energy transfer is radiation. This is a process that involves electromagnetic waves, a concept that is beyond the scope of this book. In brief, every substance at any temperature T emits electromagnetic radiation, which carries energy with it. Light is an example of electromagnetic radiation, as are radio waves, microwaves, x-rays, and gamma rays. All these types of radiation are basically the same: rapidly oscillating transverse waves that can travel through empty space as well as through substances. They differ only in the frequency of the waves. Radio waves are relatively low-frequency waves, while light has a higher frequency, and x-rays are higher frequency yet. Since these waves all travel at the same speed—the speed of light—higher frequency means shorter wavelength. The sources of the radiation are the atoms and molecules that make up any system, and which all have electrical and magnetic properties. These atoms and molecules can release some of their energy by emitting electromagnetic radiation and undergoing transitions from higher to lower energy states. Similarly, atoms and molecules can absorb electromagnetic radiation impinging on them, undergoing transitions from lower to higher energy states. For a system to be in thermal equilibrium with its surroundings it must absorb as much radiation as it emits.

A study of radiation from solid objects shows that the total amount of radiation energy (summed over all frequencies) emitted per second H_{em} from an object at uniform Kelvin temperature T, and having total surface area A, obeys the **Stefan-Boltzmann law**

$$H_{\mathrm{em}} = A\epsilon\sigma T^4 \tag{17.3}$$

where σ is a universal constant called the **Stefan-Boltzmann constant** with the value

$$\sigma = 5.67 \times 10^{-8}\ \mathrm{W/(m^2 \cdot K^4)} \tag{17.4}$$

and ϵ is a dimensionless constant, called the **emissivity**, that varies from substance to substance. The value of ϵ can vary from 0 to 1.

While (*17.3*) gives us the rate of emission of radiation by an object, we can quickly determine what the rate of absorption of radiant energy is for the same object when suspended in a large closed container whose inside walls are kept at some fixed temperature T. We first recognize that the rate of absorption is going to depend on the absorptive properties of the object, as well as on the intensity of the radiation that is impinging on it. This radiation, which fills the container like a bath, is emitted by the walls of the container. If we assume that the object is in thermal equilibrium with the walls of the container, then it absorbs radiation at the same rate as it emits radiation, $H_{\mathrm{em}} = H_{\mathrm{abs}}$, and is itself at temperature T. Then (*17.3*) leads to

$$H_{\mathrm{abs}} = A\epsilon\sigma T^4 \tag{17.5}$$

Now, (*17.5*) must hold even if the object is not at the temperature T (that is not in thermal equilibrium with the container), since the intensity of radiation in the container is determined by the temperature of the walls and not by that of the object. The presumption here is that the object is so small compared to the container that we can ignore its contribution to the "soup" of radiation within the container. It then follows that, for an object at temperature T_1 enclosed in a container with walls at temperature T_2, the *net* rate of flow of thermal energy out of the object is

$$H_{\mathrm{net}} = H_{\mathrm{em}} - H_{\mathrm{abs}} = A\epsilon\sigma({T_1}^4 - {T_2}^4) \tag{17.6}$$

H_{net} can be positive or negative, depending on whether the object or the walls is the hotter.

Note from (*17.3*) that $\epsilon \approx 1$ for a good emitter. The same ϵ appears in (*17.5*), and so a good emitter is also a good absorber. Since, in almost all cases, the radiation hitting an object is either absorbed or reflected, a poor absorber-emitter must be a good reflector, and a good absorber-emitter must be a poor reflector. At normal temperatures a good absorber-emitter appears black when one shines a light on it, since so little of the light is reflected. A perfect or ideal absorber-emitter ($\epsilon = 1$) is called a **blackbody**. Substances such as charcoal behave very much like a blackbody, but no real object is a perfect blackbody.

A small hole in a container whose inside walls are in equilibrium at some temperature T emits radiation as if it were a blackbody at temperature T. This can be understood intuitively: the small opening is a near-perfect absorber—all radiation that falls on it gets lost inside the container and has negligible chance of reflecting back out again. Since a perfect absorber is a perfect emitter, the radiation energy from the hole is that of a blackbody.

Problem 17.12.

(*a*) A sphere of radius $R = 50$ mm acts like a blackbody. If the sphere is maintained at temperature $T = 300$ K, what is the rate of radiation from it?

(*b*) How would the answer to (*a*) change if the temperature were $T = 600$ K?

(*c*) How would the answer to (*b*) change if the radius were 150 mm?

Solution

(*a*) From (*17.3*), with $\epsilon = 1$,

$$H = A\sigma T^4 = (4\pi R^2)\sigma T^4 = [4(3.14)\,(0.0025\ \mathrm{m}^2)]\left[5.67\times 10^{-8}\ \mathrm{W}/(\mathrm{m}^2\cdot \mathrm{K}^4)\right](300\ \mathrm{K})^4 = 14.4\ \mathrm{W}$$

(*b*) If the temperature doubles, H goes up by $2^4 = 16$, so, $H = 230$ W.

(*c*) If the radius triples, the area A goes up by a factor of 9, so, $H = 2070$ W.

Problem 17.13. An incandescent lamp filament, with a surface area of 100 mm^2, operates at a temperature of 2300°C. Assume that the filament acts like a blackbody.

(*a*) What is the rate of radiation from the filament?

(*b*) If the walls of the room in which the lamp operates are at 27°C, what is the rate at which the filament absorbs radiation?

(*c*) At what rate does electrical energy have to be supplied to the filament to keep its temperature constant? (Ignore conduction and convection losses from the filament.)

Solution

(*a*) $$H_{\mathrm{em}} = A\sigma T^4 = (100\times 10^{-6}\ \mathrm{m}^2)\left[5.67\times 10^{-8}\ \mathrm{W}/(\mathrm{m}^2\cdot \mathrm{K}^4)\right](2573\ \mathrm{K})^4 = 249\ \mathrm{W}$$

(*b*) $H_{\mathrm{abs}} = A\sigma T^4 = (100\times 10^{-6}\ \mathrm{m}^2)\,[5.67\times 10^{-8}\ \mathrm{W}/(\mathrm{m}^2\cdot \mathrm{K}^4)]\,(300\ \mathrm{K})^4 = 46$ mW. We see that the absorption due to the walls at room temperature is negligible.

(*c*) As energy radiated away from the filament, the filament would rapidly cool down unless the energy were continuously replenished. Thus 249 W of electrical power is required to keep the filament at constant temperature.

Problem 17.14. Assume that the steam pipe of Problem 17.9 has an emissivity of 0.60. Calculate the net rate of radiative thermal transfer from the pipe to the room, and compare it to the convective rate.

Solution

We apply (*17.6*), using the data from Problem 17.9:

$$\begin{aligned} H_{\mathrm{net}} &= (\pi DL)\epsilon\sigma(T_1{}^4 - T_2{}^4) \\ &= (3.14)\,(0.10\ \mathrm{m})\,(3.0\ \mathrm{m})\,(0.60)\left[5.67\times 10^{-8}\ \mathrm{W}/(\mathrm{m}^2\cdot \mathrm{K}^4)\right][(363\ \mathrm{K})^4 - (295\ \mathrm{K})^4] = 314\ \mathrm{W} \end{aligned}$$

We note that more thermal energy is transferred by convection than by radiation. This is generally true for steam and hot water "radiators" despite their name.

Problem 17.15. The surface area of the sun is about 2.4×10^{19} m^2, and its surface temperature is about 6000 K. Use this data to determine the total radiative power emitted by the sun, assuming it behaves like a blackbody.

Solution

$$H = A\sigma T^4 = (24 \times 10^{18}\ \mathrm{m}^2)\left[5.67 \times 10^{-8}\ \mathrm{W}/(\mathrm{m}^2 \cdot \mathrm{K}^4)\right](6000\ \mathrm{K})^4 = 1.76 \times 10^{27}\ \mathrm{W}$$

Problem 17.16. Two objects at room temperature appear black and white, respectively, when light shines on them. The lights are turned off, and the two objects are then heated to the same high temperature until one of them glows brightly. Which one glows?

Solution

Since the object that appeared black is a good absorber, it is also a good emitter ($\epsilon \approx 1$), so at high temperature it will glow brightly. The object that initially appeared white is a good reflector and poor absorber, so it is also a poor emitter ($\epsilon \approx 0$) and will be much less bright at the higher temperature.

Problems for Review and Mind Stretching

Problem 17.17. A metal cylinder, of length L = 29 cm and cross-sectional area A = 300 cm^2, has one end submerged in a shallow pool within an insulated vessel (Fig. 17-3). The pool consists of 0.600 kg of ice in equilibrium with water. The other end is kept at a constant temperature of 150°C. It is found that all the ice melts in 2.0 min. What is the conductivity of the metal, and what metal is it likely to be? Assume that there are no heat losses out the sides of the cylinder or from the vessel.

Solution

We can assume that the temperature of the submerged end remains at 0°C all through the melting process. To solve *(17.1)* for k, we need to know only the rate of heat transfer H; all the other quantities

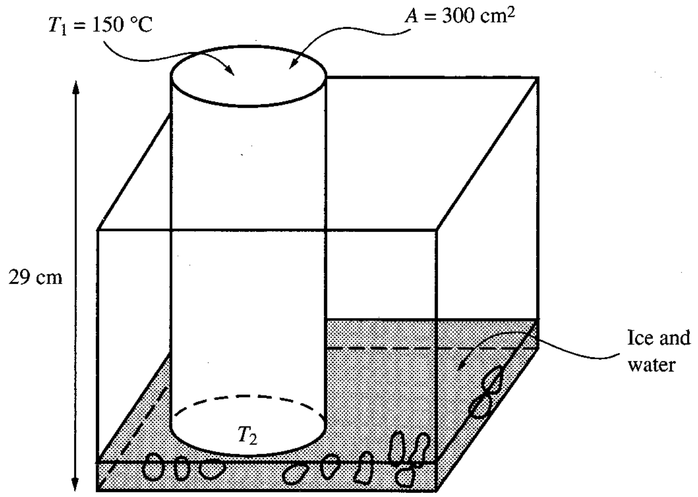

Fig. 17-3

are given. But H can be determined by figuring out how much heat must have been transferred to melt the ice and then dividing by the time taken. From Table 15.4 the heat of fusion of water is $L_f = 335$ kJ/kg. Thus,

$$H = \frac{mL_f}{\Delta t} = \frac{(0.600\ \text{kg})\,(335\ \text{kJ/kg})}{(2.0\ \text{min})\,(60\ \text{s/min})} = 1.675\ \text{kW}$$

$$k = \frac{HL}{A(T_1 - T_2)} = \frac{(1675\ \text{W})\,(0.29\ \text{m})}{(300 \times 10^{-4}\ \text{m}^2)\,(150\,°\text{C} - 0\,°\text{C})} = 108\ \text{W/(m} \cdot \text{K)}$$

Table 17.1 suggests *brass* as the metal.

Problem 17.18. Compare the R-value of a single pane of glass 1.0 cm thick with that of a double pane, which consists of two 0.40-cm-thick panes separated by a 0.2-cm-thick layer of stagnant air [$k_{\text{air}} = 0.024$ W/(m · K)].

Solution

For the solid pane

$$R = \frac{L}{k} = \frac{0.010\ \text{m}}{0.80\ \text{W/(m} \cdot \text{K)}} = 0.0125\ \text{m}^2 \cdot \text{K/W}$$

For each glass pane of the sandwich,

$$R_{\text{glass}} = \frac{0.0040\ \text{m}}{0.80\ \text{W/(m} \cdot \text{K)}} = 0.0050\ \text{m}^2 \cdot \text{K/W}$$

For the dead-air space we have to concern ourselves only with conduction:

$$R_{\text{air}} = \frac{0.002\ \text{m}}{0.024\ \text{W/(m} \cdot \text{K)}} = 0.0833\ \text{m}^2 \cdot \text{K/W}$$

The total R for the sandwich is then

$$R' = 2R_{\text{glass}} + R_{\text{air}} = 0.0933\ \text{m}^2 \cdot \text{K/W}$$

about 7.5 times larger than that of the solid glass.

Problem 17.19. A space capsule has an outer shell of surface area 100 m^2 and average R-value 4.00 m^2 · K/W. If the inner walls are to be kept at 27°C, how much wattage must the capsule heaters generate? Assume an outer-wall temperature of 200 K.

Solution

The steady-state rate at which heat leaves the interior of the capsule is

$$H = \frac{A\,\Delta T}{R} = \frac{(100\ \text{m}^2)\,(300\ \text{K} - 200\ \text{K})}{4.00\ \text{m}^2 \cdot \text{K/W}} = 2500\ \text{W}$$

This must be the rate of heating to maintain the interior at constant temperature.

Problem 17.20. Assume that the heat conducted to the outer wall of the capsule in Problem 17.19 leaves the capsule solely through radiation. Assume further that 800 W is absorbed by the capsule from "nearby" stars. What is the emissivity of the outer surface?

Solution

For steady state, the outer wall must radiate away the 2500 W from inside and reradiate the 800 W from outside, for a total output of 3300 W. The Stefan–Boltzmann law then yields

$$3300\ \text{W} = A\epsilon\sigma T^4 = (100\ \text{m}^2)\epsilon[5.67 \times 10^{-8}\ \text{W}/(\text{m}^2 \cdot \text{K}^4)]\,(200\ \text{K})^4 \qquad \text{or} \qquad \epsilon = 0.364$$

Problem 17.21.

(*a*) An outside wall of a house has an area of 100 m^2. On a day when the outside air temperature is −5.0°C, the outside surface of the wall remains at 15°C. What is the rate of heat loss through the wall to the outside?

(*b*) Assuming that the other three walls and the roof each lose heat at the same rate as the wall in part (*a*), how much power must the house heating system deliver to maintain a steady state? Ignore losses other than through the walls and roof.

Solution

(*a*) The rate of heat loss is just the rate at which heat is removed from the wall due to convection of the outside air. For a vertical wall this is $H = hA\ \Delta T$, with $h = 1.77(\Delta T)^{1/4}\ \text{W}/(\text{m}^2 \cdot \text{K})]$. For our case

$$H = [1.77(20)^{1/4}\ \text{W}/(\text{m}^2 \cdot \text{K})]\,(100\ \text{m}^2)\,(20\ \text{K}) = 7490\ \text{W}$$

(*b*) The total rate of heat loss is 5(7490 W) = 37,450 W, and this must be the rate of power delivery of the heating system.

Supplementary Problems

Problem 17.22. A small silver ingot measures 3 cm by 3 cm by 12 cm. The 9-cm^2 end faces are held at constant temperatures. T_1 and T_2, respectively, and no heat escapes out the other sides.

(*a*) Find the temperature differences $\Delta T = T_1 - T_2$ across the ingot if heat is conducted through it at the rate of 600 W.

(*b*) If the cooler end face had its temperature lowered by 20°C, how would the temperature of the hotter end face have to change to make the heat flow rate double that of (*a*)?

Ans. (*a*) 197°C; (*b*) increase by 177°C

Problem 17.23. Two 30-mm-diameter rods, one of brass and one of aluminum, are connected end to end. The aluminum rod has length 0.60 m and has its far end kept at 400°C; the brass rod has length 0.40 m and has its far end held at 20°C. Both rods are sheathed in highly insulating material. Calculate (*a*) the interface temperature, (*b*) the rate at which heat travels through the rods.

Ans. (*a*) 231°C; (*b*) 40.6 W

Problem 17.24.

(*a*) Find the conversion of the coefficient of thermal conductivity from the mixed engineering units Btu · in/(ft^2 · °F) to SI units.

(*b*) What are the appropriate units for *A*, *L*, *T*, and *H* in (*17.1*) when *k* is in engineering units?

Ans. (*a*) 1 Btu · in/(ft^2 · h · °F) = 0.144 W/(m · K); (*b*) A (ft^2), *L* (in), *T* (°F), *H* (Btu/h).

Problem 17.25. A glass window pane measuring 3 ft by 4 ft by 0.30 in has its outside surface kept at 12°F and its inside surface at 80°F. How many Btu per hour pass through the pane? (See Problem 17.24.)

Ans. 15,100 Btu/h

Problem 17.26. An insulating sandwich is made of four layers: an inner and outer layer of wood, each 1.0 cm thick; a layer of cork 3.0 cm thick; and a layer of Styrofoam 2.0 cm thick.

(*a*) Find the R-value for the sandwich.

(*b*) If the cross-sectional area of the sandwich is 2.0 m^2, what is the rate of thermal energy transfer through it when the temperatures of the two outside surfaces differ by 80°C?

Ans. (*a*) 2.95 $m^2 \cdot K/W$; (*b*) 54.2 W

Problem 17.27. A room window pane is 80 cm wide, 120 cm high, and 0.70 cm thick. The air temperature inside the room is 20 °C, while the air outside is at −20°C. Find (*a*) the rate at which heat flows through the pane; (*b*) the temperatures of the inside and outside surfaces of the glass.

Ans. (*a*) 71.9 W; (*b*) + 0.33°C and −0.33°C

Problem 17.28. A steam radiator in a room has the convection coefficient $h = 2.30(\Delta T)^{1/4}$ $W/(m^2 \cdot K)$. The surface area of the radiator is 3.0 m^2. If the room is kept at 22°C and the radiator surface is at 90°C, find the rate of convective heat transfer from the radiator to the room.

Ans. 1.35 kW

Problem 17.29. The filament of a light bulb emits 100 W of radiant energy when it is at its steady-state operating temperature of 2700 K.

(*a*) If the emissivity of the filament is 0.35, find its surface area.

(*b*) If the walls of the room are at 23°C, find the radiant energy absorbed by the filament per second.

Ans. (*a*) 0.948 cm^2; (*b*) 15.2 mW

Problem 17.30. A sphere of radius 25 mm and emissivity 0.40 is suspended by an electrical cord in an evacuated chamber whose walls are kept at 600 K.

(*a*) How much electrical power must be supplied to the sphere to keep it at 650 K? Ignore losses due to heat conduction along the wire.

(*b*) Redo the calculation if the sphere is blackened so that it behaves like a blackbody.

Ans. (*a*) 8.71 W; (*b*) 21.8 W

Problem 17.31. If the sphere of Problem 17.30(*a*) were actually supplied with 400 W of electrical power, what would its steady-state temperature be?

Ans. 1242 K

Problem 17.32. If the emissivity of the radiator in Problem 17.28 is 0.25, find the *net* radiant power that is transferred from the radiator to the room.

Ans. 416 W