Chapter 14

Fluids in Motion (Hydrodynamics)

14.1 THE NATURE OF FLUID MOTION

In the last chapter we discussed the properties of fluids at rest. Their central feature was the lack of shear forces between layers of the fluid and between the fluid and the boundary surfaces. Partly as a consequence of that fact it was found that the pressure in a container of fluid varies only with the vertical depth of the fluid.

In this chapter we will discuss the properties of fluids in motion. In general, there are shear forces between layers of fluid that move past each other and between the moving fluid and the boundary surfaces. This property of fluids is called **viscosity**, and the shear forces, which are frictional in nature, are called **viscous forces**.

For some fluids, the viscous forces can be quite small, especially when they are moving slowly. In such cases one can ignore viscosity. Such a fluid is known as a **nonviscous fluid**.

We will first consider the properties of nonviscous fluid motion, and then the case of viscosity. Before doing either, however, we discuss the nature of fluid motion itself.

Fluids in motion occur in our everyday experience: the water moving through the pipes of a house; hot air moving through the heating ducts of a building; water flowing slowly in a quiet river; water cascading through rapids and over waterfalls; air rushing past an airplane wing, as seen in the frame of reference of the airplane; and winds moving clouds through the sky. Much fluid motion is quite complex, with the flow pattern at any given point changing over time. Such motion is called **turbulent** flow. In many cases, however, the flow pattern at any point stays the same from moment to moment. Such motion is called **steady-state** or just **steady** flow; it is also called **laminar** flow.

Consider a particle of water moving in a stream. The path it takes is called the **flow line** of the particle. In steady flow, any particle that is located on the flow line of a previous particle will repeat the motion of that particle. This means that all particles on the flow line not only follow the same path but speed up and slow down in precisely the same way from point to point along the path. The motion of a fluid in steady flow thus appears smooth and regular.

Another way of characterizing a fluid flow is not to look at the motion of a single particle, but instead look at the flow pattern of the entire fluid at a given instant. By freezing the motion at a given instant, while retaining knowledge of the velocities of all the particles at that instant, one could do the following tracing. Start with any particle, and see in which direction its velocity points. Draw an imaginary infinitesimal line segment to the next particle in that direction. Next, see in which direction the velocity of this new particle points, and repeat the process. If one draws a line through all the particles so chosen, one has what is called a **streamline** of the flow. Each particle of the fluid will lie on one or another streamline. For the instant of time in question, each streamline has been chosen so that all the particles on it all have velocities tangent to it. Although in general fluid motion, the streamlines constructed at different times typically change from moment to moment, in steady flow they remain constant in time.

For steady flow the streamlines and the flow lines are identical. Consider a particle at a point on a streamline and unfreeze the motion. Since the particle is moving tangent to the streamline, it will move the infinitesimal distance to the location of the next particle on the streamline. Since the flow is steady, the particle will now move in precisely the way that the previous particle was moving when it was there, which is again tangent to the original streamline. Continuing this reasoning, the particle will trace out a path, or flow line, which is identical to the streamline. In turbulent motion, however, the streamlines and flow lines can be very different. Turbulent flow is characterized by swirling and eddies and constantly changing patterns of motion.

Problem 14.1. Show that for steady-state flow, two streamlines can never cross each other.

Solution

A particle of fluid at any point on a streamline must move tangent to the streamline for steady-state flow. If two streamlines crossed, it would mean that the particles reaching the point of crossing would have a choice of moving in either of two different directions, and the flow would not be repetitive and steady in time. This obviously cannot happen for steady flow.

Problem 14.2. Consider the case of steady flow, and consider a small cross-sectional area through which some of the fluid is flowing. If we trace the streamlines of the fluid that pass through the perimeter of this area, we get what is called a *stream tube*, or *flow tube*. Show that the flowing fluid can never cross the boundary of the tube.

Solution

The situation is depicted in Fig. 14-1. The particles of fluid moving within the stream tube are moving along streamlines which cannot cross the boundary streamlines, as shown in Problem 14.1. Thus the fluid inside cannot cross the boundary. Similarly, particles of fluid outside the stream tube move along streamlines that never cross the boundary streamlines and thus cannot enter the stream tube.

Note. In general, as a fluid flows, its density can vary from location to location. This is especially pronounced for gases, which are easily compressed. Densities can also change for liquids, but for most liquids these changes are small. If we can ignore the change in density of a liquid we say it is **incompressible**. (An incompressible fluid that has no viscosity is called an **ideal fluid**.) In the following section, we will assume steady or laminar flow, with zero viscosity, unless otherwise stated.

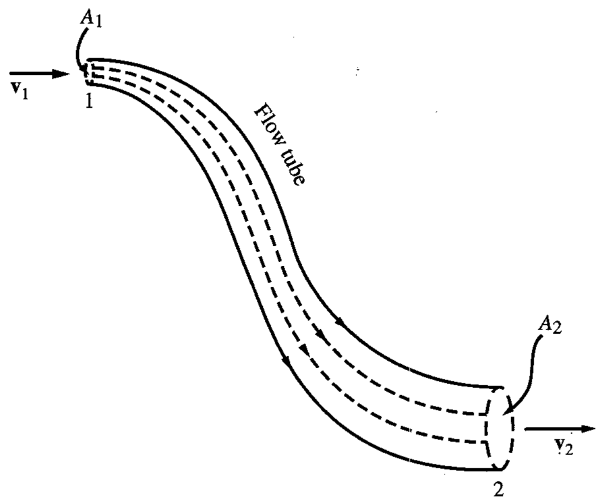

Fig. 14-1

14.2 THE LAWS OF FLUID MOTION

Equation of Continuity

Consider the flow tube of Fig. 14-1. We assume that areas at each end are small and are chosen perpendicular to the direction of flow at each end. Let $\mathbf{v}_1$ and $\mathbf{v}_2$ be the velocities of the fluid at the two ends. The mass of fluid that flows into one end of the tube in a given time interval must be the same as the mass that flows out the other end in the same time interval. This follows from the conservation of mass, and the fact that for steady flow the mass inside the flow tube between the two ends must remain constant.

Suppose that a short time interval Δt elapses. All particles that are to the left of the surface A_1 within a distance $v_1\,\Delta t$ of it will pass through the surface in this time interval, as shown in Fig. 14-2. The volume of fluid entering the tube is thus $\Delta V_1 = v_1\,\Delta t\,A_1$, and the mass is given by $\Delta m_1 = d_1\,\Delta V_1 = d_1 v_1\,\Delta t\,A_1$, where d_1 is the density of fluid at A_1. Similarly, the amount of mass leaving the tube through A_2 is $\Delta m_2 = d_2\,\Delta V_2 = d_2 v_2\,\Delta t\,A_2$, where d_2 is the fluid density at A_2. From $\Delta m_1 = \Delta m_2$, we get the equation of continuity

$$d_1 v_1 A_1 = d_2 v_2 A_2 \qquad (14.1)$$

If the fluid is incompressible, we have $d_1 = d_2$, and (*14.1*) reduces to

$$v_1 A_1 = v_2 A_2 \qquad (14.2)$$

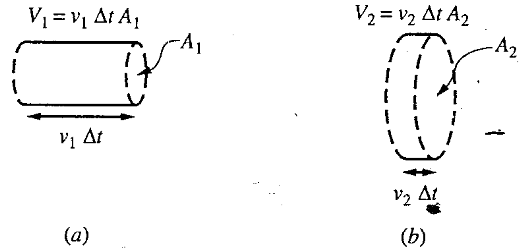

Fig. 14-2

Problem 14.3.

(*a*) Suppose that Fig. 14-1 depicts a steady flow of air. Assume $A_1 = 1.0\text{ cm}^2$, $A_2 = 3.0\text{ cm}^2$, and $v_1 = 12$ cm/s. If $d_2 = 1.3d_1$, find v_2.

(*b*) Suppose that Fig. 14-1 depicts a steady flow of water, with the same values for A_1, A_2, and v_1. Assuming water is incompressible, what is v_2?

Solution

(*a*) By (*14.1*), after dividing both sides by d_1, we get

$$(12\text{ cm/s})(1.0\text{ cm}^2) = 1.3(3.0\text{ cm}^2)v_2 \qquad \text{or} \qquad v_2 = 3.08\text{ cm/s}$$

(*b*) By (*14.2*),

$$(12\text{ cm/s})(1.0\text{ cm}^2) = (3.0\text{ cm}^2)v_2 \qquad \text{or} \qquad v_2 = 4.0\text{ cm/s}$$

Problem 14.4. Water flows through a pipe system of variable cross section as shown in Fig. 14-3(*a*). Assume that water completely fills all sections of the pipe and the flow is incompressible.

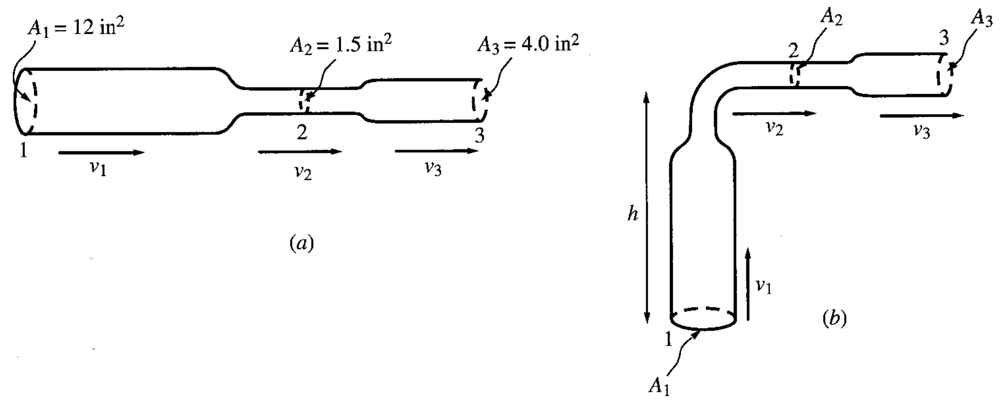

Fig. 14-3

(*a*) If $v_1 = 0.80$ ft/s, find v_2 and v_3.

(*b*) What volume, in cubic feet, passes through a cross section of the pipe each second?

(*c*) How many pounds of water pass through a cross section of the pipe per second?

(*d*) If the same pipe were bent into the shape shown in Fig. 14-3(*b*), with the same value for v_1, how will the result of part (*a*) change?

Solution

(*a*) When water flows through a pipe and completely fills the pipe, the pipe itself is, in effect, a stream tube. Applying (*14.2*) to points 1, 2, and 3, we have

$$(0.80 \text{ ft/s})(12 \text{ in}^2) = v_2(1.5 \text{ in}^2) = v_3(4.0 \text{ in}^2) \qquad \text{or} \qquad v_2 = 6.4 \text{ ft/s} \qquad v_3 = 2.4 \text{ ft/s}$$

(The equations were left in mixed units because area units cancel out).

(*b*) $\Delta V_1 = v_1 \, \Delta t \, A_1$. $\Delta V_1 / \Delta t$ = rate of volume flow $= v_1 A_1 = (0.80 \text{ ft/s})(12 \text{ in}^2)/(144 \text{ in}^2/\text{ft}^2) = 0.0667 \text{ ft}^3/\text{s}$. (Here we must use consistent units.)

(*c*) Recalling from Chap. 13 that the weight density of water is 62.5 lb/ft^3, we have

$$d_w \frac{\Delta V_1}{\Delta t} = (62.5 \text{ lb/ft}^3)(0.0667 \text{ ft}^3/\text{s}) = 4.17 \text{ lb/s}$$

(*d*) The result is the same. Nothing changes because Eq. (*14.2*) is solely a consequence of conservation of mass and fluid incompressibility.

Bernoulli's Equation

We now turn to the question of how the pressure behaves in a flowing fluid. Again we will assume an incompressible fluid with constant density d. Consider the flow tube in Fig. 14-4. Let us apply the work-energy theorem to the fluid that at a given instant of time lies between points 1 and 2. This fluid constitutes our system, and everything else is external to it. At the end of a time interval Δt, our system (the same fluid) lies between points 1′ and 2′. Since we are ignoring viscosity, the only forces other than gravity doing work on our system are those due to the pressure of the fluid to the left of our system at point 1 and to the right of our system at point 2. The force on our system at point 1 is $P_1 A_1$, acting to the right. During time Δt that force acts through a distance $v_1 \, \Delta t$, yielding the positive work $\Delta W_1 = P_1 A_1 v_1 \, \Delta t$. But, $A_1 v_1 \, \Delta t = \Delta V_1$, the volume moved through in time Δt, so $\Delta W_1 = P_1 \, \Delta V_1$.

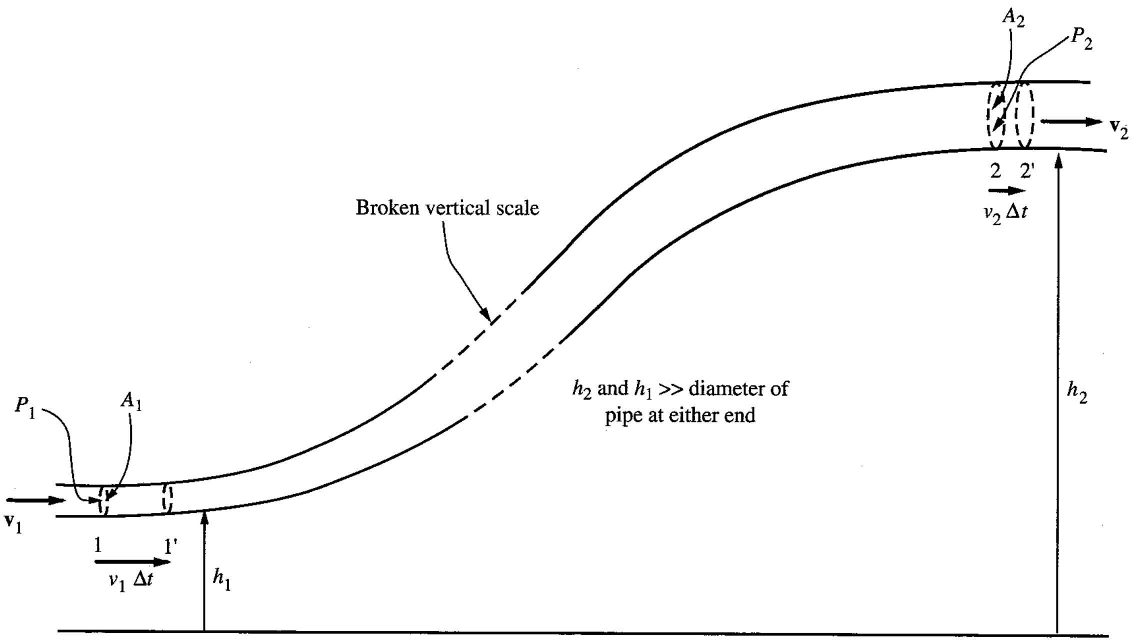

Fig. 14-4

The force from the fluid to the right of our system at point 2 is just P_2A_2 acting to the left. In time Δt the fluid boundary moves a distance $v_2\,\Delta t$ to the right. The work done is thus

$$\Delta W_2 = -P_2A_2v_2\,\Delta t = -P_2\,\Delta V_2$$

The net work done by all forces other than gravity is thus $\Delta W_T = P_1\,\Delta V_1 - P_2\,\Delta V_2$. Since the fluid is assumed incompressible, $\Delta V_1 = \Delta V_2$. Calling this common volume ΔV, we have

$$\Delta W_T = P_1\,\Delta V - P_2\,\Delta V \tag{14.3}$$

From the work-energy theorem, this must equal the net change in the kinetic and potential energy of our system in time Δt:

$$\Delta W_T = \Delta E_k + \Delta E_p \tag{14.4}$$

To find ΔE_k, we note that, in time Δt, our system has moved to the region between points 1′ and 2′. Because the flow is steady, the kinetic energy of the fluid lying between points 1′ and 2 is the same at the beginning and end of the time interval Δt. The net change is therefore due to the fact that the kinetic energy of the fluid that was originally between points 1 and 1′ has disappeared, and in its place we have the kinetic energy of the fluid that is now between points 2 and 2′. We already saw that the two volumes have common volume ΔV, so the mass in each region is $d\,\Delta V$, and we have

$$\Delta E_k = \tfrac{1}{2}d\,\Delta V\,v_2{}^2 - \tfrac{1}{2}d\,\Delta V\,v_1{}^2 \tag{14.5}$$

By the same reasoning, change in the potential energy of the system in the time interval Δt is due to the disappearance of mass between 1 and 1′ and the appearance of mass between 2 and 2′.

$$\Delta E_p = d\,\Delta V\,gh_2 - d\,\Delta V\,gh_1 \tag{14.6}$$

Substituting (*14.3*), (*14.5*), and (*14.6*) into (*14.4*) and canceling ΔV, we get

$$P_1 - P_2 = \tfrac{1}{2}dv_2{}^2 - \tfrac{1}{2}dv_1{}^2 + dgh_2 - dgh_1$$

or

$$P_1 + \tfrac{1}{2}dv_1^2 + dgh_1 = P_2 + \tfrac{1}{2}dv_2^2 + dgh_2 \qquad (14.7)$$

Equation (*14.7*) is called **Bernoulli's equation.**

Problem 14.5. In Fig. 14-5 we show a section of a horizontal pipe with water flowing through it.

(*a*) If the speed of the water at point 1 is $v_1 = 2.0$ m/s, find the velocity v_2 at point 2.

(*b*) If the pressure of the fluid at point 1 is $P_1 = 2.0 \times 10^5$ Pa, find the pressure P_2 at point 2.

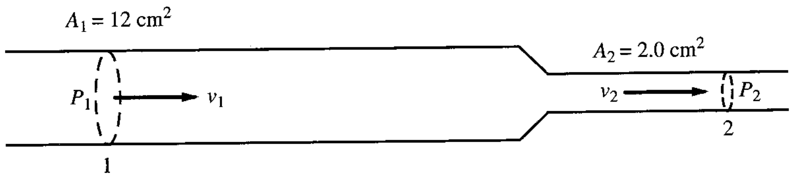

Fig. 14-5

Solution

(*a*) We use (*14.2*), the equation of continuity for incompressible fluids:

$$v_1A_1 = v_2A_2 \Rightarrow (2.0\ \text{m/s})(12\ \text{cm}^2) = v_2(2.0\ \text{cm}^2) \qquad \text{or} \qquad v_2 = 12\ \text{m/s}$$

(*b*) To find the pressure we use (*14.7*), Bernoulli's equation. Since the pipe is horizontal, and the diameters of pipes are relatively small, we can assume $h_1 = h_2$. Then the potential-energy terms drop out, leaving

$$P_1 + \tfrac{1}{2}dv_1^2 = P_2 + \tfrac{1}{2}dv_2^2$$

or
$$P_2 = 2.0 \times 10^5\ \text{Pa} + \tfrac{1}{2}(1000\ \text{kg/m}^3)(2.0\ \text{m/s})^2 - \tfrac{1}{2}(1000\ \text{kg/m}^3)(12\ \text{m/s})^2$$
$$= 2.0 \times 10^5\ \text{Pa} - 0.70 \times 10^5\ \text{Pa} = 1.30 \times 10^5\ \text{Pa}$$

Note. Problem 14.5 shows that, in a horizontal pipe, the pressure is lower where the velocity is higher.

Problem 14.6. Water flows through the pipe shown in Fig. 14-6. We examine two points, 1 and 2, a vertical distance $h = 5.0$ m apart. At point 1, the velocity of the water is $v_1 = 2.0$ m/s, the pressure is $P_1 = 2.0 \times 10^5$ Pa, and the cross-sectional area is $A_1 = 12$ cm^2.

(*a*) If A_2, the cross-sectional area at point 2 is the same as A_1, find the pressure P_2 at point 2.

(*b*) If the lower part of the pipe were narrower than shown and had a cross-sectional area at point 2 of $A_2 = 2.0$ cm^2, find the pressure at point 2.

Solution

(*a*) Since $A_1 = A_2$ the continuity equation yields $v_1 = v_2$. Then, from Bernoulli's equation (*14.7*), the velocity terms cancel out, leaving $P_1 + dgh_1 = P_2 + dgh_2$. Since $h_1 - h_2 = h$, we get

$$P_2 = P_1 + dgh \qquad (i)$$

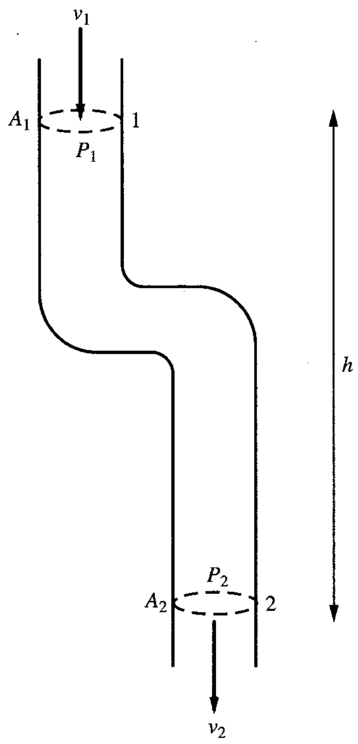

Fig. 14-6

Then
$$P_2 = (2.0 \times 10^5 \text{ Pa}) + (1000 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(5.0 \text{ m})$$
$$= 2.0 \times 10^5 \text{ Pa} + 0.49 \times 10^5 \text{ Pa} = 2.49 \times 10^5 \text{ Pa}$$

Note. Equation (*i*) is just the equation for variation of hydrostatic pressure with depth. Thus, if the velocity is equal at two points in a flowing liquid, the pressure difference is the same as if the liquid were at rest.

(*b*) Now we need to find the new velocity at point 2. Since v_1, A_1, and A_2 are exactly as in Problem 14.5, we again have $v_2 = 12.0$ m/s. Then, by (*14.7*), we have

$$P_1 + \tfrac{1}{2}dv_1{}^2 + dgh_1 = P_2 + \tfrac{1}{2}dv_2{}^2 + dgh_2 \quad \text{or} \quad P_2 = P_1 + (\tfrac{1}{2}dv_1{}^2 - \tfrac{1}{2}dv_2{}^2) + (dgh_1 - dgh_2)$$

Substituting [see Problems 14.5(*b*) and 14.6(*a*)], we have

$$P_2 = 2.0 \times 10^5 \text{ Pa} - 0.70 \times 10^5 \text{ Pa} + 0.49 \times 10^5 \text{ Pa} \quad \text{or} \quad P_2 = 1.79 \times 10^5 \text{ Pa}$$

A special case of interest is that in which the cross-sectional area on one side of a stream tube is very large compared to the other side. An example is a tank with a small opening on the side or bottom.

Problem 14.7. In Fig. 14-7 we shown a tank of cross-sectional area $A = 500 \text{ cm}^2$ open to the atmosphere and filled with a liquid of density d to a depth $h_1 = 1.5$ m. A small hole of cross-sectional area $a = 5.0 \text{ cm}^2$ is drilled in the tank at a height $h_2 = 0.4$ m from the bottom.

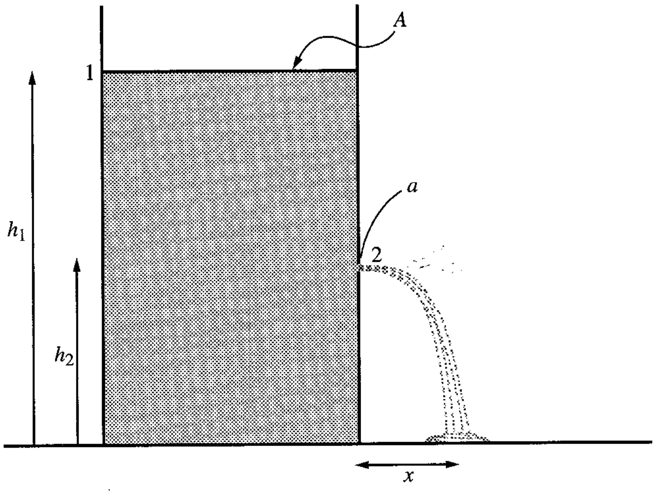

Fig. 14-7

(*a*) Find the velocity with which the liquid pours out of the hole.

(*b*) Ignoring air resistance, how far from the wall of the tank does the stream of liquid hit the ground?

Solution

(*a*) Applying (*14.7*) and noting that the liquid is open to the atmosphere at both points 1 and 2 so that $P_1 = P_2 = P_A$, we get

$$\tfrac{1}{2}dv_1{}^2 + dgh_1 = \tfrac{1}{2}dv_2{}^2 + dgh_2$$

From (*14.2*) we have $v_1A_1 = v_2A_2$ or $v_1 = v_2(A_2/A_1)$. For our case, $A_2 = a = 5.0\text{ cm}^2$ and $A_1 = A = 500\text{ m}^2$, so $A_2/A_1 = 0.01$. Therefore, $v_1 = 0.01v_2$ and $v_1{}^2 = 0.0001v_2{}^2$. Therefore, $\tfrac{1}{2}dv_1{}^2$ is completely negligible compared to $\tfrac{1}{2}dv_2{}^2$ and may be dropped. Letting $h_1 - h_2 = h$, we get

$$\tfrac{1}{2}v_2{}^2 = gh \Rightarrow v_2{}^2 = 2gh \Rightarrow v_2 = \sqrt{2gh}$$

Substituting, we get

$$v_2 = [2(9.8\text{ m/s}^2)(1.5\text{ m} - 0.4\text{ m})]^{1/2} = 4.64\text{ m/s}$$

(*b*) Since the fluid particles leave the tank with a horizontal initial velocity, the time it takes them to fall vertically from rest through a distance $h_2 = 0.40$ m is calculated as follows:

$$h_2 = \tfrac{1}{2}gt^2 \Rightarrow 0.40\text{ m} = (4.9\text{ m/s}^2)t^2 \Rightarrow t = 0.286\text{ s}$$

Then the horizontal distance is

$$x = v_2t = (4.64\text{ m/s})(0.286\text{ s}) = 1.33\text{ m}$$

Note. $v_2 = \sqrt{2gh}$ in Problem 14.7 is the velocity that a drop of water would gain in free fall from rest through the distance h (which is the distance from the top of the liquid in the tank to the point of exit). This result is known as **Torricelli's theorem**. It is valid when the liquid is open to the atmosphere at both ends and the surface area of the tank is very large compared to the area where the liquid leaves the tank.

Problem 14.8.

(*a*) In Problem 14.7, what volume of liquid leaves the tank through the opening per second?

(*b*) How would the answer to (*a*) change if the area a of the opening were doubled?

(*c*) How would the answer to (*a*) change if the hole were near the bottom of the tank?

Solution

(*a*) The volume ΔV_2 of liquid leaving the tank at point 2 in a small time Δt is just $(v_2\,\Delta t)a$. Hence,

$$\frac{\Delta V_2}{\Delta t} = v_2 a = (4.64\ \text{m/s})(5.0 \times 10^{-4}\ \text{m}^2) = 0.0023\ \text{m}^3/\text{s} = 2300\ \text{cm}^3/\text{s}$$

(*b*) If the area a were doubled, it would still be very small compared to A, the cross-sectional area of the tank, so Torricelli's theorem still holds, and the velocity v_2 would be unchanged. Then the volume flow rate would just reflect the doubled area, and $\Delta V_2/\Delta t$ would double.

(*c*) If the hole were near the bottom, $v_2 = \sqrt{2gh'}$, where $h' = h_1 = 1.5$ m. Then

$$v_2 = [2(9.8\ \text{m/s}^2)(1.5\ \text{m})]^{1/2} = 6.64\ \text{m/s}$$

and

$$\frac{\Delta V_2}{\Delta t} = (6.64\ \text{m/s})(5.0 \times 10^{-4}\ \text{m}^2) = 0.0033\ \text{m}^3/\text{s}$$

Problem 14.9.

(*a*) If the fluid in Problems 14.7 and 14.8(*a*) was water, what would be the rate at which mass left the tank?

(*b*) What would be the time rate of increase of momentum of the water as it left the tank?

(*c*) From your answer to (*b*) can you determine the reaction force on the tank exerted by the exiting water?

Solution

(*a*) From Problem 14.8(*a*) the volume rate was $\Delta V_2/\Delta t = 0.0023\ \text{m}^3/\text{s}$. Using the density of water d, the mass rate equals

$$\frac{\Delta M_2}{\Delta t} = \frac{d\,\Delta V_2}{\Delta t} = (1000\ \text{kg/m}^3)(0.0023\ \text{m}^3/\text{s}) = 2.3\ \text{kg/s}$$

(*b*) Each particle of water leaves the tank with horizontal velocity v_2. If ΔM_2 is the mass leaving in time Δt, the total momentum picked up in time Δt is $\Delta M_2 v_2$. Dividing by Δt gives the rate of increase of momentum of the water as it leaves:

$$v_2 \frac{\Delta M_2}{\Delta t} = (4.64\ \text{m/s})(2.3\ \text{kg/s}) = 10.7\ \text{kg}\cdot\text{m/s}^2 = 10.7\ \text{N to the right}$$

(*c*) Since the rate of increase of momentum of the water, as determined in (*b*), must be the force exerted on the exiting water by the water left behind, by Newton's third law the exiting water must exert an equal and opposite reaction force, $F_R = 10.7$ N, to the left.

Problem 14.10. A boy holds a garden hose as shown in Fig. 14-8. Water pours out of the hose at a speed of $v = 10$ m/s, and the cross-sectional area of the end of the hose is $a = 2.0\ \text{cm}^2$. What horizontal force must the boy exert on the hose to keep it in place?

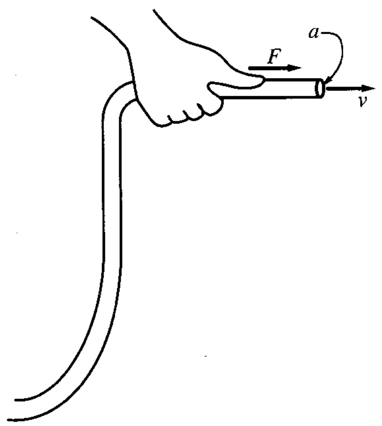

Fig. 14-8

Solution

From the previous problems we can see the essential approach to this problem. The water flowing up the vertical portion of the hose has no horizontal velocity component, so all the horizontal momentum gained by the water as it leaves the hose must have been caused by a horizontal force **F** exerted by the walls of the hose on the water as it goes around the bend. This same force is the force that the boy must exert on the hose to keep it in place. To calculate the magnitude of this force we need to determine the time rate of increase of momentum in the horizontal direction. The rate at which mass exits the hose is $\Delta M/\Delta t = d\,\Delta V/\Delta t$. Since $\Delta V/\Delta t = va$, we have $\Delta M/\Delta t = dva$. Each unit of mass picks up a horizontal velocity v. The horizontal force, which equals the time rate of increase of horizontal momentum, is then

$$F = v\,\frac{\Delta M}{\Delta t} = dv^2a = (1000\text{ kg/m}^3)(10\text{ m/s})^2(2.0\times 10^{-4}\text{ m}^2) = 20.0\text{ N}$$

Venturi Tubes

When fluids flow through a pipe system, one often wants to monitor the pressure at various points in the pipe, as well as determine the flow velocity. One way of doing this is by means of transparent tubes attached to the pipe system, with a liquid such as water or mercury enclosed. Figure 14-9 gives the configuration of some venturi tube setups.

In Fig. 14-9(a) we have a vertical tube rising out of a pipe with water flowing through it under a positive gauge pressure. The upper end of the tube is open to the atmosphere. Water rises in the tube until the height of water produces a gauge pressure that just balances the pressure in the pipe. The pressure in the pipe is determined by observing the height of the water in the tube.

In Fig. 14-9(b) a thin bent tube with mercury in it is inserted in a pipe and sealed to prevent leakage. The left end of the tube has an opening on one side, with the fluid moving by at speed v, while the right end is open to the atmosphere. The tube acts like an open-tube manometer and measures the gauge pressure of the flowing fluid.

Figure 14-9(c) shows a more sophisticated venturi setup that can directly measure the velocity of flow in the pipe. We assume the diameter of the venturi tube is very small compared to that of the pipe so that it does not change the flow velocity in the pipe very much. The left end of the tube has an opening on its top, whereas the right end of the tube opens directly into the flowing fluid. Since the fluid must go around one or the other side of the tube, there is a small region or point right in front of the tube where the fluid is at rest. Such a point is called a **stagnation point**. Using Bernoulli's

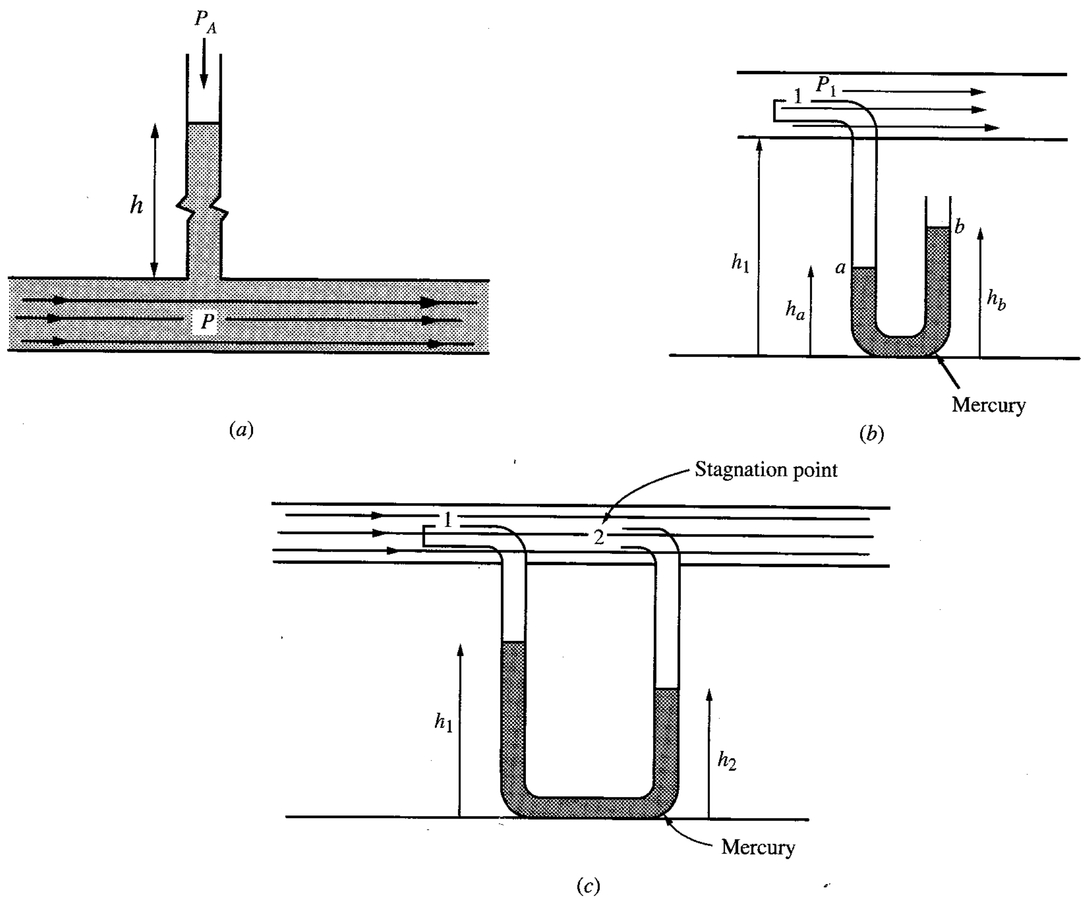

Fig. 14-9

equation, and assuming a horizontal pipe, the pressure difference between points 1 and 2 is $P_2 - P_1 = \frac{1}{2}dv_1^2$. The height difference of the mercury (corrected for the different heights of fluid above the mercury on the two sides) directly measures $P_2 - P_1$ and therefore yields the velocity v_1.

Problem 14.11.

(*a*) In Fig. 14-9(*a*) the water in the venturi tube rises 30 cm above the pipe level. Find the absolute pressure of the water in the pipe.

(*b*) In Fig. 14-9(*b*), water flows through the pipe at the same pressure as in (*a*). Assuming $h_1 = 30$ cm, find the height difference $h_b - h_a$ in the mercury.

Solution

(*a*) $P = P_A + dgh = 1.013 \times 10^5\ \text{Pa} + (1000\ \text{kg/m}^3)(9.8\ \text{m/s}^2)(0.30\ \text{m}) = 1.042 \times 10^5\ \text{Pa}$

(*b*) We find the pressure at point *a* from the left and the right and equate them:

$$P_1 + d_w g(h_1 - h_a) = P_a = P_A + d_{\text{Hg}} g(h_b - h_a)$$

Substituting,

$$1.042 \times 10^5\ \text{Pa} + (1000\ \text{kg/m}^3)(9.8\ \text{m/s}^2)(0.30\ \text{m})$$
$$= 1.013 \times 10^5\ \text{Pa} + (13{,}600\ \text{kg/m}^3)(9.8\ \text{m/s}^2)(h_b - h_a) \qquad \text{or} \qquad h_b - h_a = 0.044\ \text{m}$$

Problem 14.12. In Fig. 14-9(c) the pipe contains air moving at velocity v. The mercury height difference is noted to be 18 mm. Find the velocity of the air, assuming that the density of the air is 1.3 kg/m^3.

Solution

The pressure difference in the two sides of the venturi tube is

$$P_2 - P_1 = d_{Hg}g(h_1 - h_2) = (13{,}600\ \text{kg/m}^3)(9.8\ \text{m/s}^2)(18 \times 10^{-3}\ \text{m}) = 2400\ \text{Pa}$$

(Here we ignore the slight difference in the height of the air above the columns of mercury on the two sides). By Bernoulli's equation the difference in pressure at points 1 and 2 is also given by

$$P_2 - P_1 = \tfrac{1}{2}d_a v^2 \Rightarrow 2400\ \text{Pa} = \tfrac{1}{2}(1.3\ \text{kg/m}^3)v^2 \qquad \text{or} \qquad v = 60.8\ \text{m/s}$$

Aerodynamics

An airplane in motion is supported by the pressure difference between the top and undersides of the wing. This pressure difference is the consequence of Bernoulli's equation. In the frame of reference of the airplane, the air is rushing past the wing, which is designed to compress the streamlines above the wing much more than those under the wing. Figure 14-10 shows a cross section of the wing with the streamlines above and below.

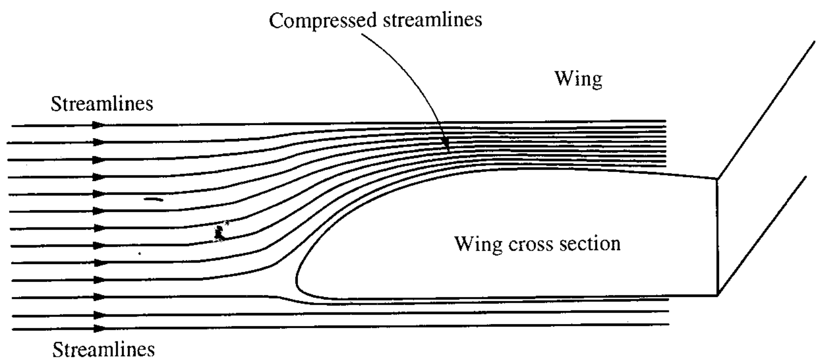

Fig. 14-10

Compression of the streamlines means that the stream tube above the wing has a smaller cross-sectional area than that in front of the plane and, from the continuity equation, the velocity of the air must therefore be greater above the wing. This greater velocity implies lower pressure than the normal pressure of the air in front of the plane. If we assume the flow lines under the wing are not compressed at all, the pressure under the wing is just the normal pressure of the air in front of the wing. Thus, there is a pressure difference on the two sides.

Problem 14.13. Consider an airplane moving through the air at velocity $v = 200$ m/s. Assume that the streamlines which move just over the top of the wing are compressed to eight-tenths their normal area, and that those under the wing are not compressed at all. ($d_a = 1.3$ kg/m^3.) Find (a) the velocity v_1 of the air just over the wing; (b) the difference in the pressure between the air just over the wing P_1 and that under the wing P_2; (c) the net upward force on both wings if the area of each wing is 40 m^2.

Solution

(*a*) Since the effective area of the stream tube is reduced to eight-tenths its original value, the velocity is increased to ten-eighths, or five-fourths its original value. So, $v_1 = 250$ m/s.

(*b*) From Bernoulli's equation, with both points at effectively the same elevation, we have $P_1 + \frac{1}{2}d_a v_1^2 = P_2 + \frac{1}{2}d_a v_2^2$. Here $v_2 = v = 200$ m/s, so

$$P_2 - P_1 = \tfrac{1}{2}(1.3\ \text{kg/m}^3)[(250\ \text{m/s})^2 - (200\ \text{m/s})^2] = 1.46 \times 10^4\ \text{Pa}$$

(*c*) The total net upward force is

$$(P_2 - P_1)A = (1.46 \times 10^4\ \text{Pa})(80\ \text{m}^2) = 1.17 \times 10^6\ \text{N}$$

Note. Similar aerodynamic effects occur when a horizontally moving ball simultaneously spins about a vertical axis. Because of drag effects the air flowing past the ball has different velocities on the two sides of the ball, leading to a pressure differential and a net sideways force on the ball. The detailed process involved are complex, but the net effect is shown schematically (top view), in the frame of reference of the center of mass of the ball (Fig. 14-11).

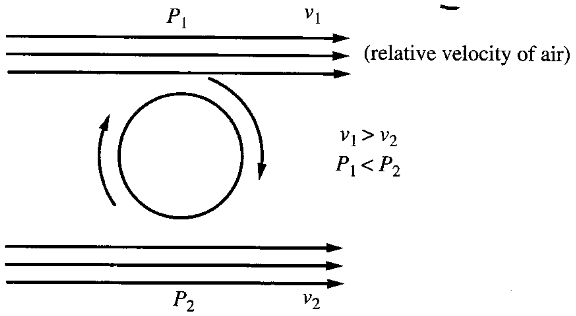

Fig. 14-11

14.3 VISCOSITY

Viscosity occurs both in steady flow and in turbulent flow and is, in effect, a frictional force between layers of fluid moving past each other. The nature of this force can best be understood by considering steady straight-line flow of a fluid. Figure 14-12(*a*) shows a fluid of thickness d between two very large plates, each of area A. We assume that the plates are large enough so that, over the time frame of the experiment, we can ignore the effects of fluid spilling out from between the plates at the edges. The bottom plate can be considered fixed, and the upper plate is being pulled to the right by a force F. The upper plate reaches equilibrium when it is moving at some velocity v. It is found that a thin layer of fluid in contact with each plate stays at rest relative to that plate. Thus, the uppermost layer of fluid moves to the right with velocity v while the bottommost layer is at rest. The velocities of the layers in between (which can be thought of as having very small thicknesses Δd) vary in proportion to their distances y from the bottom layer, as shown in Fig. 14-12(*b*). Since each layer is in equilibrium, the force F on the top surface of a given layer (pulling to the right) has the same magnitude as that on the bottom surface (pulling to the left). Thus the shear stress F/A is the same on all layers. While the absolute velocity is different for different layers, the rate of change of velocity with depth, or velocity gradient, stays the same: $\Delta v/\Delta y = v_y/y = v/d$, where v_y is the horizontal

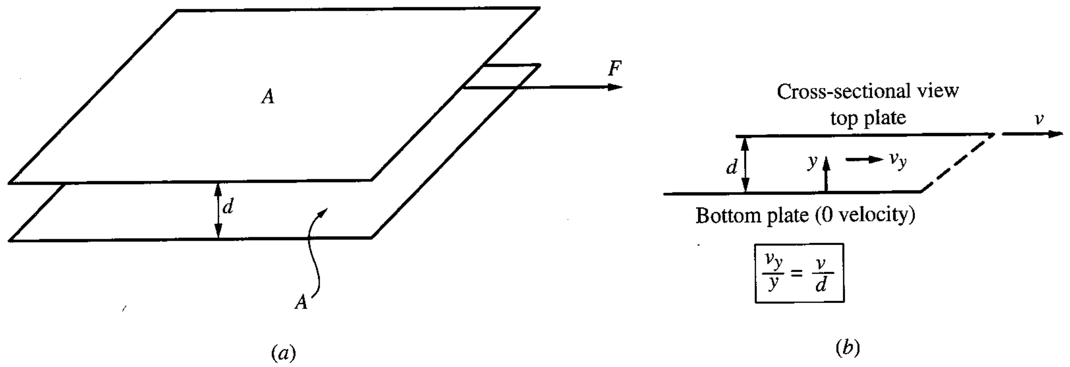

Fig. 14-12

velocity at height y. This is consistent with the observed result: For fluid in steady flow between parallel plates, the stress is proportional to the velocity gradient or

$$\frac{F}{A} = \eta \frac{v}{d} \tag{14.8}$$

where η (the Greek letter *eta*) is the proportionality constant. This constant, which varies from fluid to fluid, is called the **coefficient of viscosity**, or, just the viscosity. From Eq. (*14.8*), we can see that the dimensions of η are $N \cdot s/m^2 = Pa \cdot s$. The unit of viscosity is called the poiseuille (Pl): 1 Pl = 1 Pa · s. The viscosities of all fluids decrease with increasing temperature. A few typical viscosities are given in Table 14.1.

Table 14.1. Viscosity

Substance	Temperature, °C	η, Pl
Air	20	1.8×10^{-5}
Water	20	1.0×10^{-3}
Whole blood	38	4.0×10^{-3}
Light oil	20	2.0
Heavy oil	20	9.9

Problem 14.14. The top plate in Fig. 14-12(*b*) has a constant velocity of 20 cm/s, when a shear stress of 300 N/m^2 is applied.

(*a*) If the distance between the plates is $d = 2.3$ mm, find the viscosity of the liquid between the plates.

(*b*) If the same liquid filled the space between the plates to a distance of $d = 4.6$ mm and the stress were increased to 600 N/m^2, what would the equilibrium velocity of the upper plate be?

Solution

(*a*) $$\eta = \frac{F/A}{v/d} = \frac{300\ \text{N/m}^2(2.3 \times 10^{-3}\ \text{m})}{0.20\ \text{m/s}} = 3.45\ \text{Pl}$$

(*b*) The viscosity stays the same, but the distance d doubles, and the stress doubles. From (*14.8*), we see that the velocity quadruples under these circumstances: $v = 80$ cm/s.

Viscous Flow through a Pipe

From the information in Eq. (*14.8*), one can use the calculus to deduce some interesting results for a viscous fluid in steady flow through a pipe. In a pipe, thin concentric cylindrical shells of liquid flow at the same speed. The maximum speed v_m of the fluid is attained at the central axis of the pipe. For a pipe of radius R, the rate of volume flow through the pipe can be shown to be

$$\frac{\Delta V}{\Delta t} = \frac{v_m}{2}\,\pi R^2 \qquad (14.9)$$

[Note that Eq. (*14.9*) implies that $v_m/2$ is the average velocity of the liquid in the pipe.]

It can also be shown that the net viscous force acting on a fluid in steady flow, due to the walls of a length L of pipe, is given by

$$F = 4\pi\eta L v_m \qquad (14.10)$$

where the force F is in the direction opposite to the flow.

Since the flow through the pipe is steady, the net force on the length L of fluid must be zero. Assuming a horizontal pipe, the only other forces are due to the pressure on each end of the length L of fluid, and the difference in these pressures must give rise to a net force that balances the viscous force of Eq. (*14.10*). This implies that when a viscous fluid is in steady flow through a horizontal pipe of uniform cross section, there is a pressure difference at different points along the pipe. This is quite different than the result from Bernoulli's equation, where the pressure difference would be zero. The effect of viscosity on pressure is demonstrated in the next problem.

Problem 14.15. Find an expression for the pressure difference between two points a distance L apart in a uniform horizontal pipe carrying a liquid moving with central velocity v_m and having viscosity η.

Solution

The situation is shown in Fig. 14-13. We consider our system to be the fluid in a length L of our pipe of radius R. Since the fluid is not accelerating, the net horizontal force must be zero. The only horizontal forces on our system are due to the pressure from the liquid to the left of the system P_1, the pressure due to the liquid to the right of the system P_2, and the viscous force due to the pipe. The force due to P_1 is $F_1 = P_1\pi R^2$ acting to the right. The force due to P_2 is $F_2 = P_2\pi R^2$ acting on the left. The viscous force is given by Eq. (*14.10*): $F = 4\pi\eta L v_m$ acting to the left. For equilibrium,

$$P_1\pi R^2 - P_2\pi R^2 - 4\pi\eta L v_m = 0 \qquad \text{or} \qquad P_1 - P_2 = \frac{4\eta L v_m}{R^2} \qquad (i)$$

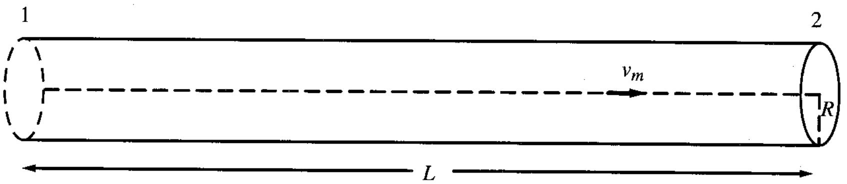

Fig. 14-13

Problem 14.16. Find the pressure drop across a 1.0-cm length of a small blood vessel 2.0×10^{-2} mm in radius, if the maximum speed of the blood in the vessel is 1.1 cm/s.

Solution

From Eq. (*i*) in Problem 14.15 (and Table 14.1), we have

$$P_1 - P_2 = \frac{4(4.0 \times 10^{-3}\ \text{Pl})(0.010\ \text{m})(0.011\ \text{m/s})}{(2.0 \times 10^{-5}\ \text{m})^2} = 4400\ \text{Pa}$$

Poiseuille's Law

By combining Eq. (*i*) of Problem 14.15 with Eq. (*14.9*) we can get a relationship between the volume flow $\Delta V/\Delta t$ and the pressure difference across a length L of pipe. From Eq. (*i*) we get

$$v_m = \frac{(P_1 - P_2)R^2}{4\eta L} \tag{14.11}$$

Substituting into Eq. (14.9) yields **Poiseuille's law**

$$\frac{\Delta V}{\Delta t} = \frac{\pi R^4(P_1 - P_2)}{8\eta L} \tag{14.12}$$

Note that the volume flow rate depends on the fourth power of the radius R of the pipe, as well as on $(P_1 - P_2)/L$, the change in pressure per unit length along the pipe.

Problem 14.17. Water flows through a horizontal pipe of radius $R = 1.0$ cm and length $L = 300$ m. If the low-pressure end is at atmospheric pressure, what must the gauge pressure at the other end be if the water flows at a rate of 20 cm^3/s? (Assume the water is at a temperature of 20 °C.)

Solution

Solving (*14.12*) for $P_1 - P_2$, we get

$$P_1 - P_2 = \frac{(2.0 \times 10^{-5}\ \text{m}^3/\text{s})(8)(1.0 \times 10^{-3}\ \text{Pl})(300\ \text{m})}{(3.14)(0.010\ \text{m})^4} = 1530\ \text{Pa}$$

Since $P_2 = P_A$, we have $P_{1g} = P_1 - P_A = 1530$ Pa.

Problem 14.18.

(*a*) Assuming the same-length pipe and the same pressure difference $P_1 - P_2$ found in Problem 14.17, what would be the new volume flow rate of the water if the radius were doubled?

(*b*) Assuming the same length of pipe as in Problem 14.17, what would the gauge pressure P_{1g} have to be to maintain the same volume flow rate as in Problem 14.17 if the radius halved?

Solution

(*a*) According to (*14.12*), the volume flow varies as the fourth power of the radius. If all other factors are the same, then doubling R increases the volume flow 16 times. Then, $\Delta V/\Delta t = 320$ cm^3/s.

(*b*) According to (*14.12*), the pressure difference varies inversely as the fourth power of the radius. Then, if the radius drops by a factor of 2, $P_1 - P_2$ must increase 16-fold. So, the new value of $P_{1g} = 16(1530\ \text{Pa}) = 2.45 \times 10^4$ Pa.

Note. The radius of a pipe has an enormous impact on the pressure necessary to maintain a certain volume flow rate. This is a determining factor in a wide range of important phenomena, from the choice of diameters of water mains to the strain on the heart due to blood vessels narrowed by cholesterol buildup.

Stokes' Law

When an object moves through a viscous fluid in such a way that the fluid is in steady flow past it, the viscous forces on the object are, to a good approximation, proportional to the relative velocity and the coefficient of viscosity. As with all frictional forces, viscous forces oppose the direction of motion. The expression for the force will vary with the shape of the object, and, in general, is difficult to determine. However, for the case of a sphere of radius r moving relative to the fluid with velocity v, we have **Stokes' law**, the simple expression

$$F = 6\pi\eta rv \qquad (14.13)$$

Problem 14.19. A lead sphere of mass m and radius r drops from rest just below the surface in a tank of oil (sp gr 0.80).

(*a*) Assuming the relative motion of the oil past the sphere is steady flow, find an expression for the acceleration of the sphere. (Ignore the effect of buoyancy of the oil.)

(*b*) Show that at some point the velocity will reach a maximum value and stay constant thereafter; find an expression for this *terminal velocity*.

Solution

(*a*) The only downward force is the weight of the lead ball mg. Since we are ignoring buoyancy, the only upward force is that due to the viscous "drag" and is given by Stoke's law. Then, choosing downward as positive, we have

$$mg - 6\pi\eta rv = ma \qquad \text{or} \qquad a = g - \frac{6\pi\eta rv}{m} \qquad (i)$$

(*b*) The velocity increases until the acceleration becomes zero. This happens when v reaches the value given by $g = 6\pi\gamma rv/m$. Once that terminal velocity v_T is reached, the ball falls at constant speed

$$v_T = \frac{mg}{6\pi\eta r} \qquad (ii)$$

Problem 14.20. Find the terminal velocity of the lead ball in Problem 14.19 if the radius of the lead ball is 2.0 cm and the viscosity of the oil is $\eta = 2.0$ Pl.

Solution

We need to determine the mass of the lead ball. Using the density of lead from Table 13.1, we get

$$m = d_L\left(\frac{4\pi r^3}{3}\right) = (11.3 \times 10^3\ \text{kg/m}^3)\frac{(4)(3.14)(2.0 \times 10^{-2})^3}{3} = 0.379\ \text{kg}$$

Substituting in the formula above for v_T of a sphere, we get

$$v_T = \frac{(0.379\ \text{kg})(9.8\ \text{m/s}^2)}{6(3.14)(2.0\ \text{Pl})(2.0 \times 10^{-2}\ \text{m})} = 4.93\ \text{m/s}$$

Problem 14.21.

(*a*) How would the answer to Problem 14.19(*a*) change if buoyancy were included?

(*b*) What is the new expression for the terminal velocity if buoyancy is included?

(*c*) What is the solution to Problem 14.20 if buoyancy is included?

Solution

(*a*) There is now an additional upward force due to buoyancy, $B = d_o g(4\pi r^3/3)$, where d_o is the density of oil. Thus, we get

$$mg - d_o g\left(\frac{4\pi r^3}{3}\right) - 6\pi\eta r v = ma$$

Solving for a and using $m = d_L(4\pi r^3/3)$, we get

$$a = g - \frac{d_o}{d_L}g - \frac{6\pi\eta r v}{m} \qquad (i)$$

(*b*) The terminal velocity is attained when $a = 0$, which yields

$$v_T = mg\,\frac{1 - d_o/d_L}{6\pi\eta r}$$

(*c*)
$$v_T = 0.379\text{ kg}\,\frac{(9.8\text{ m/s}^2)(1.0 - 0.80/11.3)}{6(3.14)(2.0\text{ Pl})(2.0\times 10^{-2}\text{ m})} = 4.58\text{ m/s}$$

Problem 14.22. Suppose that a raindrop has a radius $r = 2.0$ mm and falls from rest from a cloud. Assuming steady flow, use Stokes' law to find the terminal velocity.

Solution

We ignore the buoyancy of air and use Eq. (*ii*) of Problem 14.19(*b*) and the value of viscosity of air from Table 14.1. The mass of the drop is

$$m = d_w V = d_w\left(\frac{4\pi r^3}{3}\right) = (1000\text{ kg/m}^3)\,\frac{4(3.14)(2.0\times 10^{-3}\text{ m})^3}{3} = 3.35\times 10^{-5}\text{ kg}$$

Then
$$v_T = \frac{(3.35\times 10^{-5}\text{ kg})(9.8\text{ m/s}^2)}{6(3.14)(1.8\times 10^{-5}\text{ Pl})(2.0\times 10^{-3}\text{ m})} = 484\text{ m/s}$$

Note. This is much larger than the velocity of typical raindrops. This is a consequence of our assumption that the flow is steady and that Stoke's law holds. In fact, above a certain velocity, turbulence sets in, and our assumption is no longer valid.

Turbulence and Reynolds Number

When the velocity of a fluid in a pipe increases past a certain value, or the relative velocity of an object through a fluid increases past a certain velocity, the flow changes from steady to turbulent. Turbulent flow is characterized by local vortices of motion that randomly change from moment to moment. In such situations the viscous forces vary with velocity as v^2 or as higher powers of v. A crude but effective determination of the onset of turbulent flow is given by the **Reynolds number** R. This is a dimensionless quantity that depends on four factors: the density d of the flowing fluid, the coefficient of viscosity η, the average relative velocity of the fluid v, and the characteristic linear dimension L of the solid boundary. For flow through a pipe, L is the diameter of the pipe. For an object moving through a fluid, L can be taken as some average linear dimension of the object facing into the fluid flow. In all cases the expression for the Reynolds number is

$$R = \frac{dvL}{\eta} \tag{14.14}$$

When R exceeds a certain value for the particular geometry at hand, the flow turns from steady to turbulent. A good rule of thumb for fluids flowing through a pipe is that when R exceeds 2000, the flow becomes turbulent. Similarly, for a sphere moving through a fluid, the critical value of R is about 10.

Problem 14.23.

(*a*) Find the Reynolds number for the ball falling through oil in Problem 14.19 and 14.20. Does the flow turn turbulent before the ball reaches critical velocity?

(*b*) Repeat for the raindrop of Problem 14.22.

Solution

(*a*) Letting L equal the diameter of the ball, we have, at the terminal velocity calculated in Problem 14.20,

$$R = \frac{(800\ \text{kg/m}^3)(4.93\ \text{m/s})(0.040\ \text{m})}{2.0\ \text{Pl}} = 79$$

The flow is turbulent.

(*b*) Here the diameter is 0.0040 m, and the density of air is found from Table 13.1. Thus, $R = (1.29\ \text{kg/m}^3)(484\ \text{m/s})(0.0040\ \text{m})/(1.8 \times 10^{-5}) = 139 \times 10^3$. Clearly, turbulence sets in long before a velocity of 484 m/s is reached. As a consequence the drag force increases much more rapidly with speed, and the terminal velocity of raindrops is much lower.

Problem 14.24.

(*a*) Assuming that at $R = 10$ we get turbulence, find the transition velocity for the raindrop in Problem 14.22.

(*b*) Find the transition velocity for water flowing through a pipe of diameter 5.0 cm, at a temperature of 20°C.

Solution

(*a*) We can find v from (*14.14*):

$$10 = \frac{[(1.29\ \text{kg/m}^3)(0.0040\ \text{m})]}{1.8 \times 10^{-5}\ \text{Pl}}\, v \qquad \text{or} \qquad v = 3.49\ \text{cm/s}$$

(*b*) We proceed the same way, except that now we have

$$2000 = \frac{[(1000\ \text{kg/m}^3)(0.050\ \text{m})]}{1.0 \times 10^{-3}\ \text{Pl}}\, v \qquad \text{or} \qquad v = 0.040\ \text{m/s}$$

Problems for Review and Mind Stretching

Problem 14.25. Referring to Problem 14.4 and the pipe in Fig. 14-3(*a*), assume zero viscosity and that the pressure at point 1 is $P_1 = 4.023 \times 10^3$ lb/ft^2.

(*a*) Find the pressure at points 2 and 3.

(*b*) How would v_2 change if the diameter of the middle section of pipe doubled?

(*c*) Assuming the changes of (*b*), how would the pressure and velocity change at point 3?

Solution

(*a*) From Problem 14.4, $v_1 = 0.80$ ft/s, $v_2 = 6.4$ ft/s, $v_3 = 2.4$ ft/s. Bernoulli's equation applied to a horizontal pipe, with $d = 1.94$ slugs/ft^3, gives

$$P_1 + \tfrac{1}{2}dv_1{}^2 = P_2 + \tfrac{1}{2}dv_2{}^2 \Rightarrow 4.023 \times 10^3 \text{ lb/ft}^2 + (0.97 \text{ slug/ft}^3)(0.80 \text{ ft/s})^2$$
$$= P_2 + (0.97 \text{ slug/ft}^3)(6.4 \text{ ft/s})^2$$

so that $P_2 = 3984$ lb/ft^2. Similarly, the pressure at point 3 is given by

$$P_1 + \tfrac{1}{2}dv_1{}^2 = P_3 + \tfrac{1}{2}dv_3{}^2 \Rightarrow 4.023 \times 10^3 \text{ lb/ft}^2 + (0.97 \text{ slug/ft}^3)(0.80 \text{ ft/s})^2$$
$$= P_3 + (0.97 \text{ slug/ft}^3)(2.4 \text{ ft/s})^2$$

so that $P_3 = 4018$ lb/ft^2.

(*b*) If the diameter doubles, the area quadruples. Then from the continuity equation $v_1A_1 = v_2A_2$, we deduce that if A_2 increases by a factor of 4, v_2 decreases to one-fourth its prior value, or 1.6 ft/s.

(*c*) The values of v_3 is unchanged since the value of A_3 hasn't changed, and from the continuity equation, applied to points 1 and 3, $v_1A_1 = v_3A_3$, which implies v_3 is the same. Since v_3 hasn't changed, Bernoulli's equation applied to points 1 and 3 implies P_3 hasn't changed either.

Problem 14.26. Assume that the cross-sectional areas at points 1, 2, and 3 of the pipe shown in Fig. 14-3(*b*) have the same dimensions as those of Problem 14.25(*a*) and that P_1 and v_1 are as given in that problem. If $h = 30$ ft, calculate the pressures at points 2 and 3.

Solution

Let the zero of gravitational potential energy be chosen at point 1. Then, Bernoulli's equation for points 1 and 2 becomes

$$P_1 + \tfrac{1}{2}dv_1{}^2 + dgh_1 = P_2 + \tfrac{1}{2}dv_2{}^2 + dgh_2$$

$$4.023 \times 10^3 \text{ lb/ft}^2 + (0.97 \text{ slug/ft}^3)(0.80 \text{ ft/s})^2 + 0 = P_2 + (0.97 \text{ slug/ft}^3)(6.4 \text{ ft/s})^2$$
$$+ (62.5 \text{ lb/ft}^3)(30 \text{ ft})$$

Solving, we get $P_2 = 2109$ lb/ft^2. Noting that $h_3 = h_2$, we follow a similar procedure to get $P_3 = 2143$ lb/ft^2.

Problem 14.27. Consider the situation in Fig. 14-7 (Problem 14.7), where the liquid is water. Show that the net force F exerted on the liquid leaving through the hole is exactly twice the net hydrostatic force exerted on the same area if no hole were there (e.g., assuming the hole is capped).

Solution

From Torricelli's theorem (Problem 14.7) we have that the velocity of the liquid leaving the tank is $v = \sqrt{2gh}$. The volume of liquid leaving per second is va, and the mass leaving per second is thus dva. Since the liquid acquires a velocity v perpendicular to the tank as it leaves, the total change in momentum per unit time for liquid leaving the tank is $(dva)v = dv^2a = d(2gh)a$. From Newton's second law the force F is the change in momentum per unit time, or $F = 2dgha$. If there were no hole in the tank, the hydrostatic pressure on the same area from the inside would be $P = P_A + dgh$. The atmosphere would be pushing from the outside with atmospheric pressure P_A. The net force F' on the area a is therefore just due to the gauge pressure and is

$$F' = (P - P_A)a = dgha = \tfrac{1}{2}F$$

Problem 14.28. Water flows through a thin horizontal tube (effects of viscosity *cannot* be ignored), and pours out of the right end (Fig. 14-14).

(*a*) If h_1, the height of water in the venturi tube above point 1, is 3.0 cm, find the gauge pressure P_{g1} at point 1.

(*b*) Find the maximum velocity v_m of the water in the tube.

(*c*) Find the volume rate of flow of the water out of the pipe.

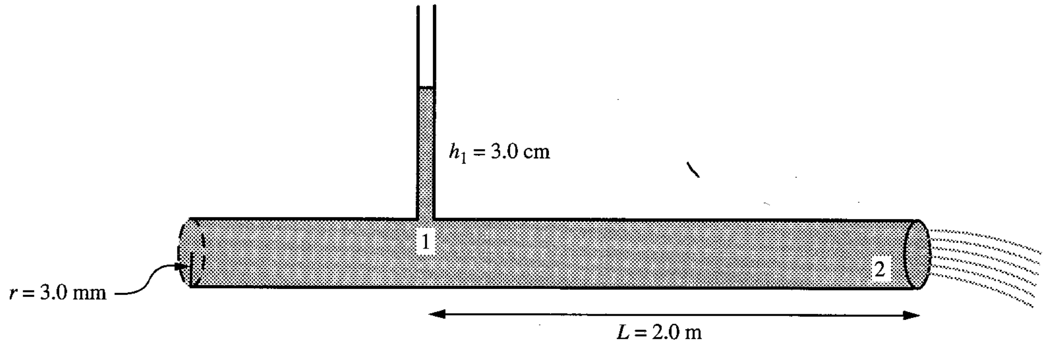

Fig. 14-14

Solution

(*a*) Since the venturi tube is open to the atmosphere, the gauge pressure is just due to the height of liquid:

$$P_{g1} = dgh_1 = (1000\ \text{kg/m}^3)(9.8\ \text{m/s}^2)(0.030\ \text{m}) = 294\ \text{Pa}$$

(*b*) To find the velocity v_m, we use Eq. (*14.11*) applied to points 1 and 2: $v_m = (P_1 - P_2)r^2/(4\eta L)$. Since $P_2 = P_A$, and $P_1 - P_A = P_{g1}$, we get

$$v_m = \frac{(294\ \text{Pa})(3.0 \times 10^{-3}\ \text{m})^2}{(4)(1.00 \times 10^{-3}\ \text{Pl})(2.0\ \text{m})} = 0.331\ \text{m/s}$$

(*c*) From Poiseuille's law (*14.12*) we have

$$\frac{\Delta V}{\Delta t} = \frac{\pi r^4(P_1 - P_2)}{8\eta L} = \frac{(3.14)(3.0 \times 10^{-3}\ \text{m})^4\,(294\ \text{Pa})}{(8)(1.00 \times 10^{-3}\ \text{Pl})(2.0\ \text{m})} = 4.67 \times 10^{-6}\ \text{m}^3/\text{s} = 4.67\ \text{cm}^3/\text{s}$$

Problem 14.29.

(*a*) Referring to Problem 14.28, what would be the effect of doubling the radius r of the tube on the velocity and on the volume flow if everything else, including the pressure P_1 remained the same?

(*b*) What is the Reynolds number R for the water flowing in the tube of Problem 14.28? Is the flow steady or turbulent?

(*c*) Find the Reynolds number if the radius were doubled. Would the flow be steady or turbulent?

Solution

(*a*) From Eq. (*14.11*), we see that doubling r quadruples v_m. From Eq. (*14.12*) we see that the volume flow rate goes as r^4, so doubling r will increase $\Delta V/\Delta t$ 16-fold.

(*b*) For our tube of radius r we have, using average velocity $v = v_m/2$:

$$R = \frac{d_w v(2r)}{\eta} = \frac{(1000\ \text{kg/m}^3)(0.1655\ \text{m/s})(6.0 \times 10^{-3}\ \text{m})}{1.00 \times 10^{-3}\ \text{Pl}} = 993$$

Since the critical Reynolds number is 2000, the flow is steady in this case.

(*c*) If the radius is doubled, then v increases fourfold, and R increases eightfold. Therefore, now $R = 7944$, and the flow is turbulent.

Supplementary Problems

Problem 14.30. Water flows without viscosity through a pipe at a rate of 12 cm^3/s. Find the velocity of the water (*a*) at a point where the cross section of the pipe is 3.0 cm^2, (*b*) at a point where the radius of the pipe is 0.50 cm.

Ans. (*a*) 4.0 cm/s; (*b*) 15.3 cm/s

Problem 14.31. If the pipe of Problem 14.30 is horizontal, find the pressure difference between the points of parts (*a*) and (*b*).

Ans. 10.9 Pa

Problem 14.32. The venturi setup of Fig. 14.9(*c*) is used to calculate the speed of a gas of density 2.3 kg/m^3 moving through the pipe. If the height difference of the mercury in the two sides of the tube is 15 mm, find the velocity of the gas.

Ans. 41.7 m/s

Problem 14.33. A large tank filled with water to a height $h_1 = 2.0$ m is fitted with a pipe at its bottom (Fig. 14-15). Assume that $h_2 = 30$ cm and that the cross-sectional areas of the vertical and horizontal portions of the pipe are $a_2 = 25\ \text{cm}^2$ and $a_3 = 15\ \text{cm}^2$. Assuming that viscosity can be ignored, find (*a*) the velocity of the water leaving the pipe at point 3; (*b*) the velocity of the water at point 2, which is *just below* the bottom of the tank; (*c*) the gauge pressure at point 2.

Ans. (*a*) 6.71 m/s; (*b*) 4.03 m/s; (*c*) 11.5 kPa

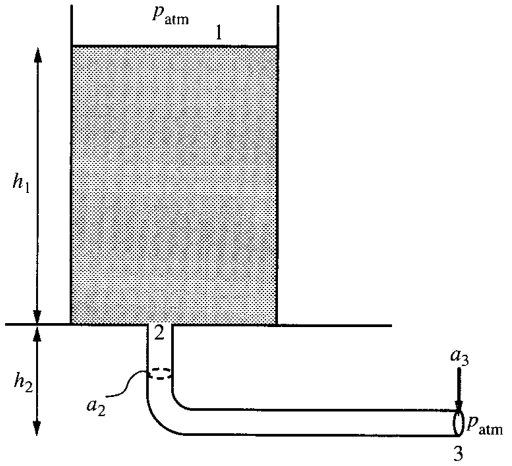

Fig. 14-15

Problem 14.34. Given the pressure p and the volume flow rate $\Delta V/\Delta t$ at a cross section of a pipe, find an expression for the instantaneous power exerted by the liquid behind the cross section on the liquid in front of it.

Ans. power $= Fv = pAv = p\,\Delta V/\Delta t)$

Problem 14.35. A **siphon**, as illustrated in Fig. 14-16, is a system in which one end of a tube rises above a tank in which it is immersed and the other end discharges fluid at a level below the surface of the liquid in the tank. Assume that the tube has a uniform cross sections and that viscosity can be ignored.

(*a*) If there is no liquid initially in the tube, the siphon will not work. Why not?

(*b*) Assume that liquid has been drawn up into the tube. How far along the tube must the liquid be before it will continue to flow on its own?

Ans. (*a*) The air in the tube will exert the same pressure as the air over the rest of the liquid surface in the tank, so the levels will remain the same; (*b*) Just below point 4 (the liquid must completely fill the tube, leaving no air gaps)

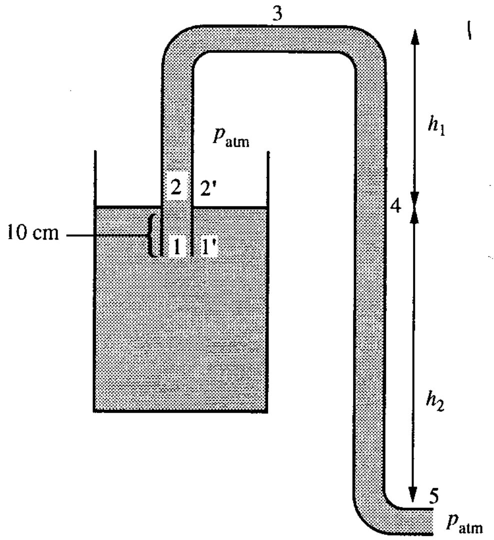

Fig. 14-16

Problem 14.36. For the siphon of Fig. 14-16, assume the liquid is gasoline (sp gr 0.90); $h_1 = 40$ cm and $h_2 = 60$ cm. The gasoline flows out of the tube at point 5. Find: (*a*) the speed with which the gasoline leaves the tube, (*b*) the gauge pressure at point 4, (*c*) the gauge pressure at point 3, (*d*) the gauge pressure at point 2.

Ans. (*a*) 3.43 m/s; (*b*) −5.29 kPa; (*c*) −8.82 kPa; (*d*) −5.29 kPa

Problem 14.37. For the situation in Problem 14.36, find the gauge pressure (*a*) at point 2′, at the same level as point 2 but outside the tube on the surface of the liquid of the tank; (*b*) at point 1, which is just inside the tube and 10 cm below the level of liquid in the tank; (*c*) at point 1′, which is at the same level as point 1 but in the bulk of the liquid. (*d*) If gasoline leaves the tube at 2.0 L/min, what is the cross-sectional area of the tube?

Ans. (*a*) 0; (*b*) −4.41 kPa; (*c*) 882 Pa; (*d*) 9.7 mm^2

Problem 14.38. A firefighter holds a hose as in Fig. 14-8. The firefighter finds that it is necessary to exert a horizontal force of 50 N on the hose to keep it still. If the cross-sectional area of the nozzle is 12 cm^2, find the velocity with which the water leaves the hose.

Ans. 6.45 m/s

Problem 14.39. Heavy oil ($\eta = 9.9$ Pl) is forced through a horizontal pipe of radius 15 mm and length 0.80 m. The high-pressure end is at a gauge pressure of 50 kPa, while the other end is open to the atmosphere.

(*a*) Find the velocity of the oil on the central axis of the pipe.

(*b*) Find the volume of oil leaving the pipe per second.

Ans. (*a*) 0.355 m/s; (*b*) 125 mL/s

Problem 14.40.

(*a*) Referring to Problem 14.39, what would the new gauge pressure have to be to push the oil through with the same velocity, if the radius were half as large and the length were twice as great?

(*b*) Repeat (*a*) if the volume flow, not the velocity, stays the same.

Ans. (*a*) 400 kPa; (*b*) 1600 kPa

Problem 14.41.

(*a*) Find the Reynolds number for the flow in Problem 14.39 and determine whether the assumption of steady flow is valid. Assume the specific gravity of the oil is 0.75

(*b*) Repeat for the situation in Problem 14.40(*a*).

(*c*) Repeat for the situation in Problem 14.40(*b*).

Ans. (*a*) 0.40 (assumption valid); (*b*) 0.20 (assumption valid); (*c*) 0.80 (assumption valid)

Problem 14.42. Suppose that an oil additive reduces the viscosity of the oil in Problems 14.39–14.41 to one-sixtieth its original value.

(*a*) Find the new value for the velocity of the oil in Problem 14.39.

(*b*) Find the new value of the Reynolds number for this flow, and determine if the flow is steady or turbulent.

(*c*) If the conditions of Problem 14.40 are imposed, find the new answers for Problems 14.41(*b*) and 14.41(*c*).

Ans. (*a*) 21.3 m/s; (*b*) 1440 (steady); (*c*) 720 (steady) and 2880 (turbulent)

Problem 14.43. A sphere of radius 2.0 mm and mass 34.0×10^{-6} kg is released from rest underwater in a deep lake. Assuming Stokes' law holds, and including the effects of buoyancy of the water, find the terminal velocity of the sphere.

Ans. 0.130 m/s

Problem 14.44. Consider the droplet of Problem 14.22. Suppose the terminal velocity is found to be 0.30 m/s. If the drag force for turbulent flow is of the form $F_D = Bv^2$, where v is the relative velocity and B is a constant depending on the viscosity and other factors, find the value of B.

Ans. $0.0036\ \text{N} \cdot \text{s}^2/\text{m}^2$

Problem 14.45. Show that the Reynolds number is dimensionless.