Chapter 13

Fluids at Rest (Hydrostatics)

13.1 INTRODUCTION

Liquids and gases are classified as fluids. In Chap. 11 we saw that solid objects are not perfectly rigid, but rather they respond to stress by distorting to some extent. In particular, if one applies a shear force to a solid, the solid responds to the stress that is created by distorting, as shown in Fig. 11-2.

Liquids resist compression in much the same way solids do, but they are unable to resist shear forces. As long as a shear force acts, the liquid will flow. Only when all shear forces are eliminated will the liquid cease to move. Indeed the fact that a liquid at rest takes the shape of its container and has a horizontal free surface is a consequence of the requirement that there be zero shear force between any two layers in the liquid, as well as between the liquid and the surfaces it is in contact with. A characteristic of liquids which distinguishes them from gases is that the molecules of a liquid are sufficiently bound to each other that the overall volume of a liquid doesn't significantly change even as the liquid changes shape to accommodate itself to different containers.

Gases, like liquids and solids, can resist compression, though much more weakly. Gases, like liquids, cannot resist shear forces, and in equilibrium no such forces are exerted by the walls of a closed container or by one part of the gas on another. Gases, however, differ from liquids in that the molecules of a gas have extremely weak interactions with each other. Gas molecules move about freely except for direct collisions with each other or with the walls of a confining container. A gas's resistance to compression comes not from intermolecular forces that resist the molecules being pushed closer together but rather from the collision of gas molecules with the walls of the container. Since the molecules of a gas have no grip on each other, a gas will expand to fill the volume of the container it is placed in.

In this chapter we will examine fluids at rest, the subject of *hydrostatics*. In the next chapter we will examine the properties of fluids in motion, the subject of *hydrodynamics*.

13.2 DENSITY AND PRESSURE OF FLUIDS

Density

The **density** d of any substance is defined as the mass per unit volume of the substance. If we have a uniform sample of materials (solid, liquid, or gas) of mass M and volume V, then

$$d = \frac{M}{V} \tag{13.1}$$

The density is an intrinsic property of a substance; that is, it does not depend on the size of the sample. Density, however, can vary with such factors as temperature and pressure. Table 13.1 gives the densities of a variety of substances at atmospheric pressure and room temperature (20°C or 68°F). The SI unit of density is the kg/m^3. Other units are g/cm^3 and $slug/ft^3$. In the foot-pound system of units it is usual to define the weight density d_w or weight per unit volume

$$d_w = \frac{W}{V} \tag{13.2}$$

Table 13.1. Densities

Material	Density, kg/m^3
Helium	0.18
Air	1.29
Benzene	880
Ethyl alcohol	810
Water	1,000
Aluminum	2,700
Brass	8,600
Gold	19,300
Ice	917
Iron	7,860
Lead	11,300
Mercury	13,600
Silver	10,500
Steel	7,860
Uranium	18,700

All figures at atmospheric pressure and room temperature; weight density of water, given for reference, is 62.5 lb/ft^3.

Since $W = Mg$, we have that

$$d_w = dg \tag{13.3}$$

The units of d_w are lb/ft^3, N/m^3, and dyn/cm^3.

Another quantity commonly used to describe densities is the specific gravity (sp gr) of a substance, which is defined as the ratio of the density of the substance to that of water:

$$\text{sp gr (X)} = \frac{d(\text{X})}{d(\text{H}_2\text{O})} \tag{13.4}$$

where X is the substance in question. This ratio is dimensionless and gives us a scale of densities relative to water. Since the density of water in CGS units is $d(H_2O) = 1.0\ g/cm^3$, it follows that the specific gravity of any substance is numerically equal to its density in g/cm^3.

We can also express specific gravity in terms of weight density. Since $d_w = dg$, we have

$$\text{sp gr (X)} = \frac{d_w(\text{X})}{d_w(\text{H}_2\text{O})} \tag{13.5}$$

[Note that $d_w(H_2O) = 62.5\ lb/ft^3$.]

Problem 13.1. A cubical block 10 cm on a side has a mass $M = 2700$ g.

(*a*) Find the density in kg/m^3; if the block is a pure substance listed in Table 13.1, what is it?

(*b*) Find the specific gravity of the substance. If the weight density of water is $d_w = 62.5\ \text{lb/ft}^3$, find the weight density of the substance in lb/ft^3.

(*c*) What would the mass of the block be if it was made of iron?

Solution

(*a*) $d = M/V = (2.70\ \text{kg})/(0.10\ \text{m})^3 = 2.70 \times 10^3\ \text{kg/m}^3$; the substance is aluminum.

(*b*) sp gr $= d(\text{Al})/d(H_2O) = (2.70 \times 10^3\ \text{kg/m}^3)/(1.00 \times 10^3\ \text{kg/m}^3) = 2.70$ [or $d_{\text{Al}} = (2700\ \text{g})/(10\ \text{cm})^3 = 2.70\ \text{g/cm}^3 \Rightarrow (\text{sp gr})_{\text{Al}} = 2.70$.]; $d_w = (\text{sp gr})d_w(H_2O) = 2.70(62.5\ \text{lb/ft}^3) = 169\ \text{lb/ft}^3$.

(*c*) If the block were of iron, we have $d(\text{I}) = M_{\text{I}}/V$, so that $M_{\text{I}} = d(\text{I})V$. From Table 13.1, we get $M_{\text{I}} = (7.86 \times 10^3\ \text{kg/m}^3)(0.10\ \text{m})^3 = 7.86\ \text{kg}$.

Pressure

The pressure P on any surface is defined as the force per unit area acting perpendicular to that surface. We assume here that the forces are spread out smoothly over some area, instead of acting at discrete points. This is illustrated in Fig. 13-1. To be precise, the pressure at a point is defined by considering the total infinitesimal force ΔF acting perpendicular to an infinitesimal area ΔA around the point in question. Then

$$P = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} \tag{13.6}$$

The pressure can, in general, vary from location to location along a surface. Unlike force, pressure is not defined as a vector. For our purposes we can think of pressure as a scalar quantity, keeping in mind that the action of the pressure on a given surface is always perpendicular to the surface. As discussed in Chap. 11, the international unit of pressure is the pascal (Pa): 1 Pa = 1 N/m^2. Other commonly used units of pressure are the dyn/cm^2, lb/ft^2, and lb/in^2.

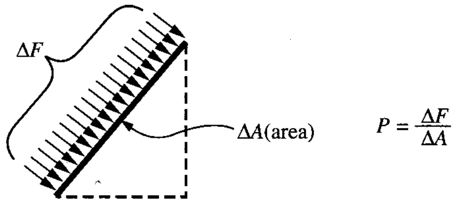

Fig. 13-1

We now examine some basic properties of the pressure in a fluid at rest (**hydrostatic pressure**). In Fig. 13-2 we examine a number of real and imaginary boundaries within such a fluid. In Fig. 13-2(*a*) we consider a small, infinitesimally thin and massless, disklike object placed somewhere in the fluid. We wish to compare the pressure due to the fluid on each side of the disk. The force on the disk due to the molecules of fluid pushing on its upper side is the same as the force those molecules would exert on the molecules of fluid on the other side of the disk if the disk were not there. From Newton's third law the molecules on the other side must exert an equal and opposite force. Since the area of the disk is the same as viewed from either side, we conclude that the pressure on each side of the disk is the same. Thus:

(*a*) *For any point in a fluid at rest, the pressure on one side of a small surface is the same as the pressure on the other side.*

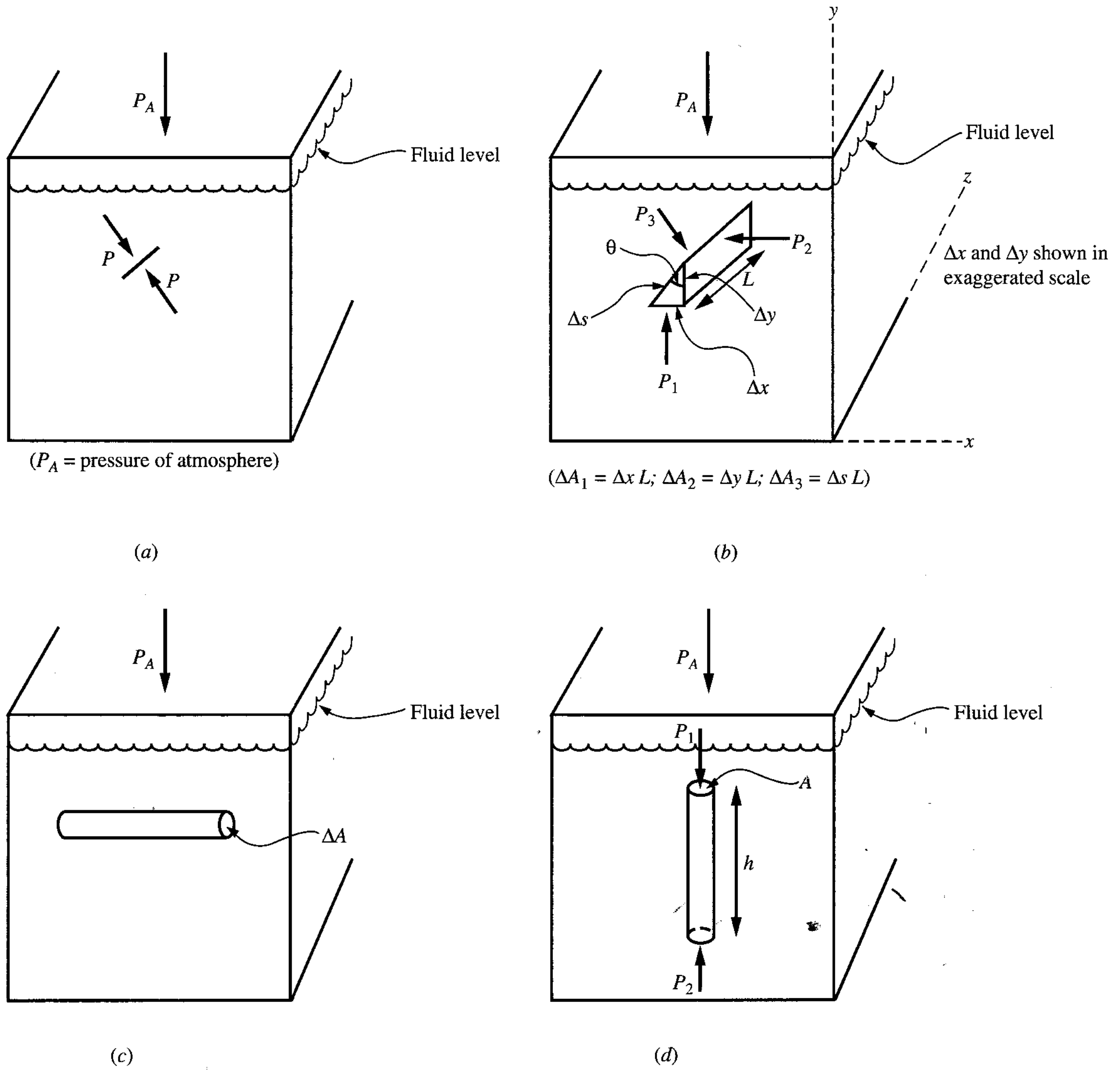

Fig. 13-2

In Fig. 13-2(*b*) we have an imaginary triangular-shaped boundary enclosing some of the fluid. We assume the boundary is infinitesimal in the x and y directions, as shown, and extends a distance L in the z direction. The fluid inside the boundary is in equilibrium, so the x components of force on it add up to zero, as do the y components. Since there can be no shear forces, the forces in the xy plane are due only to the pressure perpendicular to the three surfaces ΔA_1, ΔA_2, and ΔA_3, which extend in the z direction, as well as due to gravity. (The forces on the end faces are only in the z direction and don't contribute to the x and y directions.) The magnitudes of the forces are $F_1 = P_1\,\Delta A_1 = P_1\,\Delta x\, L$, $F_2 = P_2\,\Delta A_2 = P_2\,\Delta y\, L$, $F_3 = P_3\,\Delta A_3 = P_3\,\Delta s\, L$, and $W = Mg = dVg$, where d = density of water and V is the triangular volume $V = \frac{1}{2}\Delta x\,\Delta y\, L$, so $W = \frac{1}{2} dg\Delta x\,\Delta y\, L$. If Δx and Δy are chosen small enough, W is negligibly small compared to the other forces because W depends on a product of two infinitesimals, while F_1, F_2, and F_3, each depend on one infinitesimal. We assume that Δx and Δy are chosen small enough so that W is negligible compared to the other forces. Then, for equilibrium in the y direction [from Fig. 13-2(*b*)],

$$F_3 \sin\theta = F_1 \qquad \text{or} \qquad P_3\,\Delta s \sin\theta = P_1\,\Delta x$$

Since $\Delta s \sin\theta = \Delta x$, we have $P_3 = P_1$. Similarly, for the x direction,

$$F_3 \cos\theta = F_2 \qquad \text{or} \qquad P_3\,\Delta s \cos\theta = P_2\,\Delta y$$

Since $\Delta s \cos\theta = \Delta y$, we have $P_3 = P_2$. Thus, $P_1 = P_2 = P_3$, and, since the angle θ was chosen arbitrarily, we conclude:

(*b*) *The pressure at a given point in a fluid at rest has a definite value that represents the force per unit area on a small surface placed at that point, oriented in any arbitrary direction.*

In Fig. 13-2(*c*) we consider a thin, imaginary, horizontal cylindrical boundary, with end faces of area ΔA. If we consider the fluid inside the cylinder to be our system, equilibrium requires that the sum of the horizontal forces on the cylinder add up to zero. The only horizontal forces are due to the pressure on the two end faces, since there can be no shear force along the length of the cylinder. Since ΔA is the same on both sides, we must have that the pressures at the two ends are equal in magnitude, so we conclude:

(*c*) *The pressure in a fluid at rest is the same at all points on a horizontal plane.*

In Fig. 13-2(*d*) we consider an imaginary vertical cylinder of height h and cross section A. Again, the fluid inside is in equilibrium, so the sum of the vertical forces must add up to zero. The vertical forces are those due to gravity, the pressure P_1 exerted downward on the top of the cylinder, and the pressure P_2 exerted upward on the bottom of the cylinder. Letting d be the density of the fluid, we have $P_1A + dVg = P_2A$. But, since $V = hA$, we have $P_1A + dghA = P_2A$, and

$$P_2 - P_1 = dgh \tag{13.7}$$

(Here we have assumed that the density remains essentially constant at all depths in the cylinder. Unless h is very large this is a good approximation for most liquids, since liquids are not easily compressed.)

Our overall conclusions about hydrostatic pressure can be summed up as follows: *The pressure in a fluid at rest varies only with the depth, in accord with Eq. (13.7), and at any given depth in a fluid the pressure exerted by the fluid in any direction is the same.*

Problem 13.2. In Fig. 13-3, we show a rectangular tank filled with water to a depth of 20 m. The horizontal cross-sectional area of the tank is $A = 25\ \text{m}^2$, and the pressure on the top of the liquid is due to the atmosphere and is $P_A = 1.01 \times 10^5$ Pa. Find (*a*) the pressure at point 2, at a depth of 12 m, as shown; (*b*) the pressure exerted by the water on the wall of the container at point 3; (*c*) the pressure exerted by the water on the bottom of the container at point 4.

Solution

(*a*) From Eq. (*13.7*), we have

$$\begin{aligned} P_2 = P_A + dgh &= (1.01 \times 10^5\ \text{Pa}) + (1000\ \text{kg/m}^3)(9.8\ \text{m/s}^2)(12\ \text{m}) \\ &= 1.01 \times 10^5\ \text{Pa} + 1.18 \times 10^5\ \text{Pa} = 2.19 \times 10^5\ \text{Pa} \end{aligned}$$

(*b*) Points 3 and 2 are at the same level, so $P_3 = P_2 = 2.19 \times 10^5$ Pa.

(*c*) $P_4 = P_A + dgh' = (1.01 \times 10^5\ \text{Pa}) + (1000\ \text{kg/m}^3)(9.8\ \text{m/s}^2)(20\ \text{m}) = 1.01 \times 10^5\ \text{Pa} + 1.96 \times 10^5\ \text{Pa} = 2.97 \times 10^5$ Pa.

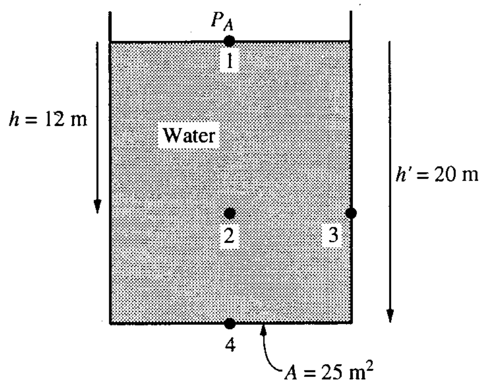

Fig. 13-3

Problem 13.3.

(*a*) For the situation of Problem 13.2, find the total force exerted by the water on the bottom of the tank.

(*b*) Do a similar calculation for the force on one of the sides of the tank, where the water covers an area $A_{side} = 50\ m^2$.

Solution

(*a*) Since the pressure is the same everywhere along the bottom of the tank, the total downward force exerted by the water on the bottom is $F = PA = (2.97 \times 10^5\ Pa)(25\ m^2) = 74.3 \times 10^5$ N.

(*b*) To find the total force on the side of the container, one would have to take into account the fact that the pressure varies with depth. In general this type of problem would require the use of calculus. In this case, however, Eq. (*13.7*) tells us that the pressure varies linearly with the depth so that the average pressure P_{av} is

$$P_{av} = \frac{P_{top} + P_{bottom}}{2} = \frac{P_A + P_4}{2} = \frac{1.01 \times 10^5\ Pa + 2.97 \times 10^5\ Pa}{2} = 1.99 \times 10^5\ Pa$$

The force would then be

$$F_{side} = P_{av}A_{side} = (1.99 \times 10^5\ Pa)(50\ m^2) = 99.5 \times 10^5\ N$$

Problem 13.4. It is often useful to refer to the difference between the actual, or absolute, pressure, P in a fluid, and the pressure exerted by the atmosphere P_A, which pervades the surface of the earth. This difference is called the **gauge pressure** P_g. Thus, $P_g = P - P_A$.

(*a*) Find the gauge pressure at points 2 and 4 of Problem 13.2.

(*b*) Find the gauge pressure at the top of the liquid surface in Problem 13.2.

(*c*) Assuming that the walls of the tank in Problem 13.3 are surrounded by air, show that the net force on the side of the tank due to all fluids, both inside and out, is equivalent to the force due to the gauge pressure of the water alone.

Solution

(*a*) From Problem 13.2, we have

$$P_{g2} = 1.18 \times 10^5\ Pa \qquad \text{and} \qquad P_{g4} = 1.96 \times 10^5\ Pa$$

(*b*) At the top surface of the water the only pressure is atmospheric, so $P_{g,\text{top}} = 0$.

(*c*) In Problem 13.3, we calculated the outward force on a side of the tank exerted by the water inside. In addition, however, the air outside the tank exerts an inward force on the side of magnitude $F' = P_A A_{\text{side}} = (1.01 \times 10^5\ \text{Pa})(50\ \text{m}^2) = 50.5 \times 10^5$ N. The *net* force due to all fluids is thus

$$F_{\text{net}} = F - F' = 99.5 \times 10^5\ \text{N} - 50.5 \times 10^5\ \text{N} = 49 \times 10^5\ \text{N, outward.}$$

To see that this is equal to the force due to the gauge pressure of the water alone, we obtain the average gauge pressure: $P_{g,\text{av}} = \frac{1}{2}(0\ \text{Pa} + 1.96 \times 10^5\ \text{Pa}) = 0.98 \times 10^5$ Pa. Then we have $F_g = P_{g,\text{av}} A_{\text{side}} = (0.98 \times 10^5\ \text{Pa})(50\ \text{m}^2) = 49 \times 10^5$ pA, which is just F_{net} obtained above.

Problem 13.5. In Fig. 13-4 we show an unusually shaped container filled with a liquid of density d.

(*a*) Show that Eq. (*13.7*) still holds for any two depths in the liquid.

(*b*) If $h_3 = 15$ m, and the liquid is benzene, find the pressure and the gauge pressure at the bottom of the container.

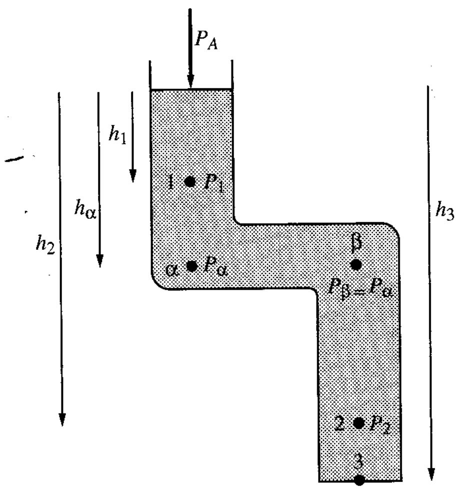

Fig. 13-4

Solution

(*a*) Choosing arbitrary points 1 and 2, we wish to show that $P_2 = P_1 + dg(h_2 - h_1)$. The arguments that led to Eq. (*13.7*) are valid for the points 1 and α, since they lie vertically above one another. Then $P_\alpha = P_1 + dg(h_\alpha - h_1)$. Furthermore, $P_\beta = P_\alpha$, since they are at the same horizontal level. We also know that

$$P_2 = P_\beta + dg(h_2 - h_\beta) \qquad \text{or} \qquad P_2 = P_\alpha + dg(h_2 - h_\alpha)$$

Substituting our previous expression for P_α, we get

$$P_2 = P_1 + dg(h_\alpha - h_1) + dg(h_2 - h_\alpha) = P_1 + dg(h_2 - h_1)$$

(*b*) From Eq. (*13.7*) we have

$$P_3 = P_A + dgh_3 = (1.01 \times 10^5\ \text{Pa}) + (8.8 \times 10^2\ \text{kg/m}^3)(9.8\ \text{m/s}^2)(15\ \text{m})$$

or
$$P_3 = 1.01 \times 10^5\ \text{Pa} + 1.29 \times 10^5\ \text{Pa} = 2.30 \times 10^5\ \text{Pa}$$

Similarly, $P_{g3} = 1.29 \times 10^5$ Pa.

13.3 SOME PRACTICAL RESULTS

We now look at a number of practical consequences of the results we have obtained for fluids at rest. These include such rules as "water seeks its own level," Pascal's principle, and the hydraulic press, all developed in the framework of the problems presented below.

Problem 13.6. A set of Pascal vases, a number of unusually shaped containers open to the atmosphere at the top and connected by a horizontal tube at the bottom, is shown in Fig. 13-5. Show that when liquid is poured into the system, the liquid rises to the same level in all the containers. This phenomenon is described by the observation that "water seeks it own level."

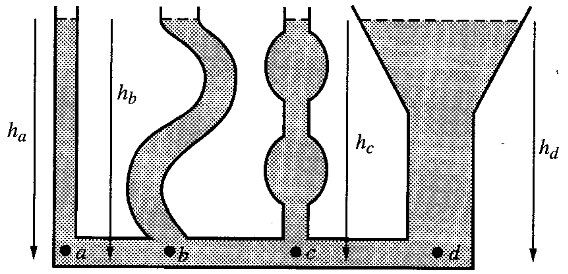

Fig. 13-5

Solution

Consider the pressures at the bottom of each container (points a, b, c, and d, respectively). From Eq. (*13.7*), and an obvious extension of the results of Problem 13.5(*b*), we see that each such pressure is just the sum of atmospheric pressure and the value of dgh for the container, where h is the depth from the top of the liquid to the bottom of the container. For example, for the second container we have $P_b = P_A + dgh_b$. For the liquid to be in equilibrium for the entire interconnected system, the pressure at each point along the bottom horizontal tube must be the same. Thus, $P_a = P_b = P_c = P_d$, which implies that $h_a = h_b = h_c = h_d$.

Problem 13.7. Show that in any container filled with a liquid, a change in pressure ΔP at any one point in the liquid leads to the same change in pressure at any other point in the liquid. This is commonly referred to as Pascal's principle.

Solution

We see from Eq. (*13.7*) that the pressure difference between any two points in a liquid depends only on the height difference of the two points. It follows that a change in pressure at one of the points must give rise to the same change in pressure at the other point, to ensure that the pressure difference between the two points will remain the same. This is the principle behind the hydraulic press.

Problem 13.8. Figure 13-6 shows a simple schematic setup for a **hydraulic press**. A nearly incompressible oil completely fills the container. One side of the container has a large cross-sectional area A and has a close-fitting light piston on top of the oil. The other side has a much smaller cross-sectional area a and also has a close-fitting light piston on top of the oil. By Problem 13.6, the height of the oil is the same in both sides of the container. Assume that a large weight W is placed on the large piston as shown, and that a pin holds the small piston in place.

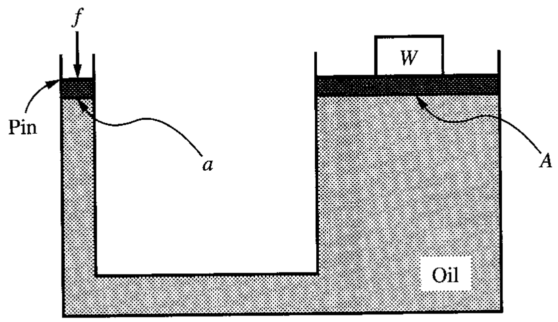

Fig. 13-6

(*a*) What is the change in pressure caused at every point in the fluid as a consequence of the weight W?

(*b*) What force f must be exerted downward on the small piston to ensure that it won't rise when the pin is removed?

(*c*) If $A = 150\ \text{in}^2$ and $a = 1.0\ \text{in}^2$, how much weight can be held up by a force $f = 25$ lb exerted downward on the small piston?

Solution

(*a*) The weight W is a downward force spread over an area A of the large piston. It therefore gives rise to an additional downward pressure on the oil of $\Delta p = W/A$. This additional pressure is transmitted to all points of the oil (Problem 13.7).

(*b*) Since there is an additional upward pressure $\Delta P = W/A$ exerted on the small piston by the oil, the total additional upward force of the oil on this piston is $(\Delta P)a = Wa/A$. Thus, the downward force f needed to counterbalance this is just $f = Wa/A$.

(*c*) Using the results of (*b*), we have

$$f = 25\ \text{lb} = W\left(\frac{1.0\ \text{in}^2}{150\ \text{in}^2}\right) = \frac{W}{150} \qquad \text{or} \qquad W = 3750\ \text{lb}$$

Problem 13.9. Referring to Problem 13.8(*c*) assume that a weight $W = 3750$ lb is being supported and that f is increased to slightly over 25 lb so that the small piston starts to move down.

(*a*) What distance h must the piston move to raise the weight 1 in?

(*b*) How much work is done by the force f, and where does this expended energy go?

(*c*) If one wants a hydraulic press that can lift a weight through a substantial distance, what change in design would make sense?

Solution

(*a*) Since the oil is incompressible, the volume decrease on one side must equal the volume increase on the other side. If the large piston moves up 1.0 in, the volume increase on this side is $(1.0\ \text{in})(150\ \text{in}^2) = 150\ \text{in}^3$. This corresponds to the volume reduction on the other side:

$$ha = 150\ \text{in}^3 \qquad \text{or} \qquad h(1.0\ \text{in}^2) = 150\ \text{in}^3 \qquad \text{or} \qquad h = 150\ \text{in}$$

(*b*) Work $= fh = (25\ \text{lb})(150\ \text{in})/(12\ \text{in/ft}) = 313\ \text{ft}\cdot\text{lb}$. This appears in the form of increased potential energy of the weight W: $\Delta E_p = (3750\ \text{lb})(1.0\ \text{in})/(12\ \text{in/ft}) = 313\ \text{ft}\cdot\text{lb}$.

(*c*) Clearly, it is not practical to push down on the small piston through huge distances. A more practical approach would be as follows. Replace the movable small piston by a valve that lets additional oil into the system. An oil pump can then pump oil from a reserve tank into the thin tube by exerting a gauge pressure slightly greater than $\Delta P = W/A$. The weight W will rise as the new oil is pumped in. When the weight rises to the appropriate height, the oil pump is shut off, and the valve is closed. To lower the weight, another valve can be opened, letting the oil slowly drain back into the reserve tank.

13.4 MEASUREMENT OF PRESSURE

A variety of instruments is available to measure pressure in a gas or a liquid. The use of liquids as the measuring device is discussed within the framework of the problems below.

Problem 13.10. The **open-tube manometer** is a simple instrument for measuring the pressure of gases or liquids in a container. The instrument (Fig. 13-7) consists of a flexible tube filled with a dense liquid, usually mercury. One end is open to the atmosphere, while the other end is exposed to a container filled with, for example, a hot gas at pressure P, so the mercury on the two sides of the tube is at different heights.

(*a*) Find an expression for the pressure P in terms of the density of mercury and the height difference $h_1 - h_2$.

(*b*) If $h_1 - h_2 = 85$ cm, find the gauge pressure of the gas.

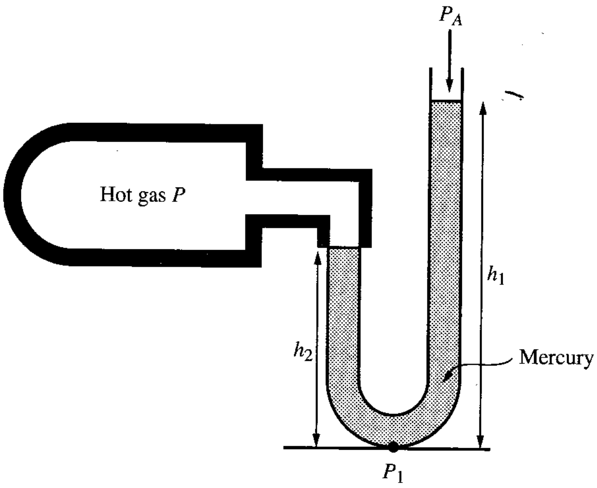

Fig. 13-7

Solution

(*a*) The condition for equilibrium is that the pressure at the bottom of the tube P_1 be the same whether determined from the left or right sections of the tube:

$$P + d_{\mathrm{Hg}} g h_2 = P_A + d_{\mathrm{Hg}} g h_1 \qquad \text{or} \qquad P = P_A + d_{\mathrm{Hg}} g (h_1 - h_2)$$

(*b*) From (*a*),

$$P = P_A + (13.6 \times 10^3\ \mathrm{kg/m^3})(9.8\ \mathrm{m/s^2})(0.85\ \mathrm{m}) = P_A + 113 \times 10^3\ \mathrm{Pa}$$

The gauge pressure is just $P - P_A = 113 \times 10^3$ Pa.

Problem 13.11. Referring to Problem 13.10, if the gas in the container cooled off and the pressure P started to drop, the mercury would start to rise in the left side of the tube. To stop this from happening one lowers the right side in such a way as to keep the height of the column of mercury on the left side fixed.

(*a*) Assuming the gauge pressure of the gas drops to half of its value from Problem 13.10, find the new value of $h_1 - h_2$.

(*b*) If the gas is allowed to cool further, it is found that the new equilibrium position is such that $h_1 < h_2$. What is the significance of that fact?

Solution

(*a*) From Problem 13.10(*b*), the new gauge pressure is

$$P - P_A = 56.5 \times 10^3 \text{ Pa} = (13.6 \times 10^3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(h_1 - h_2) \quad \text{or} \quad h_1 - h_2 = 0.424 \text{ m}$$

(*b*) This indicates that $P - P_A$ is negative, so P is less than atmospheric pressure.

Problem 13.12. A pressure gauge that measures the pressure of the atmosphere itself is called a **barometer**. A simple mercury barometer is shown in Fig. 13-8. A tall cylindrical tube open on one end is first filled with mercury. It is then carefully inverted into an open container of mercury and is supported by a stand with a clamp. The mercury starts to come out of the tube but comes to equilibrium at some height h above the mercury surface in the container. The space above the mercury is a near vacuum, filled only with low-pressure, mercury vapor.

(*a*) Find the atmospheric pressure P_A in terms of the height h.

(*b*) Determine the height h when the atmospheric pressure P_A has its normal value of 1.013×10^5 Pa.

Solution

(*a*) The pressure at the surface level of the mercury in the container must have the same value as the pressure at that level in the tube. The value of the pressure at that level in the tube is just $d_{Hg}gh$, while in the container it is just atmospheric pressure P_A. Thus, $P_A = d_{Hg}gh$.

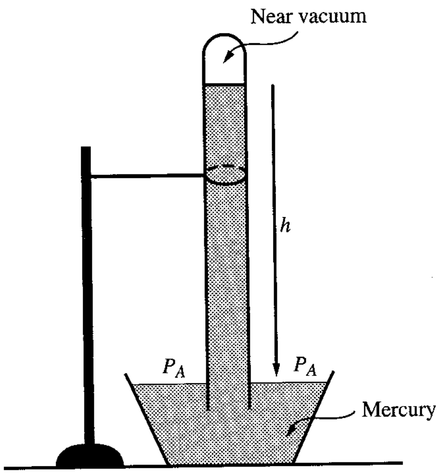

Fig. 13-8

(*b*) From (*a*),

$$1.013 \times 10^5 \text{ Pa} = (13.6 \times 10^3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)h \qquad \text{or} \qquad h = 0.760 \text{ m}$$

More commonly, the height is given in cm: $h = 76.0$ cm. It is quite common to actually quote the pressure in terms of the height of the supported column of mercury. Thus, the statement that atmospheric pressure is 75.5 cm of mercury means that the atmosphere supports a column of mercury 75.5 cm high.

13.5 ARCHIMEDES' PRINCIPLE

Another significant result of hydrostatics is Archimedes' principle, which will be discussed within the context of the following problems.

Problem 13.13. A cubical block of iron of side $a = 15$ cm is suspended by a cord in a container of water, as shown in Fig. 13-9.

(*a*) Show that the net upward force on the block due to the water has a magnitude equal to the weight of an amount of water that would occupy the volume of the block. (This volume is the volume of the liquid displaced by the block, also known as the *displaced* volume, and this net upward force is known as the *buoyant* force B due to the liquid. The fact that the buoyant force equals the weight of the displaced liquid is called **Archimedes' principle**: the law of buoyancy.)

(*b*) Find the value of the buoyant force exerted on the block by the water.

(*c*) Find the tension in the cord when the system is in equilibrium.

Solution

(*a*) The vertical forces exerted on the block by the water are the forces F_1 and F_2, on the top and bottom faces due to the corresponding pressures P_1 and P_2. The net upward force is then

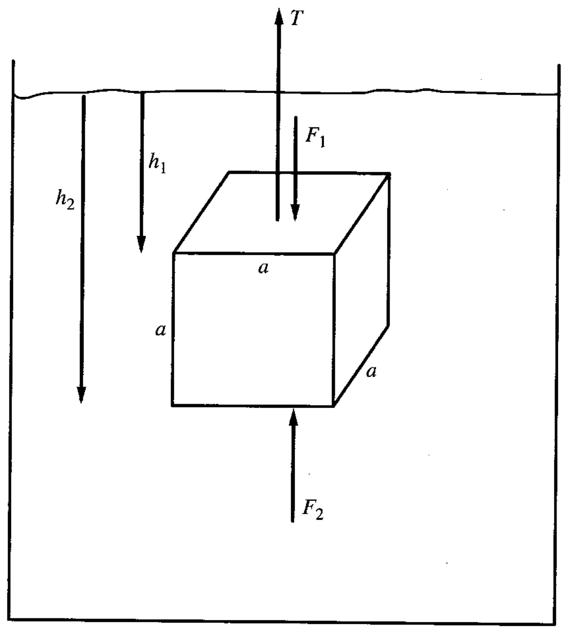

Fig. 13-9

$B = F_2 - F_1 = P_2a^2 - P_1a^2 = (P_2 - P_1)a^2$. But, from Eq. (*13.7*), $P_2 - P_1 = dga$, where d is the density of water, so $F_2 - F_1 = (dga)a^2 = dga^3$, which is the required result.

(*b*) $B = (1000\ \text{kg/m}^3)(9.8\ \text{m/s}^2)(0.15\ \text{m})^3 = 33.1\ \text{N}$.

(*c*) If W is the weight of the block, then $T + B = W$. $W = d_1ga^3 = (7.86 \times 10^3\ \text{kg/m}^3)(9.8\ \text{m/s}^2) \times (0.15\ \text{m})^3 = 260\ \text{N}$. Then

$$T = W - B = 260\ \text{N} - 33\ \text{N} = 227\ \text{N}$$

Problem 13.14.

(*a*) Show that Archimedes' principle holds for an arbitrarily shaped object submerged in an arbitrary fluid.

(*b*) Find the statement of Archimedes' principle for a floating object.

Solution

(*a*) The situation is shown in Fig. 13-10(*a*). At first glance it would seem almost impossible to get the desired result, since unlike the cube of Problem 13.13, the pressure acts at a variety of different angles, and has a variety of magnitudes, over the surface of the object. To get the buoyant force one would need to add up all the vertical components of the myriad different forces acting on all the infinitesimal surface elements of the object, a procedure that requires the calculus. Still, there is a much simpler way to obtain the result, one which Archimedes himself used many centuries before the invention of the calculus.

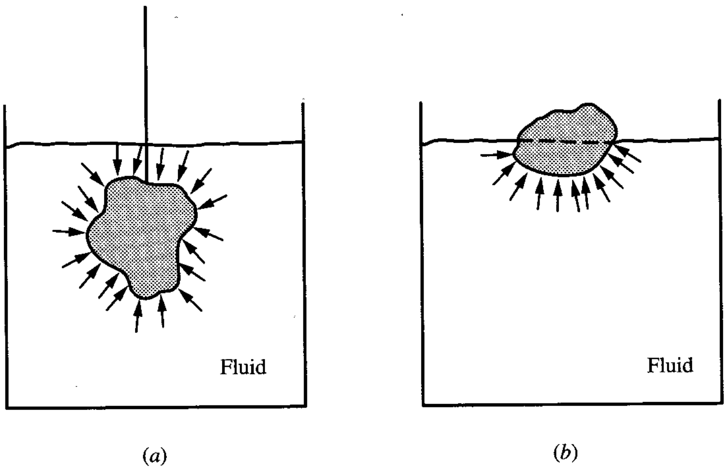

Fig. 13-10

A hint as to the approach to take comes from our realization that there is no net horizontal force exerted by a liquid on an object that is submerged. This can be understood as follows. Suppose we replace the object by an identical volume of the same type liquid as is found in the container. We consider an imaginary boundary in the exact shape of the object separating this liquid from the rest of the liquid of the container. The old and new liquid together constitute a uniform liquid at rest in the container, and is therefore in equilibrium. The horizontal forces due to the liquid outside the boundary acting on the liquid inside the boundary must therefore add up to zero. Since the liquid outside the boundary is exactly the same as it was when the object was there, the force it exerts is unchanged, and the horizontal force on the object must have been zero also.

This same reasoning works for the vertical forces. For the liquid inside the boundary to be in equilibrium, the vertical forces due to the liquid outside must just balance the weight of the liquid inside. Again, these forces are unchanged if the object itself were inside the boundary, so we have our result: the net buoyant force of the liquid on the object is upward and equals the weight of the amount of liquid that would fill the space of the object, in other words, the amount of liquid displaced by the object. This is Archimedes' Principle.

(*b*) Figure 13-10(*b*) depicts a floating object. We can use the same reasoning as before to get the expression for the buoyant force. Now, however, the force exerted by the liquid is due only to the pressure at points below the surface. Consider the object replaced by an amount of liquid which fills the space occupied by the part of the object which was submerged, i.e., the part of the object below the dashed line along the surface of the liquid, as shown. Again, we assume an imaginary boundary in the shape of the submerged portion of the object.

Since the old and new liquid together constitute a uniform liquid with a horizontal surface boundary at the top, it is a liquid in equilibrium. Let us ignore the effects of atmospheric pressure above the liquid surface for the moment. Then, the force exerted on the "new" liquid inside the boundary by the "old" liquid outside must just balance the weight of the liquid inside. This is just the weight of liquid that displaces the submerged part of the object. Since the outside liquid is exactly the same whether the "new" liquid or the object is within the boundary, it must exert the same force on each. Thus, the buoyant force on the object is still the weight of liquid displaced by the object. To include the effects of atmospheric pressure, we note that it pushes down on the object from above and pushes up on the object (through the increased pressure of the liquid) from below. These two effects can be shown to cancel, as they did in the case of the wall in Problem 13.4(*c*), so that Archimedes' principle holds also for a floating object.

Problem 13.15. An iron ball of mass $M = 10$ kg is submerged in a container of liquid X by means of a cord as shown in Fig. 13-11. The tension T in the cord is found to be 65 N. Find (*a*) the buoyant force B acting on the ball; (*b*) the density of the liquid d_X.

Solution

(*a*) For equilibrium, $T + B = W$. The weight of the ball $W = Mg = 98$ N. Therefore,

$$B = W - T = 98 \text{ N} - 65 \text{ N} = 33 \text{ N}$$

(*b*) $B = d_X gV$, where V is the volume of the iron ball. Since we know B and g, we could obtain d_X if we knew V. To obtain V, we use the density of iron d_I from Table 13.1, and we note that $d_I V = M$:

$$(7.86 \times 10^3 \text{ kg/m}^3)V = 10 \text{ kg} \qquad \text{or} \qquad V = 0.00127 \text{ m}^3$$

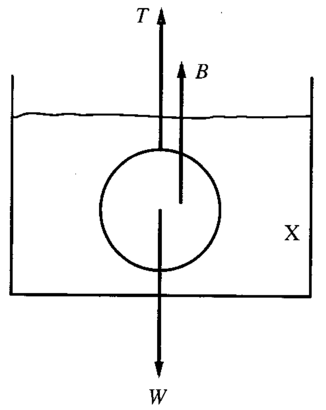

Fig. 13-11

Then, substituting in $B = d_X gV$, we get

$$33\ \text{N} = d_X(9.8\ \text{m/s}^2)(0.00127\ \text{m}^3) \qquad \text{or} \qquad d_X = 2.65 \times 10^3\ \text{kg/m}^3$$

Problem 13.16. The iron ball of Problem 13.15 is removed from liquid X and placed in a container of mercury, where it is found to float. What fraction of the volume of the ball is submerged?

Solution

For floating, the buoyant force (weight of the displaced liquid) just balances the weight of the ball:

$$d_I gV = d_{Hg} gV_s$$

where V and V_s are the total volume of the ball and the volume submerged in the mercury, respectively. Hence, the fraction submerged is

$$\frac{V_s}{V} = \frac{d_I}{d_{Hg}} = \frac{7.86}{13.6} = 0.578$$

Problem 13.17. A block of wood of mass 30 kg floats in seawater of density $d_{sw} = 1025\ \text{kg/m}^3$. When a child of mass 40 kg stands on the block, the block just barely floats with its top surface level with the water. Find (*a*) the volume V, and (*b*) the density d_{wd}, of the wood.

Solution

(*a*) From equilibrium considerations we have

$$B = W_{wd} + W_{ch} = (30\ \text{kg} + 40\ \text{kg})(9.8\ \text{m/s}^2) \qquad \text{or} \qquad B = 686\ \text{N}$$

We also know that

$$B = d_{sw} gV \Rightarrow 686\ \text{N} = (1025\ \text{kg/m}^3)(9.8\ \text{m/s}^2)V \qquad \text{or} \qquad V = 0.0683\ \text{m}^3$$

(*b*)

$$d_{wd} = \frac{M_{wd}}{V} = \frac{30\ \text{kg}}{0.0683\ \text{m}^3} = 439\ \text{kg/m}^3$$

Problem 13.18. A beaker filled with water rests on a scale that reads $F = 50.0$ N. Next, an aluminum block of volume $V = 800\ \text{cm}^3$ is suspended from a cord in the water, as shown in Fig. 13-12. Find (*a*) the weight of the aluminum and the buoyant force exerted on it by the water; (*b*) the tension T in the cord; (*c*) the new reading F' of the scale.

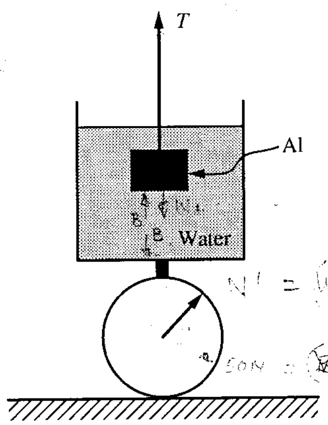

Fig. 13-12

Solution

(*a*) $$W_{Al} = d_{Al}gV = (2.70 \times 10^3\ \text{kg/m}^3)(9.8\ \text{m/s}^2)(800 \times 10^{-6}\ \text{m}^3) = 21.2\ \text{N}$$
$$B = d_w gV = (1000\ \text{kg/m}^3)(9.8\ \text{m/s}^2)(800 \times 10^{-6}\ \text{m}^3) = 7.84\ \text{N}$$

(*b*) From equilibrium of the aluminum, $T + B = W_{Al}$, so that

$$T = 21.2\ \text{N} - 7.8\ \text{N} = 13.4\ \text{N}$$

(*c*) By Newton's third law, the aluminum exerts a downward force of magnitude B on the liquid. The scale must balance the weight of the beaker and water, which was given to be 50.0 N, plus the downward reaction force to the buoyant force $B = 7.8$ N. The result is that the scale reads $F' = 57.8$ N.

Problem 13.19. A rectangular block of ice of cross-sectional area A and depth h floats at the interface of oil and water as shown in Fig. 13-13. Find the fraction of the ice that is in the water. (Assume $d_{oil} = 800\ \text{kg/m}^3$, and see Table 13.1 for d_{ice} and d_{water}.)

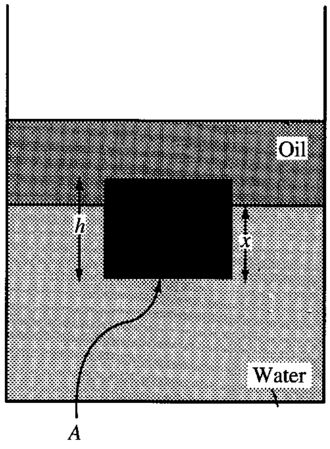

Fig. 13-13

Solution

The buoyant force is just the weight of displaced liquid, which is now a combination of oil and water. If x is the depth that the ice sinks in the water we have that the buoyant force $B = d_{water}gAx + d_{oil}gA(h - x)$. This force must balance the weight of the ice $W_{ice} = d_{ice}gAh$. Dividing by gA, we get

$$d_{ice}h = d_{water}x + d_{oil}(h - x)$$

Simplifying and then substituting, we get

$$d_{ice} = d_{oil} + (d_{water} - d_{oil})\frac{x}{h} \quad \text{or} \quad 917 = 800 + 200\frac{x}{h} \quad \text{or} \quad \frac{x}{h} = 0.585 = \text{fraction in water}$$

13.6 HYDROSTATICS OF GASES

As pointed out at the beginning of the chapter, the pressure due to a gas is a consequence of the bombardment of the surfaces of a confining container by the free-moving gas molecules. On the other hand, the actual value of the pressure of the atmosphere near the earth's surface is a consequence of the weight of the ocean of air above the surface. Thus the pressure near the ground is higher than that a thousand feet up in much the same way that the pressure of water at the bottom of a tank is greater than that near the top of the tank. The pressure difference of the air, however, cannot be expressed in the simple form $P_2 - P_1 = d_{air}gh$ because the density of a gas changes significantly with increasing pressure, so d_{air} is not even approximately constant over the height of the atmosphere. Nonetheless, although the formula is more complicated, the pressure increases in a definite way as one gets closer and closer to ground level, and this increase in pressure is accompanied by a greater density of air. The pressure of the atmosphere at ground level is exerted on all surfaces and is added to the hydrostatic pressure of liquids that are in contact with it.

The pressure exerted by the air at any given location is directly due to the bombardment of molecules and not directly due to the push of the air from above. This can be seen by considering an empty paper cup sitting on a table. If a steel plate is put across the top of the cup, there is no contact between the air above the cup and the air in the cup. Yet, the pressure of the air in the cup remains the same (otherwise the sides of the cup would get crushed in by the pressure of the air outside).

Problem 13.20.

(*a*) Find an expression for the force F exerted by the pressure of the atmosphere on the curved surface of a solid hemispherical object of radius R, as shown in Fig. 13-14(*a*).

(*b*) If $R = 2.0$ in, find the value of the force F. ($P_A = 14.7\ \text{lb/in}^2$.)

(*c*) Two hollow hemispheres of radius $R = 10.0$ cm are brought together and make an airtight seal. The air inside is pumped out through a valve so that the pressure inside drops to zero. How large a force must be applied to each hemisphere to pull them apart?

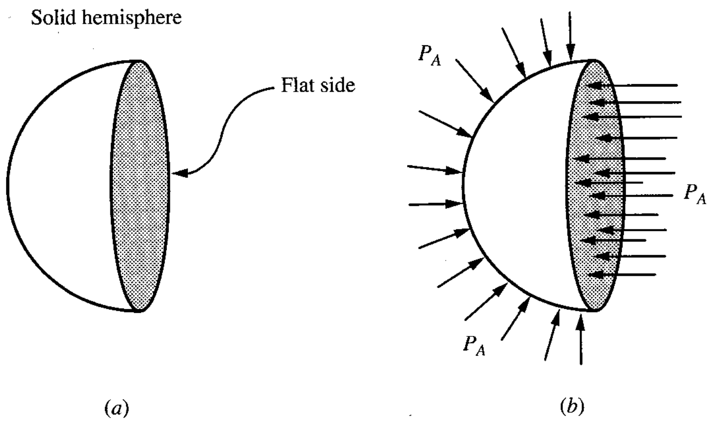

Fig. 13-14

Solution

(*a*) The pressure at any point on the hemisphere pushes perpendicular to the surface, as shown in Fig. 13-14(*b*). Thus, although the magnitude of the pressure is constant and equal to P_A, the net force due to that pressure is the resultant of myriad forces on the surface pointing in different directions. We can nonetheless obtain this resultant force simply by the following reasoning. The overall force of the atmosphere on the hemisphere must be zero, since otherwise the hemisphere would be accelerated, and we know this does not happen. Therefore, the net force on the curved surface must exactly balance the net force on the flat surface. This latter force is $P_A(\pi R^2)$ along the symmetry axis of the hemisphere. The force on the curved portion must therefore have the same magnitude, but point in the opposite direction.

(*b*) $F = (14.7 \text{ lb/in}^2)(3.14)(2.0 \text{ in})^2 = 185 \text{ lb}$.

(*c*) The force due to atmospheric pressure pressing in on each half is

$$F = P_A(\pi R^2) = (1.013 \times 10^5 \text{ Pa})(3.14)(0.10 \text{ m})^2 = 3.180 \text{ N}$$

This is the force that must be applied to pull the two halves apart.

Just as a liquid can exert a buoyant force, so can the air in the atmosphere. Although the buoyant force of the air is generally small (because the density of air is low), for hot-air balloons and blimps (dirigibles), which have large volume, the buoyant force of the air is the basis for their operation.

Problem 13.21. A blimp has a frame and cargo that together weigh 10,000 N. Assume that the blimp is filled with helium at room temperature. What volume V must the blimp have to just float in the air near the earth's surface?

Solution

The blimp and its contents are in equilibrium when it floats, so the buoyant force B equals the total weight of the blimp. Assuming that almost the entire volume V of the blimp is taken up by the helium, we have

$$d_{\text{He}}gV + 10{,}000 \text{ N} = d_{\text{air}}gV \qquad \text{or} \qquad V = \frac{10{,}000 \text{ N}}{d_{\text{air}}g - d_{\text{He}}g}$$

Substituting in the densities we get

$$V = \frac{10{,}000 \text{ N}}{(1.29 \text{ kg/m}^3 - 0.18 \text{ kg/m}^3)(9.8 \text{ m/s}^2)} = 919 \text{ m}^3$$

13.7 SURFACE TENSION AND CAPILLARITY

Surface Tension

A droplet of water has a spherical shape because the surface of a liquid tends to pull itself in as if it were an elastic membrane. This phenomenon, known as **surface tension**, is ultimately caused by the pull of the molecules *below* the surface of the liquid on the molecules *at* the surface. This tends to pull the surface into a smooth and compact layer. Surface tension gives rise to such phenomena as a steel needle being able to float on a water surface [Fig. 13-15(*a*)], even though the density of the metal object is much greater than that of water. Other examples of surface tension are shown in Fig. 13-15(*b*) and (*c*). The surface tension γ is defined as the force per unit length exerted by a liquid surface on an object, along its boundary of contact with the object. This force is parallel to the liquid surface and perpendicular to the boundary line of contact. For a straight boundary of length L and a total force F we have $\gamma = F/L$. In Fig. 13-15(*b*) we show a soap membrane supporting a slide wire of weight W and length b. The surface tension pulls perpendicular to the wire and along the membrane. Because the soap membrane has two surfaces, the total upward force is $F = 2b\gamma$. Since the wire is in equilibrium, $W = F$, and so $\gamma = W/2b$.

Unlike an actual elastic membrane, the surface tension does not increase as the surface is stretched. Thus, if the soap membrane of Fig. 13-15(*b*) were stretched by pulling the wire down to a

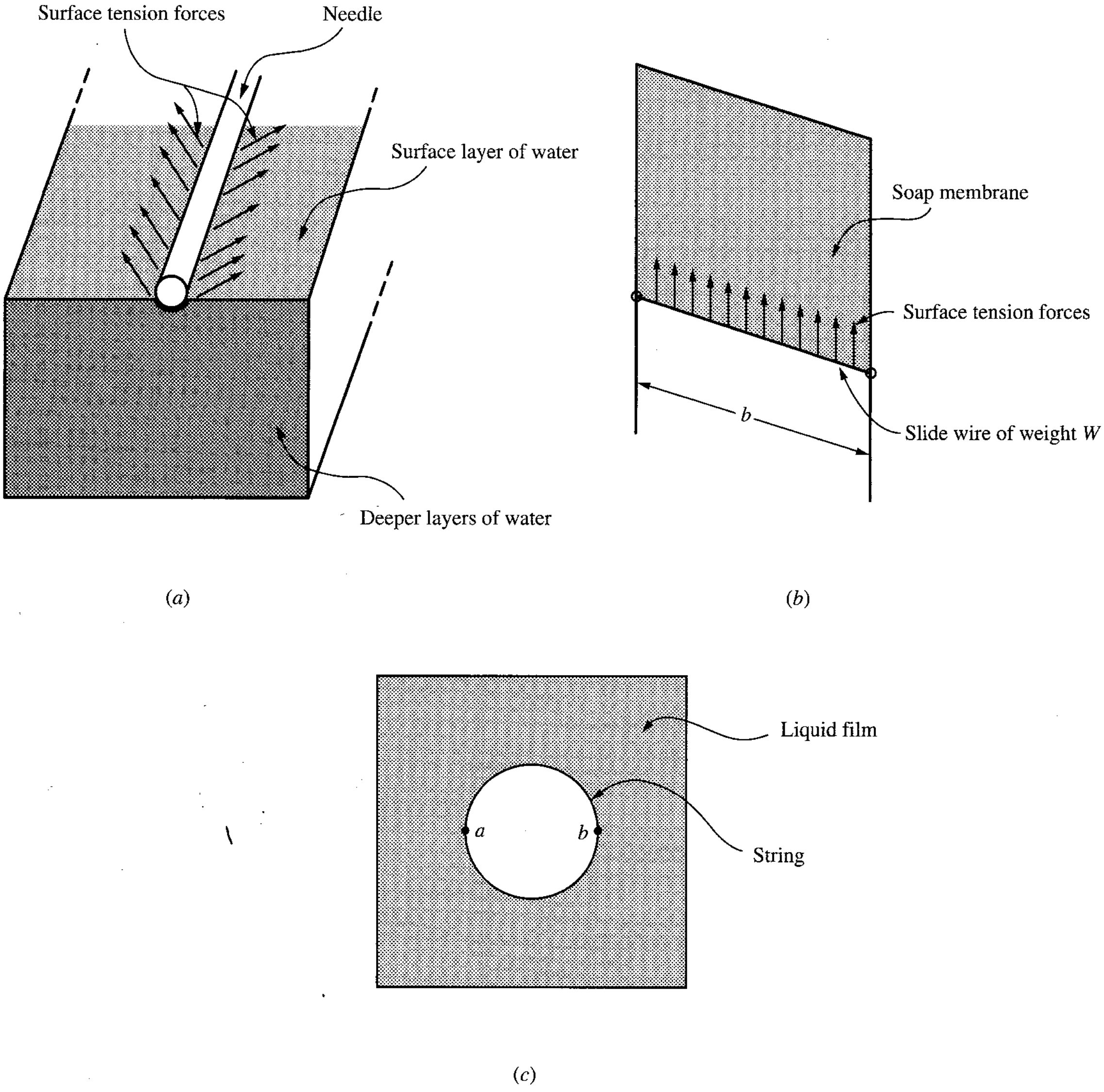

Fig. 13-15

lower position, the wire would remain in equilibrium in the new position upon release, since γ, and hence F, will not have changed. The value of the surface tension does not change because more molecules enter the surface from the interior as the membrane is stretched, and the intermolecular spacings stay the same. Surface tension does change when the liquid temperature or pressure changes, or when chemical substances are brought into contact with the surface.

Problem 13.22. In Fig. 13-15(c) a light string is pulled into a tight circular shape of radius $r = 3.0$ cm by the surface tension of a liquid film that extends between the string and the rectangular frame. If the surface tension of the film is $\gamma = 0.02$ N/m, find the tension in the cord. [*Hint*: It can be shown that the net force due to the film (which has two sides) pulling on the upper half of the string between points a and b is just $4r\gamma$ in the upward direction.]

Solution

In Fig. 13-16 we show half the string between points a and b. The upper arrows depict the surface tension pulling perpendicular to the string all around the string from a to b. From the hint we know that the net effect of the surface tension is an upward force of magnitude $4r\gamma$. The two downward arrows depict the tension in the string, which is exerted by the lower half of the string on the upper half at points a and b. Since the upper half of the string is in equilibrium, and its weight can be neglected, we have

$$4r\gamma = 2T \qquad \text{or} \qquad T = 2r\gamma = 2(0.030\ \text{m})(0.02\ \text{N/m}) = 0.0012\ \text{N}$$

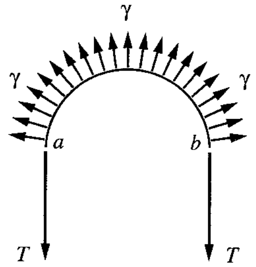

Fig. 13-16

Problem 13.23. A soap bubble of radius 0.60 cm filled with air has a surface tension in its inner and outer surfaces of $\gamma = 0.04$ N/m. Find the gauge pressure P_g of the air in the bubble. [*Hint*: From Problem 13.20(*a*), the net force on a hemisphere of radius r due to a uniform pressure P is $\pi r^2 P$ in the direction along the symmetry axis of the hemisphere.]

Solution

We consider the upper hemisphere of the bubble, as shown in Fig. 13-17. The surface tension pulls downward all around the rim with a force per unit length γ. Then the total downward force is $2\pi r\gamma$ due to each film surface (the inside and the outside). The total is thus $4\pi r\gamma$. This downward force is counterbalanced by the force due to the pressure of the air inside the bubble less the force due to the atmospheric pressure outside. From the hint, this is just an upward force of $\pi r^2(P - P_A) = \pi r^2 P_g$. For equilibrium we must have

$$4\pi r\gamma = \pi r^2 P_g \qquad \text{or} \qquad P_g = \frac{4\gamma}{r} = \frac{4(40 \times 10^{-3}\ \text{N/m})}{0.60 \times 10^{-2}\ \text{m}} = 26.7\ \text{Pa}$$

Capillarity

When a liquid is in contact with a surface of a container, there is a competition between the force of attraction of the molecules of the container on the molecules of the liquid (adhesion) and the force

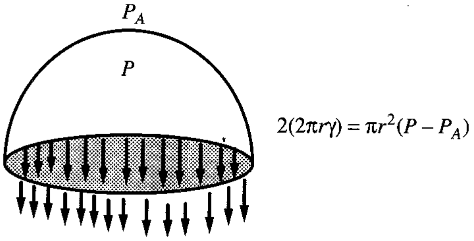

Fig. 13-17

of attraction of the molecules of the liquid on each other (cohesion). If the adhesion forces are greater than the cohesion forces, the liquid surface will bend upward at the point of contact because of the liquid surface being pulled toward the wall of the container. This is illustrated in Fig. 13-18(*a*) for the case of water in a glass tube. The adhesive forces dominate in this case, and the water surface makes an angle θ with the walls of the container at the point of contact. The angle θ, called the contact angle, has an important effect when the liquid is in a very thin tube of radius r. Since the surface tension of the liquid is parallel to the surface of the liquid, by Newton's third law the glass exerts an upward force on the liquid all around the rim of the tube given by $F = 2\pi r\gamma \cos\theta$. This pulls the liquid up above its normal height in the tube, until the weight of the excess height h of liquid just balances the surface tension force F. If d is the density of the liquid, we have for the weight $w = dg\pi r^2 h$. Equating F and w, we get

$$h = \frac{2\gamma \cos\theta}{dgr} \tag{13.8}$$

As can be seen, the smaller the radius r, the greater the capillary height h. Thus, for very thin tubes, the liquid can rise, in apparent defiance of gravity, to substantial heights, as shown in Fig. 13-18(*b*).

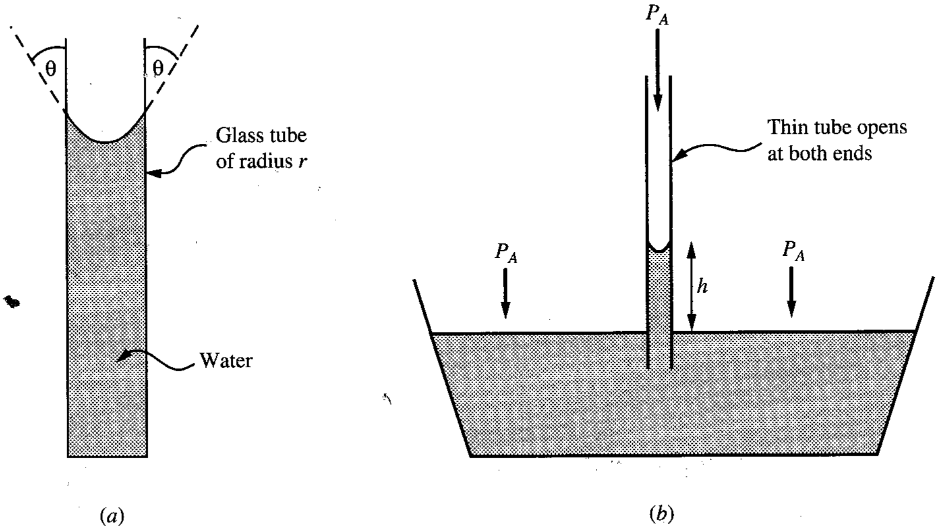

Fig. 13-18

Blood flow in small blood vessels and flow of nutrients in plants are examples of this phenomenon in nature. The general name for the elevation or depression of a liquid surface under such conditions is **capillarity**, or **capillary action**.

Problem 13.24. Assume that in the Pascal vases of Fig. 13-5, the straight tube is very thin, with radius $r = 2.0$ mm. Assuming $\gamma = 0.07$ N/m and a contact angle of 20°, how high above the normal level will the water rise in this tube?

Solution

Calling the excess height Δh, from Eq. (*13.8*) we have

$$\Delta h = \frac{2(0.07\ \text{N/m})\cos 20^\circ}{(1000\ \text{kg/m}^3)(9.8\ \text{m/s}^2)(0.0020\ \text{m})} = 0.0067\ \text{m} = 0.67\ \text{cm}$$

Problem 13.25. In the case of mercury in a glass tube, the cohesive forces are greater than the adhesive forces, so the mercury surface is pulled down, as shown in Fig. 13-19(*a*). The contact angle in this case is obtuse, and the surface tension forces pull down on the liquid, making it drop below the normal level in the tube. The U-shaped container shown in Fig. 13-19(*b*) is filled with mercury. Find the height difference of the mercury in the two vertical tubes, if $\gamma_{Hg} = 0.545$ N/m and contact angle $\theta = 140°$.

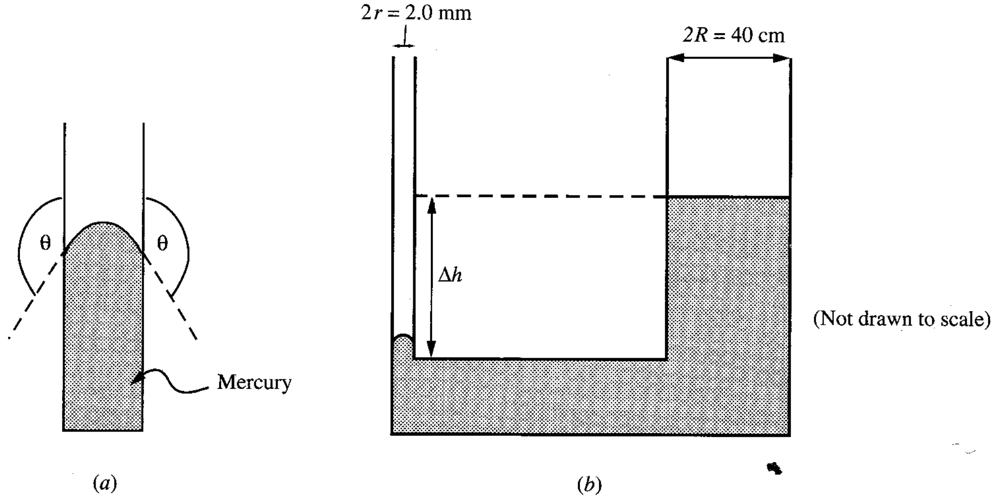

Fig. 13-19

Solution

Recalling from Eq. (*13.8*) that the capillary height varies inversely as the radius of the tube, we can ignore any change from normal height in the wide tube. For the thin tube, (*13.8*) yields

$$\Delta h = \frac{2(0.545\ \text{N/m})\cos 140°}{(1360\ \text{kg/m}^3)(9.8\ \text{m/s}^2)(0.0010\ \text{m})} = -0.063\ \text{m} = -6.3\ \text{cm}$$

Problems for Review and Mind Stretching

Problem 13.26. A bent tube containing oil (of sp gr 0.750) and water is shown in Fig. 13-20. The tube cross sections are wide enough so that capillary action is negligible. The heights h_1 and h_2 are the height of the water and oil, respectively, above the oil-water interface. If $h_2 = 15$ cm, find the height difference Δh between the liquid in the two sides of the tube.

Solution

Since the pressure must be the same on both sides at the level of the interface, we have $P_A + d_w g h_1 = P_A + d_o g h_2$. Simplifying, we get $d_w h_1 = d_o h_2$ or

$$h_1 = \frac{d_o}{d_w} h_2 = (0.75)(15\ \text{cm}) = 11.3\ \text{cm} \qquad \text{and} \qquad \Delta h = h_2 - h_1 = 3.7\ \text{cm}$$

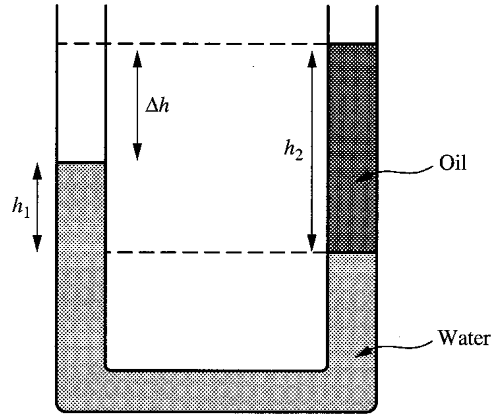

Fig. 13-20

Problem 13.27. Figure 13-21(*a*) depicts a dam of length $L = 100$ ft holding back the waters of a lake (with steep side walls) that is 60 ft deep.

(*a*) Find the force exerted by the water alone on the dam.

(*b*) If the dam were constructed so that its base just rests on the ground, with a coefficient of static friction between dam and ground of $\mu_s = 0.50$, what minimum weight W_{min} must the dam have if it is not to slip?

(*c*) Instead of a lake, assume the dam is holding back water filling a 3.0-in-wide ditch running the whole length of the dam, as shown in Fig. 13-21(*b*). Assume the water fills the ditch to a height of 60 ft. How would the answers to parts (*a*) and (*b*) change?

Solution

(*a*) The force of the water alone on the dam is due to the gauge pressure of the water, P_g. (This ignores the added effect of the atmospheric pressure on the water.) To find this force, we multiply the average pressure of the water on the dam by the area over which it acts: $F = P_{g,av}A$. Here, $P_{g,av} = \frac{1}{2}(0 + dgh)$, so

$$F = \frac{(62.5\ \text{lb/ft}^3)(60\ \text{ft})}{2}(60\ \text{ft})(100\ \text{ft}) = 11{,}250{,}000\ \text{lb}$$

(*b*) The net horizontal force due to the air and water on the dam is just the force of the water alone, as calculated in part (*a*). This is because the effect of the atmosphere pushing on the curved portion of the dam just balances the added force due to the pressure of the atmosphere on the water [see, e.g., Problem 13.4(*c*)]. The maximum static frictional force is just $f_s = \mu_s N$. Since $N = W$, we have $f_s = \mu_s W$. For equilibrium we must have $\mu_s W_{min} = F$, or

$$W_{min} = \frac{F}{\mu_s} = \frac{11{,}250{,}000\ \text{lb}}{0.5} = 22{,}500{,}000\ \text{lb} = 11{,}250\ \text{tons}.$$

(*c*) Since the pressure of the water depends only on depth, the answers to parts (*a*) and (*b*) are the same! The width of the column of water does not matter.

Problem 13.28. A hydraulic lift is to raise an automobile weighing $W = 12{,}000$ N. The radius of the lifting tube is $R = 30$ cm.

(*a*) What increase in pressure is necessary to lift the auto?

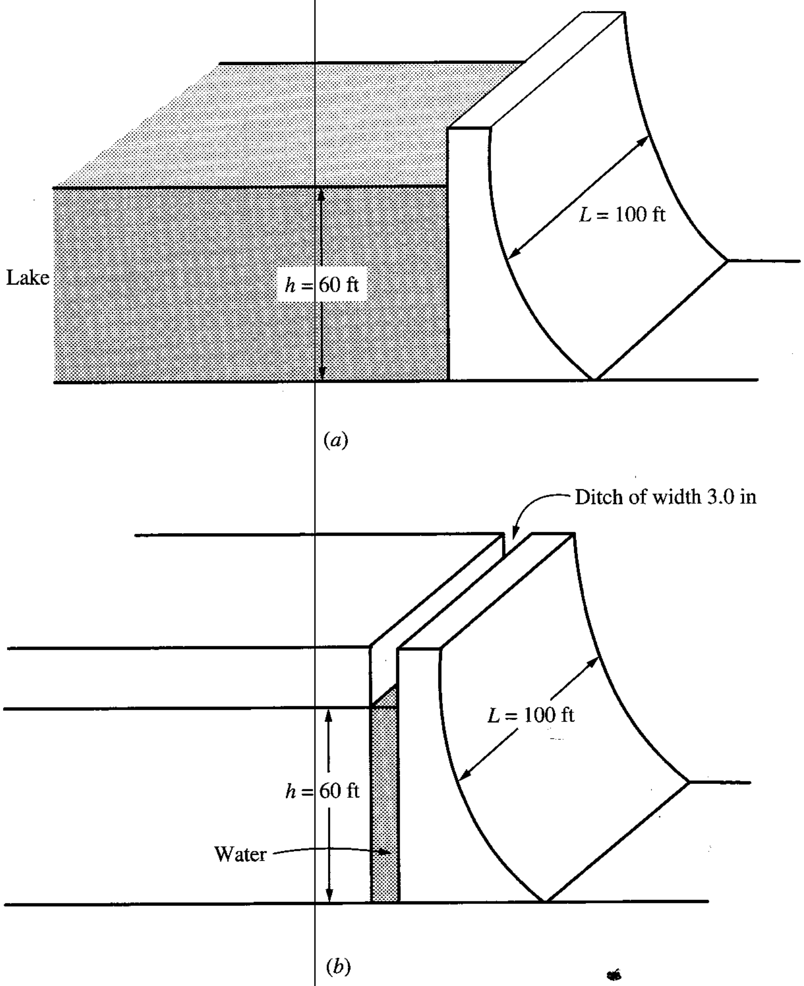

Fig. 13-21

(*b*) If the maximum force that can be generated on the thin tube side of the lift is $F = 80$ N, what is the maximum radius r_{max} that the thin tube can have to lift the auto?

Solution

(*a*)
$$\Delta P = \frac{W}{A} = \frac{W}{\pi R^2} = \frac{12{,}000 \text{ N}}{(3.14)(0.30 \text{ m})^2} = 42{,}500 \text{ N/m}^2$$

(*b*) $F/a_{max} = \Delta P \Rightarrow F = \Delta P a_{max} \Rightarrow 80 \text{ N} = (42{,}500 \text{ N/m}^2)(3.14) r_{max}^2 \Rightarrow r_{max} = 0.0245$ m.

Problem 13.29. A block of wood floats in water with two-thirds of its volume V submerged in water. What fraction of its volume is submerged when it floats in benzene?

Solution

From Table 13.1, $d_B = 880$ kg/m^3, while for water $d_w = 1000$ kg/m^3. Whether floating in water or benzene, the buoyant force just balances the weight of the block W. Thus, $W = d_w g V_w = d_B g V_B$, where V_w and V_B are the displaced (submerged) volumes in the water and benzene, respectively. From this we get $d_B V_B = d_w V_w$, and dividing both sides by the volume of the block:

$$d_B \frac{V_B}{V} = d_w \frac{V_w}{V} \qquad \text{or} \qquad \frac{V_B}{V} = \left(\frac{d_w}{d_B}\right)\left(\frac{V_w}{V}\right) = \left(\frac{1.0}{0.880}\right)\left(\frac{2}{3}\right) = 0.758$$

Problem 13.30. Two soap bubbles are formed from the same soap mixture, one having twice the radius of the other. What is the ratio of the gauge pressures of the air inside the two bubbles?

Solution

Assume the smaller and larger radii are r and R, respectively. As in Problem 13.23, we have $\pi r^2 P_{g1} = 4\pi r\gamma$, or

$$P_{g1} = \frac{4\gamma}{r}$$

Similarly, we have $\pi R^2 P_{g2} = 4\pi R\gamma$, or

$$P_{g2} = \frac{4\gamma}{R}$$

Taking the ratio P_{g1}/P_{g2} we get $R/r = 2.0$.

Supplementary Problems

Problem 13.31. The specific gravity of mercury is 13.6. Find (*a*) the volume of 16 kg of mercury; (*b*) the weight density of mercury, in lb/ft^3.

Ans. (*a*) 11.8×10^{-4} m^3; (*b*) 850 lb/ft^3

Problem 13.32.

(*a*) Find the absolute pressure at the bottom of a tall cylinder filled with a column of water 3.00 m high. Assume the top of the cylinder is open to the atmosphere, with $P_A = 1.013 \times 10^5$ Pa.

(*b*) How high a column of benzene must be poured in on top of the water to double the gauge pressure at the bottom of the cylinder?

Ans. (*a*) 1.307×10^5 Pa; (*b*) 3.41 m

Problem 13.33. A light tank in the shape shown in Fig. 13-22 is filled with water to a height of 40 cm. The area of the bottom of the tank is 100 cm^2, and the cross-sectional area of the narrow neck is 4.0 cm^2. The tank rests on a scale.

(*a*) What is the reading of the scale?

(*b*) What is the total downward force on the bottom of the tank due to the water alone? (Neglect the effect of the atmosphere above the water.)

Ans. (*a*) 20.4 N; (*b*) 39.2 N

Prlblem 13.34. Referring to Problem 13.33, shouldn't the answers to parts (*a*) and (*b*) be the same, and if not, why?

Ans. No. The water also exerts an upward force on the upper surface of the tank of area A'. The difference between the force on the bottom and this force is the net force of the water on the tank and must equal the weight of the water.

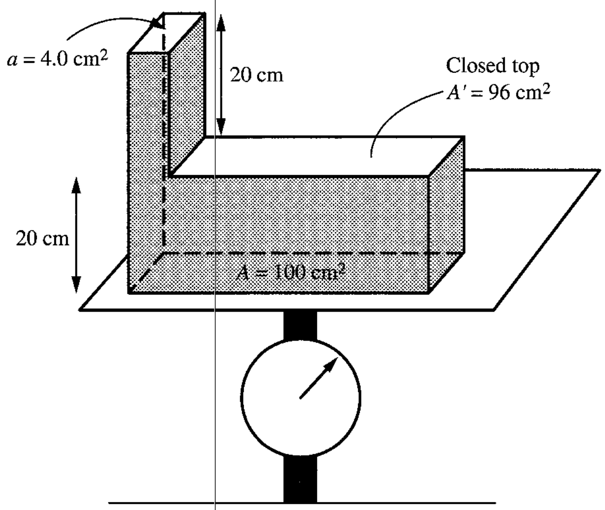

Fig. 13-22

Problem 13.35. If the barometer of Fig. 13-8 used water instead of mercury, how high would the column of water have to be?

Ans. 10.3 m

Problem 13.36.

(*a*) What force, in pounds, is exerted by atmospheric pressure on the outside surface of an 8.0- by 4.0-ft door of a house?

(*b*) Why doesn't the door cave in?

Ans. (*a*) 67,700 lb; (*b*) because an equal and opposite force is exerted due to air pressure on the other side of the door

Problem 13.37. Assume that in the hydraulic press of Fig. 13-6, the diameters of the two vertical tubes are 2.0 in and 2.0 ft, respectively. How large a mass M, in kilograms, resting on the large piston can be lifted by a force $f = 35{,}000$ dyn.

Ans. 5.14 kg

Problem 13.38. If a stone feels exactly (*a*) one-half, (*b*) three-fourths as heavy when held under water as when in air, find its density.

Ans. (*a*) 2000 kg/m^3; (*b*) 4000 kg/m^3

Problem 13.39. A nugget weighing 9.355 N is hung from the end of a sensitive balance and dipped until completely submerged in water contained in a cylinder of cross-sectional area 75 cm^2. The water is observed to rise in the cylinder, and the balance reads 8.870 N.

(*a*) Find the density of the nugget and its likely composition.

(*b*) How high does the water rise in the cylinder?

Ans. (*a*) 19,300 kg/m^3, pure gold; (*b*) 0.660 cm

Problem 13.40. A plank of wood, of area $A = 2000\ cm^2$ and thickness 6.0 cm floats freely on seawater ($d_{sw} = 1024\ kg/m^3$), with 4.0 cm submerged. When an aluminum weight is hung from a cord below the center of the plank, the plank is observed to just float with its top surface even with water.

(*a*) What is the density of the wood?

(*b*) What is the volume of the aluminum?

Ans. (*a*) 683 kg/m^3; (*b*) 0.00244 m^3

Problem 13.41. A ball (sp gr 0.35) is submerged in freshwater to a depth of 2.0 m and released.

(*a*) Find the acceleration of the ball, ignoring viscous forces.

(*b*) How fast will the ball be moving as it reaches the surface?

Ans. (*a*) 18.2 m/s^2; (*b*) 8.5 m/s

Problem 13.42. A large spherical reflecting ball of volume 4.0 m^3 and mass 50 kg is suspended from a light cable in a disco ballroom.

(*a*) What is the buoyant force on the ball due to the air?

(*b*) What is the tension in the cable?

Ans. (*a*) 50.6 N; (*b*) 439 N

Problem 13.43. A hot-air balloon consists of a large airtight canvas bag attached by ropes to a hanging gondola with passengers and equipment. The weight of all the above items is 2700 N. Assuming that the volume inside the bag is 400 m^3, find the density of the heated air inside the bag necessary for the balloon to float in equilibrium. Ignore the volume of the gondola and its contents.

Ans. 0.60 kg/m^3

Problem 13.44. Suppose the needle of Fig. 13-15(*a*) is of steel, with a length of 30 mm and a radius of 0.20 mm. The surface tension of water is 0.07 N/m. Find (*a*) the weight of the pin; (*b*) the total vertical component of force that must be exerted by the surface tension on each side of the needle to keep it in equilibrium; (*c*) the angle this force makes with the vertical. (Assume buoyancy effects can be ignored.)

Ans. (*a*) 290 μN; (*b*) 145 μN; (*c*) 4.0°

Problem 13.45. A glass tube of inner radius 0.30 mm is dipped into a container of water. How high does the water rise in the tube assuming that $\gamma = 0.07$ N/m and the contact angle is 0°?

Ans. 48 mm

Problem 13.46. A drop of water of radius 2.0 mm has the expected spherical shape because of surface tension ($\gamma = 0.07$ N/m). Evaluate the gauge pressure of the water inside the drop. [*Hint*: In a droplet, unlike a bubble, only one liquid surface exists.]

Ans. 70 Pa