# Chapter 8

# Impulse and Linear Momentum

## 8.1 IMPULSE

### *Impulse of a Constant Force*

If a constant force $\mathbf{F}$ acts on an object for a time $t$, then the impulse $\mathbf{I}$ due to the force $\mathbf{F}$ is defined as

$$\mathbf{I} = \mathbf{F}t \tag{8.1}$$

Note that this is a vector equation, and $\mathbf{I}$ points along $\mathbf{F}$ since $t$ is a positive scalar. If we restrict ourselves to two-dimensional problems (the $xy$ plane), Eq. (*8.1*) is equivalent to the component equations

$$I_x = F_x t \qquad \text{and} \qquad I_y = F_y t \tag{8.2}$$

Figure 8-1($a$) shows a simple situation in which a constant horizontal force $\mathbf{F}$ pulls a block along a tabletop. The impulse $\mathbf{I}$, due to $\mathbf{F}$, is depicted for the given time interval $t$. The units of impulse are those of force times time and therefore are N · s, dyn · s, or lb · s, depending on the system of units being used.

If more than one constant force acts on the block for the time $t$, then the total impulse is just the vector sum of the impulses. Thus, in Fig. 8-1($b$), the same block is shown again, but this time with the other forces acting: the weight, the normal force, and a kinetic frictional force. The total impulse is

$$\mathbf{I}_T = \mathbf{I}_F + \mathbf{I}_w + \mathbf{I}_N + \mathbf{I}_f = \mathbf{F}t + \mathbf{w}t + \mathbf{N}t + \mathbf{f}t = (\mathbf{F} + \mathbf{w} + \mathbf{N} + \mathbf{f})t$$

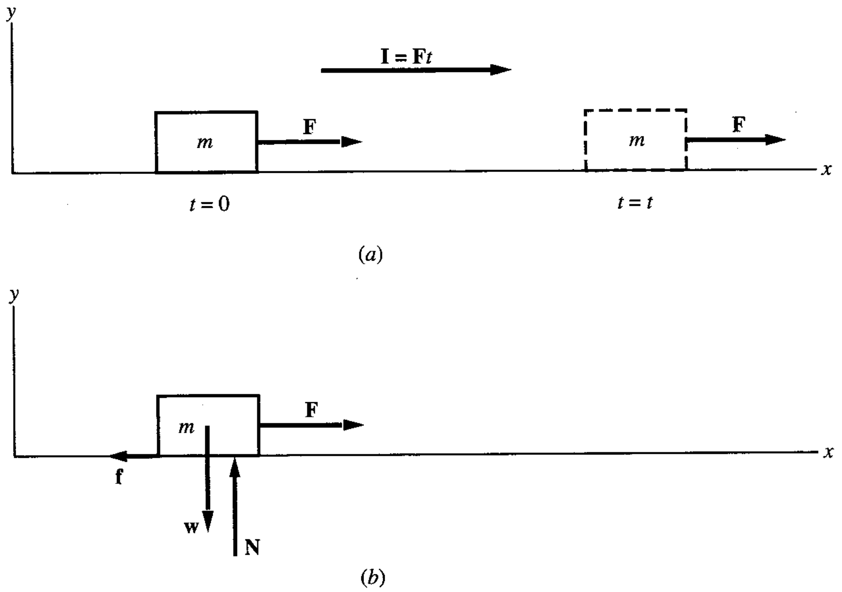


**Fig. 8-1**

Since the term in parentheses on the right is just the resultant force, we have that the total impulse equals the impulse of the resultant force.

From Fig. 8-1(*b*) we see that the $x$ and $y$ component equations for $\mathbf{I}_T$ are

$$(I_T)_x = (F - f)t \qquad \text{and} \qquad (I_T)_y = (N - w)t$$

Since there is equilibrium in the $y$ direction, we have $N = w$ and $(I_T)_y = 0$. The total impulse is thus in the $x$ direction and involves a positive contribution from **F** and a negative contribution from **f**.

**Problem 8.1.**

(*a*) Assume that the force **F** in Fig. 8-1(*a*) has a magnitude of 100 N and the time $t = 8$ s. Find the impulse due to **F** in the 8-s interval.

(*b*) Suppose that after time $t = 8$ s the force **F** was changed to 300 N and acted on the block for an additional 8 s. What would the impulse be in the second 8 s?

(*c*) Suppose that after this second 8-s interval, the force **F** were changed again, this time to 400 N acting in the negative $x$ direction, and that the force acted for an additional 4 s. What would the impulse be in this third time interval?

**Solution**

(*a*) The impulse clearly is in the $x$ direction: $I_x = Ft = (100 \text{ N})(8 \text{ s}) = 800 \text{ N} \cdot \text{s}$.

(*b*) Again the impulse is in the $x$ direction: $I_x = (300 \text{ N})(8 \text{ s}) = 2400$ N.

(*c*) Here the impulse clearly points in the negative $x$ direction: $I_x = (-400 \text{ N})(4 \text{ s}) = -1600$ N.

**Problem 8.2.** In Fig. 8-1(*b*) assume that $F = 50$ lb, $w = 30$ lb, and the coefficient of kinetic friction is $\mu_k = 0.5$. Find the total impulse in a 6-s time interval.

**Solution**

The impulses due to the weight and normal force are equal and opposite and therefore add up to zero. The impulses due to $F$ and $f$ are along the $x$ axis, so the total impulse is in the horizontal direction. The $x$ component of the impulse due to $F$ is $(I_F)_x = (50 \text{ lb})(6 \text{ s}) = 300 \text{ lb} \cdot \text{s}$. The frictional force is given by $f = \mu_k N = 0.5(30 \text{ lb}) = 15$ lb, so the impulse due to $f$ is $(I_f)_x = (-15 \text{ lb})(6 \text{ s}) = -90 \text{ lb} \cdot \text{s}$. The total impulse is then $(I_T)_x = (I_F)_x + (I_f)_x = 300 \text{ lb} \cdot \text{s} - 90 \text{ lb} \cdot \text{s} = 210 \text{ lb} \cdot \text{s}$. [Alternatively, we could first find the resultant force $F_T$: $(F_T)_x = F_x + f_x = 50 \text{ lb} - 15 \text{ lb} = 35$ lb. Then $(I_T)_x = (F_T)_x t = (35 \text{ lb})(6 \text{ s}) = 210 \text{ lb} \cdot \text{s}$, as before.]

***Note.*** There is an interesting similarity between the definition of impulse of a constant force and that of work of a constant force (as defined in Chap. 6). Work was defined as a force times a distance, and impulse is defined as a force times a time. We will see below that, just as for the case of work, the definition of impulse can be extended to the case of a variable force acting on an arbitrarily moving object. In addition, in our study of work, we saw that one could relate the work done to the change in kinetic energy (which is related to the change in velocity) over the interval in which the work was performed. We will soon see that impulse can also be related, through a quantity called momentum, to a change in velocity over the period during which the impulse is performed.

There is, however, a very fundamental difference in the nature of the definitions of work and of impulse. Work is defined as a scalar quantity; it has no direction. Impulse, on the other hand, is defined as a vector quantity. Whereas kinetic energy involves only the magnitude of the velocity, the quantity involving velocity to which impulse is related involves the velocity vector.

### *Impulse of a Variable Force*

We now extend our concept of impulse to a variable force acting on an object moving on an arbitrary path. In Fig. 8-2 we depict the situation. We consider the time interval from the initial time $t_{\text{init}} = t_1$ to the final time $t_{\text{final}} = t_N$ and divide the path of the particle up into $N - 1$ small segments corresponding to small time intervals $\Delta t$. Thus, for example, $\Delta t_1 = t_2 - t_1$, $\Delta t_2 = t_3 - t_2, \ldots,$ $\Delta t_i = t_{i+1} - t_i, \ldots, \Delta t_N = t_N - t_{N-1}$. The forces $\mathbf{F}_1, \mathbf{F}_2, \ldots, \mathbf{F}_i, \ldots, \mathbf{F}_{N-1}$ correspond to the average values of the forces in each of the corresponding small time intervals. Then by definition

$$\mathbf{I} = \lim_{\Delta t \to 0} \sum \mathbf{F}_i \, \Delta t_i \tag{8.3}$$

where the sum is over all the $N - 1$ intervals between $t_1$ and $t_N$, and the limit means that all the $\Delta t$'s get infinitesimally small (and, correspondingly, the number of intervals between $t_1$ and $t_N$ get infinitely large). In terms of $x$ and $y$ components,

$$I_x = \lim_{\Delta t \to 0} \sum (F_x)_i \, \Delta t_i \qquad I_y = \lim_{\Delta t \to 0} \sum (F_y)_i \, \Delta t_1 \tag{8.4}$$

If $F_x$ and $F_y$ are known functions of the time, Eqs. (*8.4*) can be understood graphically. In Fig. 8-3 we depict an example of $F_x$ as a function of time by plotting $F_x$ vs. $t$. (A similar depiction could be made

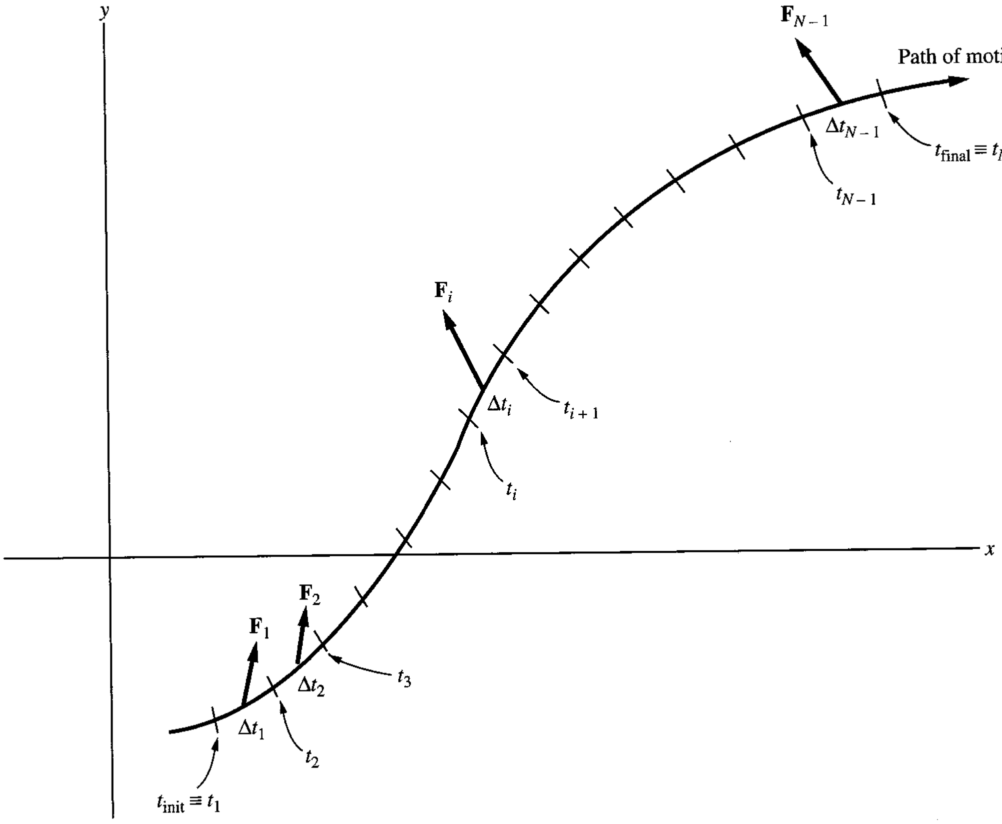


**Fig. 8-2**

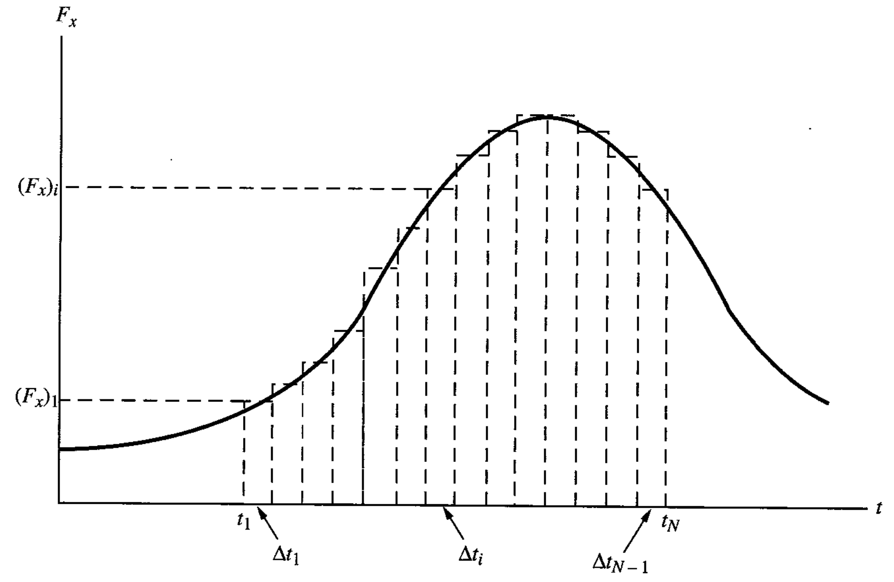


**Fig. 8-3**

for $F_y$.) By dividing the $t$ axis into small intervals $\Delta t$, we see that the sum in the $x$ component of (8.4) is approximated by the sum of the areas of the rectangles; in the limit as the $\Delta t$'s go to zero this becomes the area under the $F_x$ vs. $t$ curve between the initial and final times. Thus, the $x$ *component of the impulse* is the *area under the $F_x$ vs. $t$ curve*. A similar result holds for the $y$ component. (Note the analogy between this and the $F \cos \theta$ *vs.* $s$ curve for work in Chap. 6.)

**Problem 8.3.** Referring to Problem 8.1, find the total impulse exerted by the force **F** over the combined 20-s time interval of parts (*a*), (*b*), and (*c*).

**Solution**

The force **F** takes on three values during the 20-s interval. Labeling the intervals 1, 2, and 3 we have $F_1 = 100$ N, $\Delta t_1 = 8$ s; $F_2 = 300$ N, $\Delta t_2 = 8$ s; and $F_3 = -400$ N, $\Delta t_3 = 4$ s. Then $I_x = F_1\,\Delta t_1 + F_2\,\Delta t_2 + F_3\,\Delta t_3 = 800\ \text{N}\cdot\text{s} + 2400\ \text{N}\cdot\text{s} - 1600\ \text{N}\cdot\text{s} = 1600\ \text{N}\cdot\text{s}$. This is just the algebraic sum of the impulses in the individual intervals.

**Problem 8.4.** Figure 8-4 depicts the time curves for the $x$ component (*a*) and the $y$ component (*b*) of a force acting on an object from $t = 0$ to $t = 20$ s.

(*a*) Find $I_x$ for the 20-s interval.

(*b*) Find $I_y$ for the 20-s interval.

**Solution**

(*a*) $I_x$ equals the area under the $F_x$ vs. $t$ curve. We break the area up into two intervals 1 and 2.

$$(I_x)_1 = (60\ \text{N})\,(10\ \text{s}) = 600\ \text{N}\cdot\text{s} \qquad (I_x)_2 = \tfrac{1}{2}(60\ \text{N})(10\ \text{s}) = 300\ \text{N}\cdot\text{s}$$

$$I_x = (I_x)_1 + (I_x)_2 = 900\ \text{N}\cdot\text{s}$$

(*b*) $I_y$ is the area under the $F_y$ vs. $t$ curve. We break the area up into the four intervals shown. $(I_y)_1 = (40\ \text{N})(5\ \text{s}) = 200\ \text{N}\cdot\text{s}$. To get $(I_y)_2$ and $(I_y)_3$ we note that areas 2 and 3 are similar

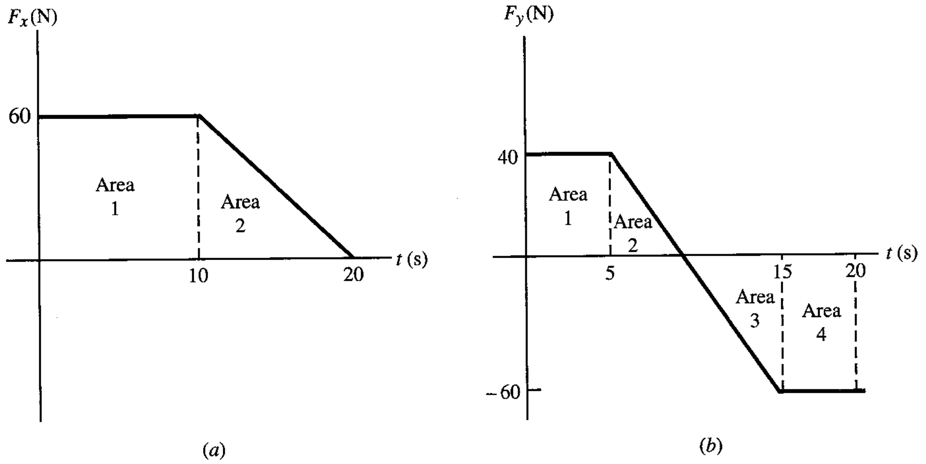


**Fig. 8-4**

triangles whose heights are in the ratio of 4 to 6. The bases must be in the same ratio, which makes the bases 4 s and 6 s, respectively, so the curve crosses the axis at $t = 9$ s. Then $(I_y)_2 = \frac{1}{2}(40\text{ N})(4\text{ s}) = 80\text{ N}\cdot\text{s}$. Similarly, $(I_y)_3 = \frac{1}{2}(-60\text{ N})(6\text{ s}) = -180\text{ N}\cdot\text{s}$. Next, $(I_y)_4 = (-60\text{ N})(5\text{ s}) = -300\text{ N}\cdot\text{s}$. Finally,

$$I_y = (200 + 80 - 180 - 300)\text{N}\cdot\text{s} = -200\text{ N}\cdot\text{s}$$

## 8.2 MOMENTUM AND THE IMPULSE-MOMENTUM THEOREM

### *Case of Constant Force*

If there is no friction in the setup of Fig. 8-1(*a*), then the force **F** is the resultant force on the system, and $F = ma$. The acceleration of the block is then constant, so $v = v_0 + at$, where $t$ is the time measured from the instant when the velocity is $v_0$. If we change our notation so that $v_0 = v_i$ at time $t_i$ and $v = v_f$ at the later time $t_f = t_i + t$, our equation becomes

$$v_f = v_i + a(t_f - t_i) \qquad \text{or} \qquad a(t_f - t_i) = v_f - v_i$$

Multiplying this last equation by the mass of the block, we have

$$ma(t_f - t_i) = mv_f - mv_i \qquad \text{or} \qquad F(t_f - t_i) = mv_f - mv_i$$

Recalling that $F(t_f - t_i) = I$, the total impulse on the block in the time interval $(t_f - t_i)$, we have

$$I = mv_f - mv_i \tag{8.5a}$$

The quantity $mv$ is called the **linear momentum** or just the **momentum**. Therefore, (*8.5a*) states that impulse is equal to the change in linear momentum or

$$I = \Delta(mv) \tag{8.5b}$$

Note that the units of momentum (in SI) are: kg · m/s. This is the same as the units of impulse: $\text{N}\cdot\text{s} = (\text{kg}\cdot\text{m/s}^2)\text{s} = \text{kg}\cdot\text{m/s}$. Other units of momentum are the g · cm/s and the slug · ft/s.

The concept of linear momentum can be generalized to two or three dimensions. By definition, if an object of mass $m$ is moving at a given instant of time with velocity **v**, then

$$\text{Linear momentum} \equiv m\mathbf{v} \tag{8.6}$$

We will now show that Eqs. (*8.5*) can be generalized to arbitrary motion of an object under the action of a variable resultant force. In other words, if **I** represents the total impulse on an object of mass $m$ between times $t_i$ and $t_f$, and $\mathbf{v}_i$ and $\mathbf{v}_f$ are the velocities at the initial and final points of the interval, we will have

$$\mathbf{I} = m\mathbf{v}_f - m\mathbf{v}_i = \Delta(m\mathbf{v}) \tag{8.7}$$

To demonstrate this result we recall that $\mathbf{F} = m\mathbf{a}$ can be expressed approximately in the form $\mathbf{F} \approx m\,\Delta\mathbf{v}/\Delta t$ for very small $\Delta t$, since $\mathbf{a} = \lim_{\Delta t \to 0} \Delta\mathbf{v}/\Delta t$. Then we must have

$$\mathbf{F}\,\Delta t \approx m\,\Delta\mathbf{v} \tag{8.8}$$

We now return to Fig. 8-2 and label the velocity at each time ($t_1, t_2, \ldots, t_i, \ldots, t_N$) with the same index: $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_i, \ldots, \mathbf{v}_N$. For the change in velocity in the corresponding infinitesimal time intervals $\Delta t_1, \Delta t_2$, etc., we have $\Delta\mathbf{v}_1 = \mathbf{v}_2 - \mathbf{v}_1$, $\Delta\mathbf{v}_2 = \mathbf{v}_3 - \mathbf{v}_2$, etc. Then, applying Eq. (*8.8*) to each interval $\Delta t$, we have $\mathbf{F}_1\,\Delta t_1 = m\,\Delta\mathbf{v}_1$, $\mathbf{F}_2\,\Delta t_2 = m\,\Delta\mathbf{v}_2$, etc. Adding up all the $\mathbf{F}\,\Delta t$ terms, we get $\mathbf{I} = \sum \mathbf{F}_i\,\Delta t_i = \sum m\,\Delta\mathbf{v}_i = m(\sum \Delta\mathbf{v}_i)$. Adding up all the $\Delta\mathbf{v}$'s just gives the overall change in vector velocity, $\mathbf{v}_N - \mathbf{v}_1$. Thus, $\mathbf{I} = m\mathbf{v}_N - m\mathbf{v}_1$, which is just Eq. (*8.7*), with $\mathbf{v}_1 = \mathbf{v}_i$ and $\mathbf{v}_N = \mathbf{v}_f$.

**Problem 8.5.** Suppose that the block in Problem 8.1 moves on a frictionless surface so that $F$ is the resultant force. Assume the mass of the block is 5.0 kg.

(*a*) If the block has a velocity of 40 m/s at the beginning of the first time interval, what is its velocity at the end of that interval (8 s later)?

(*b*) What is the velocity at the end of the full 20 s?

**Solution**

(*a*) From part (*a*) of Problem 8.1 we have $I_x = 800\ \text{N}\cdot\text{m}$. Then, using (*8.5a*) we get

$$800\ \text{N}\cdot\text{m} = (5.0\ \text{kg})v_f - (5.0\ \text{kg})\,(40\ \text{m/s}) \qquad \text{or} \qquad v_f = 200\ \text{m/s}$$

(*b*) We could solve this by letting $v_f$ of part (*a*) be $v_i$ for the second time interval and apply Eq. (*8.5a*) again to solve for $v_f$ at the end of the second time interval. Then we would repeat the process for the last time interval. Instead we can use the general result (*8.7*) for the entire interval. The overall impulse in the full 20-s interval was calculated in Problem 8.3, so we have

$$1600\ \text{N}\cdot\text{s} = (5.0\ \text{kg})v_f - (5.0\ \text{kg})\,(40\ \text{m/s}) \qquad \text{or} \qquad v_f = 360\ \text{m/s}$$

**Problem 8.6.** Suppose in Problem 8.4 the force **F** represents the resultant force on a particle of mass 2.0 kg moving in the $xy$ plane. Assume that at $t = 0$ the particle has a velocity of magnitude 100 m/s making an angle of 30° above the positive $x$ axis. Find the magnitude and direction $\theta$ of the velocity at the end of the 20-s interval.

**Solution**

Here we split Eq. (*8.7*) into its $x$ and $y$ components:

$$I_x = mv_{fx} - mv_{ix} \qquad I_y = mv_{fy} - mv_{iy} \tag{i}$$

From Problem 8.4 we get $I_x = 900\ \text{N}\cdot\text{s}$ and $I_y = -200\ \text{N}\cdot\text{s}$; from the data, $v_{ix} = (100\ \text{m/s})\cos 30° = 86.6\ \text{m/s}$ and $v_{iy} = (100\ \text{m/s})\sin 30° = 50.0\ \text{m/s}$. Applying (*i*), we get

$$900\ \text{N}\cdot\text{s} = (2.0\ \text{kg})v_{fx} - (2.0\ \text{kg})\,(86.6\ \text{m/s}) \qquad \text{or} \qquad v_{fx} = 537\ \text{m/s}$$

$$-200\ \text{N}\cdot\text{s} = (2.0\ \text{kg})v_{fy} - (2.0\ \text{kg})(50.0\ \text{m/s}) \qquad \text{or} \qquad v_{fy} = -50.0\ \text{m/s}$$

Then $v_f = (v_{fx}^2 + v_{fy}^2)^{1/2} = 539$ m/s and $\tan\theta = |50/537| = 0.0931$. Solving for $\theta$ we get $\theta = 5.32°$. The vector $\mathbf{v}_f$ is in the fourth quadrant, has magnitude 539 m/s, and points 5.32° below the positive $x$ axis.

### *Short Impulses*

One of the most useful applications of the concept of impulse is to cases where very large forces act for very short time intervals. Consider, for example, a baseball game in which a batter hits the ball straight back at the pitcher. This is essentially a one-dimensional problem, with $I = \Delta(mv) = mv_f - mv_i$. Choose the positive direction from the batter toward the pitcher. For a given initial and final velocity the impulse is completely determined, even though we don't know the specifics of the force the bat exerts on the ball at any instant of time. Nonetheless we can draw a rough graph of the force vs. the time (Fig. 8-5). The solid line depicts the actual force exerted by the bat on the ball. Time $t_a$ represents the instant when the pitched ball first makes contact with the bat. As the bat makes firmer contact with the ball the force rises rapidly to some maximum value. Thereafter the ball starts to separate from the bat, and the force drops rapidly until time $t_c$ is reached, when the ball completely loses contact with the bat. The entire time interval $t_c - t_a$ is only thousandths of a second. Whatever the exact shape of the curve, we know that the impulse is the area under the curve. For a given impulse, the shorter the time interval over which the force acts, the higher the peak force must be, since the area under the $F$ vs. $t$ curve must stay the same. Another quantity that we can determine is the average force $F_{av}$ giving rise to the impulse. This is defined as the constant force which, if acting for the same time interval, would give rise to the same impulse. The force $F_{av}$ has a magnitude such that the area under the dashed rectangle in Fig. 8-5 is equal to that under the actual curve.

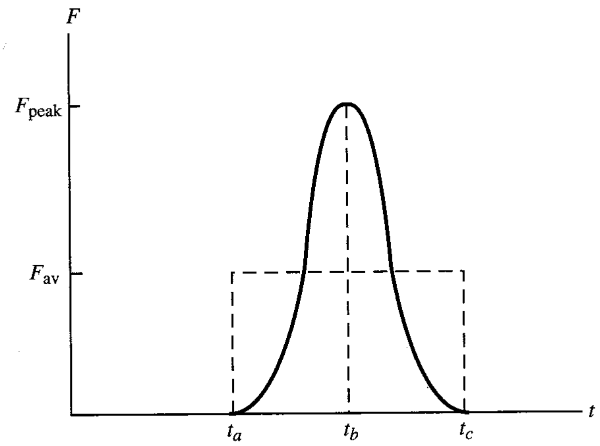


**Fig. 8-5**

**Problem 8.7.** A baseball, of mass 0.20 kg, is pitched at 40 m/s and is hit straight back at the pitcher at 90 m/s. Assume the positive $x$ axis points toward the pitcher.

(*a*) Find the impulse exerted by the bat on the ball.

(*b*) If the ball is in contact with the bat for 0.0035 s, find the average force exerted on the ball.

(*c*) How would the result of part (*b*) change if the contact time were one-third as long?

**Solution**

(*a*) $$I = mv_f - mv_i = (0.20\text{ kg})\,[(90\text{ m/s}) - (-40\text{ m/s})] = 26\text{ N}\cdot\text{s}$$

(We have used the fact that $v_f$ is positive and $v_i$ is negative.)

(*b*) $$F_{av}\,\Delta t = I$$

so that

$$F_{av}(0.0035\text{ s}) = 26\text{ N}\cdot\text{s} \quad \text{or} \quad F_{av} = 7430\text{ N}$$

(*c*) Since the impulse is fixed, if $\Delta t$ is one-third as long, $F_{av}$ must be three times as great, so $F_{av}$ = 22,300 N.

**Problem 8.8.** A baseball catcher pulls her glove back as she catches the ball, rather than holding it stiffly. Explain as precisely as possible why this is advantageous.

**Solution**

The impulse that the catcher must impart in stopping the ball is fixed by the ball's initial momentum. Therefore, the longer she takes to bring the ball to rest, the smaller the average force she must exert, and hence the smaller the reaction force on her hand. It is to her advantage to lengthen the time of contact of ball with glove as much as possible.

**Problem 8.9.** A bullet hits a bone in the body. The bone is known to shatter if the peak force exerted on it exceeds 3000 N; otherwise the bone just brings the bullet to a stop. Assume the bullet has a mass of 10 g and is traveling at a speed of 500 m/s.

(*a*) If the bone does not shatter, what is the total impulse delivered to it by the bullet?

(*b*) Assuming that the average force exerted on the bone is one-third of the peak force, what is the shortest stopping time for the bullet?

**Solution**

(*a*) From Newton's third law it is easy to see that the impulse exerted by the bone on the bullet is equal and opposite to the impulse exerted by the bullet on the bone. Thus finding the former also gives the latter; in fact they both have the same magnitude. For our case

$$|I| = |\Delta(mv)| = |(0 - mv_i)| = (0.010\text{ kg})\,(500\text{ m/s}) = 5.0\text{ N}\cdot\text{s}$$

(*b*) We know that

$$I = 5\text{ N}\cdot\text{s} = F_{av}\,\Delta t = \tfrac{1}{3}F_{peak}\,\Delta t \qquad (i)$$

Since the largest $F_{peak}$ without shattering is 3000 N, we put 3000 N in Eq. (*i*) to get the minimum $\Delta t$:

$$5\text{ N}\cdot\text{s} = (1000\text{ N})\,\Delta t_{min} \quad \text{or} \quad \Delta t_{min} = 0.0050\text{ s} = 5\text{ ms}$$

**Problem 8.10.** Assume that in Problem 8.7 the batter hit the same pitch so that the ball left the bat with the same speed as before, but this time it was aimed 50 ° above the horizontal (so that it sailed directly over the pitcher's head).

(*a*) Choosing the $x$ axis toward the pitcher and the $y$ axis vertically upward, find the $x$ and $y$ components of the impulse exerted by the bat on the ball.

(*b*) Find the magnitude $I$ and direction $\theta$ of the impulse exerted by the bat on the ball.

(*c*) Assuming the bat is in contact with the ball for the same length of time as in part (*b*) of Problem 8.7, find the magnitude and direction of the average force.

**Solution**

(*a*) We need the $x$ and $y$ components of the initial and final velocities. For the incoming pitch we have $v_{ix} = -40$ m/s, $v_{iy} = 0$. For the batted ball, immediately after it leaves the bat, we have $v_{fx} = (90\text{ m/s})\cos 50° = 57.9$ m/s, $v_{fy} = (90\text{ m/s})\sin 50° = 68.9$ m/s. Then for the impulse we have

$$I_x = mv_{fx} - mv_{ix} = (0.20\text{ kg})\,[(57.9\text{ m/s}) - (-40\text{ m/s})] = 19.6\text{ N}\cdot\text{s}$$
$$I_y = mv_{fy} - mv_{iy} = (0.20\text{ kg})\,[(68.9\text{ m/s}) - (0\text{ m/s})] = 13.8\text{ m/s}$$

(*b*) $I = (I_x^2 + I_y^2)^{1\backslash 2} = [(19.6)^2 + (13.8)^2]^{1/2}$ N · s = 24.0 N · s. If $\theta$ is the angle of elevation of the impulse above the $x$ axis, we have

$$\tan\theta = \frac{13.8}{19.6} = 0.704 \qquad \text{or} \qquad \theta = 35.1°$$

(*c*) Since $\mathbf{I} = \mathbf{F}_{\text{av}}\,\Delta t$, the vector $\mathbf{F}_{\text{av}}$ has the same direction as $\mathbf{I}$, and

$$F_{av} = \frac{I}{\Delta t} = \frac{24.0\text{ N}\cdot\text{s}}{0.0035\text{ s}} = 6860\text{ N}$$

***Note.*** In the previous collision problems we have ignored the contribution of the force of gravity to the impulse during the collisions. This is because the impulse lasts such a short time that the contributions of an "ordinary" force such as gravity to the impulse will be very small when compared to the contribution of the huge (but short-lived) contact forces over the same time interval. In Problem 8.10, for example, the contact time is thousandths of a second. The force of gravity is about 2 N; the average force due to the bat, by contrast, is almost 7000 N and is thus the dominant contributor to the impulse during the collision. Once the ball leaves the bat, however, the force of gravity must be taken into account.

## 8.3 CONSERVATION OF LINEAR MOMENTUM

### *Case of Two Objects*

Suppose we have two objects with no external forces acting on them, moving under their own mutual attraction or repulsion, as shown in Fig. 8-6. The force $\mathbf{F}_{ab}$ represents the force of object $A$ on object $B$, and $\mathbf{F}_{ba}$ represents the force of object $B$ on $A$. Then, by Newton's third law, $\mathbf{F}_{\text{ab}} = -\mathbf{F}_{ba}$ at any instant of time. Definition (*8.3*) then shows us that, over any time interval, $\mathbf{I}_{ab} = -\mathbf{I}_{ba}$. From the general impulse-momentum theorem, applied successively to each object, we get

$$\mathbf{I}_{ab} = \Delta(m_b\mathbf{v}_b) = m_b\mathbf{v}_{bf} - m_b\mathbf{v}_{bi} \qquad \text{and} \qquad \mathbf{I}_{ba} = \Delta(m_a\mathbf{v}_a) = m_a\mathbf{v}_{af} - m_a\mathbf{v}_{ai}$$

Then

$$m_b\mathbf{v}_{bf} - m_b\mathbf{v}_{bi} = -(m_a\mathbf{v}_{af} - m_a\mathbf{v}_{ai}) \qquad \text{or} \qquad m_a\mathbf{v}_{af} + m_b\mathbf{v}_{bf} = m_a\mathbf{v}_{ai} + m_b\mathbf{v}_{bi} \tag{8.9}$$

The second of Eqs. (*8.9*) may be restated as total final momentum = total initial momentum. Thus, for our two objects, momentum is conserved, no matter what the forces between the two objects may be.

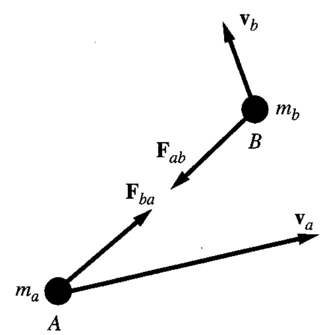


**Fig. 8-6**

*Note.* We will see later on, when we discuss center of mass, that conservation of momentum can be generalized to a system of any number of objects when no external forces (forces from outside the system) are acting on the system. It can even be extended to a system of objects that *do* have external forces acting on them, as long as the resultant of all the external forces adds up to zero.

For two-dimensional motion, Eq. (*8.9*) can be broken into its $x$- and $y$-component equations

$$m_a\mathbf{v}_{afx} + m_b\mathbf{v}_{bfx} = m_a\mathbf{v}_{aix} + m_b\mathbf{v}_{bix} \qquad (8.10a)$$

$$m_a\mathbf{v}_{afy} + m_b\mathbf{v}_{bfy} = m_a\mathbf{v}_{aiy} + m_b\mathbf{v}_{biy} \qquad (8.10b)$$

### *Collisions in One Dimension*

Equations (*8.9*) and (*8.10*) are particularly useful in dealing with problems of collisions between objects, where the dominant forces are the forces of the collision itself and the external forces can be neglected (see the note following Problem 8.10). In this section we will discuss one-dimensional collisions, (i.e., those that occur in a straight line).

**Problem 8.11.** Two blocks are moving in the same direction along the $x$ axis on a horizontal frictionless surface, as shown in Fig. 8-7. The blocks collide head-on (so that there is no change in the line of motion of either object). Find a relationship between the velocities of the two blocks after the collision.

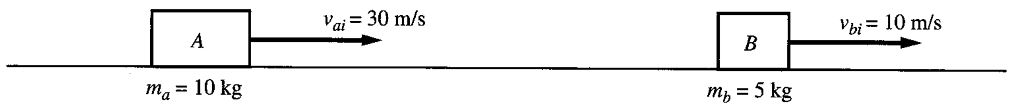


**Fig. 8-7**

**Solution**

Since there are no external forces in the $x$ direction, momentum is conserved along the $x$ axis, and we apply Eq. (*8.10a*) (but drop the $x$ subscript for convenience).

$$(10\ \text{kg})\,(30\ \text{m/s}) + (5.0\ \text{kg})\,(10\ \text{m/s}) = (10\ \text{kg})v_{af} + (5.0\ \text{kg})v_{bf}$$

which reduces to $2v_{af} + v_{bf} = 70$ m/s.

**Problem 8.12.** Assume the same situation in Fig. 8-7 except that now block $B$ is initially moving to the left at 20 m/s. Find the new relationship between the final velocities of the two blocks.

**Solution**

The situation is similar to that of Problem 8.11, and we again apply Eq. (*8.10a*):

$$(10\ \text{kg})\,(30\ \text{m/s}) + (5.0\ \text{kg})\,(-20\ \text{m/s}) = (10\ \text{kg})v_{af} + (5.0\ \text{kg})v_{bf}$$

or $2v_{af} + v_{bf} = 40$ m/s.

*Note.* In a collision problem in which the initial velocities are known, momentum conservation gives us a relation between the final velocities, but not enough information to completely solve for those velocities, unless additional information is provided.

### *Elastic Collisions*

An elastic collision is one in which the total kinetic energy of the colliding objects is the same just before and just after the collision:

$$\tfrac{1}{2}m_a v_{af}{}^2 + \tfrac{1}{2}m_b v_{bf}{}^2 = \tfrac{1}{2}m_a v_{ai}{}^2 + \tfrac{1}{2}m_b v_{bi}{}^2 \tag{8.11}$$

The implication of such a collision is that no thermal energy is generated during the collision and no energy is lost to the surroundings. Instead, as the two objects crush up against each other they are somewhat compressed, and, like springs, they store up potential energy which is released back in the form of kinetic energy as they separate. Since, by assumption, there are no energy losses, mechanical energy is conserved. Any external potential energy, such as that of gravity, is assumed to remain unchanged during the extremely short duration of the collision process.

While truly elastic collisions are believed to occur on the atomic scale, on the macroscopic scale they are always an approximation to the actual situation, since there are inevitably some thermal losses. Nonetheless, they are often excellent approximations to some collision processes.

Equation (*8.11*) is a quadratic equation in the velocities and therefore is often cumbersome to use. It turns out that by combining Eq. (*8.11*) with Eq. (*8.10a*) one can derive a much simpler equation involving the velocities that holds for one-dimensional elastic collisions:

$$(v_{af} - v_{bf}) = -(v_{ai} - v_{bi}) \tag{8.12a}$$

You may recognize from our discussion of relative motion in an earlier chapter that the expressions in parentheses on the left and right are just the velocities of object $A$ relative to object $B$, $v_{ab}$, after the collision and before the collision, respectively. Then Eq. (*8.12a*) simply says that for a one-dimensional elastic collision the relative velocity of approach of the two objects is equal and opposite to their relative velocity of separation:

$$v_{abf} = -v_{abi} \tag{8.12b}$$

**Problem 8.13.** Prove that Eq. (*8.12a*) follows from Eqs. (*8.11*) and (*8.10a*), as was stated above.

**Solution**

We again drop the $x$ subscript in Eq. (*8.10a*) since the whole problem is in one dimension. Rearranging Eqs. (*8.10a*) and (*8.11*) so that terms with the same mass appear on the same side of the equation, we get

$$m_a(v_{af} - v_{ai}) = m_b(v_{bi} - v_{bf}) \tag{i}$$

$$\tfrac{1}{2}m_a(v_{af}{}^2 - v_{ai}{}^2) = \tfrac{1}{2}m_b(v_{bi}{}^2 - v_{bf}{}^2) \tag{ii}$$

Noting that $A^2 - B^2 = (A - B)(A + B)$ for any $A$ and $B$, we can rewrite Eq. (*ii*) as

$$m_a(v_{af} - v_{ai})(v_{af} + v_{ai}) = m_b(v_{bi} - v_{bf})(v_{bi} + v_{bf}) \tag{iii}$$

Using Eq. (*i*), we see that Eq. (*iii*) simplifies to

$$v_{af} + v_{ai} = v_{bi} + v_{bf} \tag{iv}$$

Bringing all final velocities to the left and all initial velocities to the right, we finally get

$$v_{af} - v_{bf} = -(v_{ai} - v_{bi})$$

**Problem 8.14.** Assume that the two blocks in Problem 8.11 had an elastic collision.

(*a*) Find the final velocities of the two blocks, using Eq. (*8.12a*) for elastic collisions.

(*b*) Verify that your answer truly corresponds to an elastic collision.

**Solution**

(*a*) In Problem 8.11 the conservation of momentum yields

$$2v_{af} + v_{bf} = 70 \text{ m/s} \tag{i}$$

Applying Eq. (*8.12a*), we have $v_{af} - v_{bf} = -(30 \text{ m/s} - 10 \text{ m/s})$ or

$$v_{af} - v_{bf} = -20 \text{ m/s} \tag{ii}$$

We now solve the two simultaneous Eqs. (*i*) and (*ii*) for the final velocities, getting

$$v_{af} = 16.7 \text{ m/s} \qquad \text{and} \qquad v_{bf} = 36.7 \text{ m/s}$$

(*b*) To show that our results are consistent with an elastic collision, we calculate the actual total kinetic energy before and after the collision, getting

$$E_{ki} = \tfrac{1}{2}(10 \text{ kg})(30 \text{ m/s})^2 + \tfrac{1}{2}(5 \text{ kg})(10 \text{ m/s})^2 = 4750 \text{ J}$$
$$E_{kf} = \tfrac{1}{2}(10 \text{ kg})(16.7 \text{ m/s})^2 + \tfrac{1}{2}(5 \text{ kg})(36.7 \text{ m/s})^2 = 4761 \text{ J}$$

These check to within rounding errors in our final velocities.

**Problem 8.15.** Assuming that the collision described in Problem 8.12 is an elastic collision, find the final velocities of the two blocks.

**Solution**

From Problem 8.12 we already have the results of momentum conservation:

$$2v_{af} + v_{bf} = 40 \text{ m/s} \tag{i}$$

From Eq. (*8.12a*) we get $v_{af} - v_{bf} = -(30 \text{ m/s} - [-20 \text{ m/s}])$ or

$$v_{af} - v_{bf} = -50 \text{ m/s} \tag{ii}$$

Solving Eqs. (*i*) and (*ii*) for $v_{af}$ and $v_{bf}$, we get

$$v_{af} = -3.33 \text{ m/s} \qquad \text{and} \qquad v_{bf} = 46.7 \text{ m/s}$$

Note that in this problem the final velocity of block $A$ is in the negative $x$ direction.

**Problem 8.16.** Two blocks of equal mass make a head-on elastic collision, with one of the blocks initially at rest. Show that just after the collision the initially moving block will be at rest, and the block initially at rest will have exactly the same velocity as the initially moving block.

**Solution**

From Eq. (*8.10a*) we have $mv_{af} + mv_{bf} = mv_{ai} + 0$, which reduces to

$$v_{af} + v_{bf} = v_{ai} \tag{i}$$

From Eq. (*8.12a*) we have

$$v_{af} - v_{bf} = -v_{ai} \tag{ii}$$

Solving for the final velocities (e.g., by equating the sum of the left sides of the two equations to the sum of the right sides) yields

$$v_{af} = 0 \qquad \text{and} \qquad v_{bf} = v_{ai}$$

**Problem 8.17.** Use Problem 8.16 to explain the behavior of the hanging steel-ball devices sold in novelty shops (Fig. 8-8).

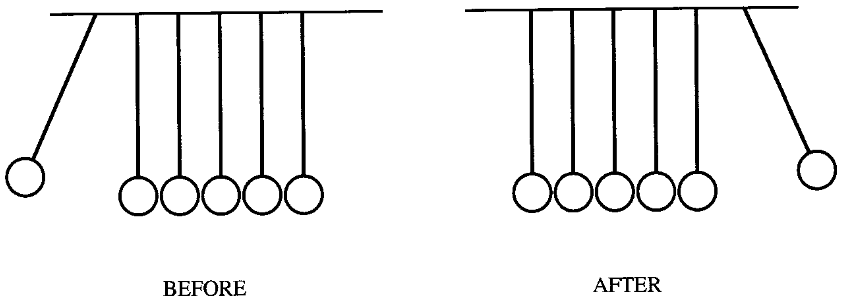


**Fig. 8-8**

**Solution**

All the steel balls have the same mass. Steel balls collide *almost* elastically, so we assume the collisions between the individual balls are elastic. Also, we have conservation of momentum in the horizontal direction, because the tensions in the supporting strings are vertical during the collision process. Finally, though the balls actually may be in contact when they are in their rest positions, we may assume that they are slightly separated, to make Problem 8.16 more readily applicable.

When the first ball is moved to the side and released, it builds up a certain velocity with which it horizontally collides with its neighbor ball, which is at rest. Then, by Problem 8.16, the first ball comes to rest and the second ball picks up its velocity, but not for long. The second ball almost immediately collides with the third ball, and by the same reasoning it comes to rest and the third ball picks up the velocity. This process continues until the next-to-last ball hits the last ball. The next-to-last ball comes to rest, and the last ball moves off with the same velocity which the first ball had. It thus rises to the same height from which the first ball was let go. After reaching the highest point it descends and starts the collision process going again, with the same speed, but in the opposite direction. The balls in the middle always appear at rest because they have a negligible distance to travel from the time they are hit to the time they once again come to rest.

### *Inelastic Collisions*

Any collision that is not elastic is called **inelastic**. An inelastic collision is characterized by a certain disappearance of kinetic energy in the collision process. In general, unless one knows precisely how much thermal energy is generated in the collision, one cannot write down an auxiliary equation to use together with the momentum conservation equation (*8.10a*) to solve for the unknown velocities. The best one can do is to account for the thermal energy loss in collisions of different objects by means of an empirical quantity called the **coefficient of restitution** $e$. The value of $e$ is defined as the ratio of the magnitude of the relative velocity after the collision to that before the collision:

$$e = \left| \frac{v_{abf}}{v_{abi}} \right| = \frac{-(v_{af} - v_{bf})}{v_{ai} - v_{bi}} \tag{8.13}$$

We see from Eqs. (*8.12*) that for an elastic collision, $e = 1$. Generally speaking, the smaller the $e$ value, the more thermal energy is generated and hence the more kinetic energy is lost. The lowest possible value of $e$ is $e = 0$, which corresponds to $v_{af} = v_{bf}$. This means that the two objects move as one after the collision; in other words, they stick together. The case of $e = 0$ is often called a *totally inelastic collision*. The value of the coefficient of restitution depends very heavily on the nature of the materials colliding, as well as other factors.

**Problem 8.18.**

(*a*) Redo Problem 8.14(*a*) if $e = 0.80$.

(*b*) What fraction of the initial kinetic energy is lost in this collision.

**Solution**

(*a*) Now momentum conservation and the definition of $e$ give

$$2v_{af} + v_{bf} = 70 \text{ m/s}$$
$$v_{af} - v_{bf} = -(0.80)\,(30 \text{ m/s} - 10 \text{ m/s}) = -16 \text{ m/s}$$

Solve as before, to find

$$v_{af} = 18.0 \text{ m/s} \qquad \text{and} \qquad v_{bf} = 34.0 \text{ m/s}$$

(*b*) The initial kinetic energy was found in Problem 8.14(*b*) to be 4750 J. For the final kinetic energy we now have

$$E_{kf} = \tfrac{1}{2}(10 \text{ kg})\,(18.0 \text{ m/s})^2 + \tfrac{1}{2}(5 \text{ kg})\,(34.0 \text{ m/s})^2 = 4510 \text{ J}$$

Then $\qquad \text{Fraction lost} = 1 - (\text{fraction remaining}) = 1 - \dfrac{E_{kf}}{E_{ki}} = 0.051 = 5.1\%$

**Problem 8.19.** If the collision described in Problem 8.12 is an inelastic collision with $e = 0.5$, find (*a*) the final velocities of the two blocks, (*b*) the total thermal energy generated in the collision.

**Solution**

(*a*) From Problem 8.12 and Eq. (*8.13*) we have

$$2v_{af} + v_{bf} = 40 \text{ m/s}$$
$$v_{af} - v_{bf} = -(0.5)\,[30 \text{ m/s} - (-20 \text{ m/s})] = -25 \text{ m/s}$$

Solving by addition, we get

$$v_{af} = 5.0 \text{ m/s} \qquad \text{and} \qquad v_{bf} = 30.0 \text{ m/s}$$

(*b*) The thermal energy generated is just $E_{ki} - E_{kf}$:

$$E_{ki} = \tfrac{1}{2}(10\text{ kg})(30\text{ m/s})^2 + \tfrac{1}{2}(5.0\text{ kg})(20\text{ m/s})^2 = 5500\text{ J}$$
$$E_{kf} = \tfrac{1}{2}(10\text{ kg})(5.0\text{ m/s})^2 + \tfrac{1}{2}(5.0\text{ kg})(30\text{ m/s})^2 = 2375\text{ J}$$
$$\text{Thermal energy} = E_{ki} - E_{kf} = 3125\text{ J}$$

**Problem 8.20.** Repeat Problem 8.18 for a totally inelastic collision.

**Solution**

For this case the two objects stick together, and we immediately know that $v_{af} = v_{bf} \equiv v_f$. While we could proceed as in Problem 8.18 it is more informative to insert $v_f$ directly into the momentum equation:

$$m_a v_{ai} + m_b v_{bi} = (m_a + m_b)v_f \qquad \text{or} \qquad (10\text{ kg})(30\text{ m/s}) + (5.0\text{ kg})(10\text{ m/s}) = (15\text{ kg})v_f \qquad (i)$$

or $v_f = 23.3$ m/s. To obtain the fractional loss in kinetic energy, we again note that from Problem 8.18 (or Problem 8.14) we have $E_{ki} = 4750$ J. We get

$$E_{kf} = \tfrac{1}{2}(m_a + m_b){v_f}^2 = \tfrac{1}{2}(15\text{ kg})(23.3\text{ m/s})^2 = 4070\text{ J}$$

Then, the fractional loss equals 1 − (4070/4750) = 0.143 = 14.3%.

### *The Ballistic Pendulum*

A ballistic pendulum is a device that is used to measure the velocities of small swift projectiles, such as bullets. A typical schematic of such a device is shown in Fig. 8-9. A bullet of mass $m$ is fired horizontally into a block of mass $M$ and embeds itself in the block. The block, which is suspended from the ceiling by vertical cords, then rises, as indicated by the dotted lines, through some measurable vertical height $h$. From this information one can deduce the initial speed $v_i$ of the bullet.

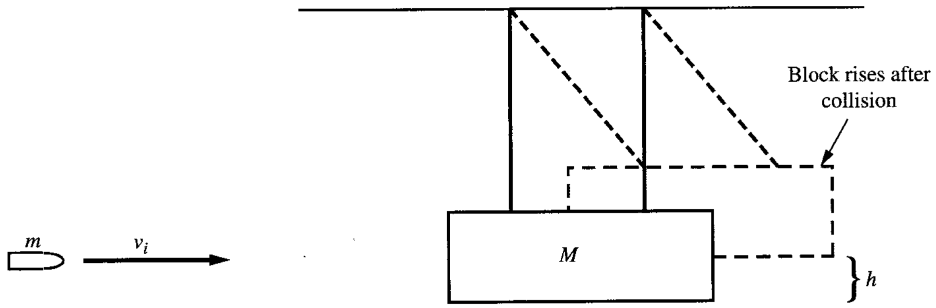


**Fig. 8-9**

**Problem 8.21.** For a ballistic pendulum (Fig. 8-9), assume $m$ = 10 g, $M$ = 3990 g, and $h$ = 3.0 cm.

(*a*) Find the velocity $V_f$ of the combined block-bullet body just after the collision.

(*b*) Find the initial velocity of the bullet.

**Solution**

(*a*) Here we use the fact that from the time immediately after the collision through the rise time, mechanical energy is conserved. This is a consequence of the fact that the tensions in the cords can do no work since the points of contact with the block move on the arcs of circles perpendicular to the cords. At the moment just after the collision we have $\frac{1}{2}(m + M)V_f^2 = (m + M)gh$. Solving for $V_f$ we get

$$V_f = (2gh)^{1/2} = [2(9.8\ \text{m/s}^2)\ (0.030\ \text{m})]^{1/2} \qquad \text{or} \qquad V_f = 0.767\ \text{m/s}$$

Observe that momentum is not conserved when the block is rising owing to the combined actions of the external forces of gravity and cord tension.

(*b*) During the collision process we assume that conservation of momentum holds in the horizontal direction. This follows since the block does not move an appreciable distance during the collision time, and the tensions in the cords and force of gravity do not act in the horizontal direction. Then $mv_i = (m + M)V_f$. Substituting the known values, we get $v_i = 307$ m/s.

**Problem 8.22.** Suppose that the bullet in Problem 8.21 passes straight through the block and emerges out the other side with velocity $v_f = 250$ m/s. Assume that the block still rises through a height $h = 3.0$ cm. Find the initial velocity of the bullet. (Ignore the effect of any splinters that emerge with the bullet.)

**Solution**

The velocity of the block just after the bullet leaves it is again determined by conservation of mechanical energy, and we again have $V_f = 0.767$ m/s. Then, applying momentum conservation during the brief penetration time, we have $mv_i = mv_f + MV_f$, so

$$10v_i = 10(250\ \text{m/s}) + 3990(0.767\ \text{m/s}) \qquad \text{or} \qquad v_i = 556\ \text{m/s}$$

**Problem 8.23.** How much kinetic energy was lost in the brief collision time of (*a*) Problem 8.21(*b*)? (*b*) Problem 8.22?

**Solution**

(*a*) $E_{ki} = \frac{1}{2}(0.010\ \text{kg})\ (307\ \text{m/s})^2 = 471\ \text{J} \qquad E_{kf} = \frac{1}{2}(4.000\ \text{kg})\ (0.767\ \text{m/s})^2 = 1.18\ \text{J}$

for a loss of $E_{ki} - E_{kf} \approx 470$ J. (Thus almost all the kinetic energy was lost to thermal energy even though momentum was conserved!)

(*b*) Here, $E_{ki} = \frac{1}{2}(0.010\ \text{kg})\ (556\ \text{m/s})^2 = 1546\ \text{J}$

and $E_{kf} = \frac{1}{2}(0.010\ \text{kg})\ (250\ \text{m/s})^2 + \frac{1}{2}(3.990)\ (0.767\ \text{m/s})^2 = 314\ \text{J}$

for a loss of 1232 J.

### *Collisions in Two Dimensions*

Much of the discussion for one-dimensional collisions carries over to the two-dimensional case. Conservation of momentum, (*8.9*), is valid as a vector equation or can be expressed as a pair of component equations. Analogous to (*8.12*), we have as the definition of the coefficient of restitution

$$e = \frac{|\mathbf{v}_{abf}|}{|\mathbf{v}_{abi}|} = \frac{|\mathbf{v}_{af} - \mathbf{v}_{bf}|}{|\mathbf{v}_{ai} - \mathbf{v}_{bi}|} \qquad (8.12\ bis)$$

(See Eq. (*3.11*).] It can be shown that in a two-dimensional *elastic* collision,

$$|\mathbf{v}_{abf}| = |\mathbf{v}_{abi}| \qquad (8.11\ bis)$$

or $e = 1$ for an elastic collision. For a sticky collision we again have $e = 0$, just as in the one-dimensional case.

**Problem 8.24.** Two pucks of equal mass $m$ have an elastic collision on a frictionless horizontal table, as shown in Fig. 8-10. Assume that puck $B$ is initially at rest, and puck $A$ has initial speed $v_{Ai} = 4.5$ ft/s and final speed $v_{Af} = 2.5$ ft/s. Choose your $x$ axis along $\mathbf{v}_{Ai}$.

(*a*) Show that the $y$ components of the two final velocities are equal and opposite.

(*b*) Find the final speed of puck $B$.

(*c*) The angle $\theta$ can be shown to be 56.2°. Find the angle $\phi$.

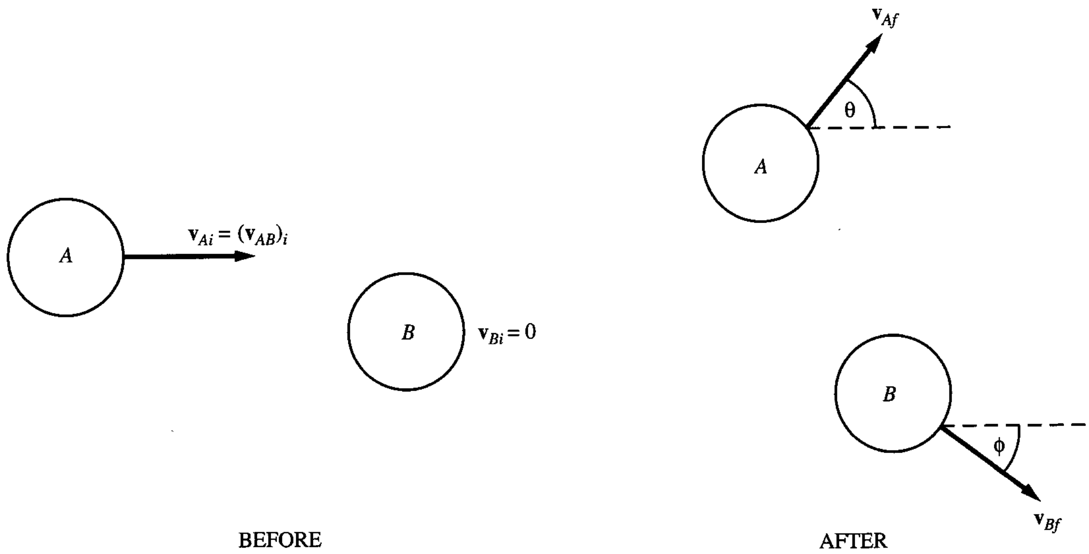


**Fig. 8-10**

**Solution**

(*a*) The initial momentum in the $y$ direction is zero, so

$$0 = mv_{(Af)y} + mv_{(Bf)y} \Rightarrow v_{(Af)y} = -v_{(Bf)y}$$

(*b*) From kinetic energy conservation, we have

$$\tfrac{1}{2}mv_{Ai}^2 = \tfrac{1}{2}mv_{Af}^2 + \tfrac{1}{2}mv_{Bf}^2$$

or $$v_{Bf}^2 = v_{Ai}^2 - v_{Af}^2 = (4.5\text{ ft/s})^2 - (2.5\text{ ft/s})^2 = 14.0\text{ ft}^2/\text{s}^2$$

Solving, we get $v_{Bf} = 3.74$ ft/s.

(*c*) From part (*a*) $v_{Af} \sin\theta = -(-v_{Bf} \sin\phi) = v_{Bf} \sin\phi \Rightarrow 2.5 \sin 56.2° = 3.74 \sin\phi \Rightarrow \sin\phi = 0.556 \Rightarrow \phi = 33.7°$. (Alternatively,

$$mv_{Ai} = mv_{Af}\cos\theta + mv_{Bf}\cos\phi \Rightarrow 4.5 = 2.5\cos 56.2° + 3.74\cos\phi \Rightarrow \cos\phi = 0.831 \Rightarrow \phi = 33.8°$$

which is our previous result to within rounding errors.) One can in fact show that the sum of the angles, $\theta + \phi$, which represents the angle between $\mathbf{v}_{Af}$ and $\mathbf{v}_{Bf}$, is exactly 90° for any elastic collision between objects of equal mass, one of which is initially at rest.

**Problem 8.25.** Two blocks, of masses $m_A = 5.0$ kg and $m_B = 12.0$ kg, are initially moving at right angles to each other on a frictionless horizontal surface, as shown in Fig. 8-11. Assume the blocks have a totally inelastic collision and that $v_{Ai} = 30$ m/s and $v_{Bi} = 15$ m/s.

(*a*) Find the magnitude and direction of the final velocity $\mathbf{v}_f$ of the combination.

(*b*) How much thermal energy is generated in this collision?

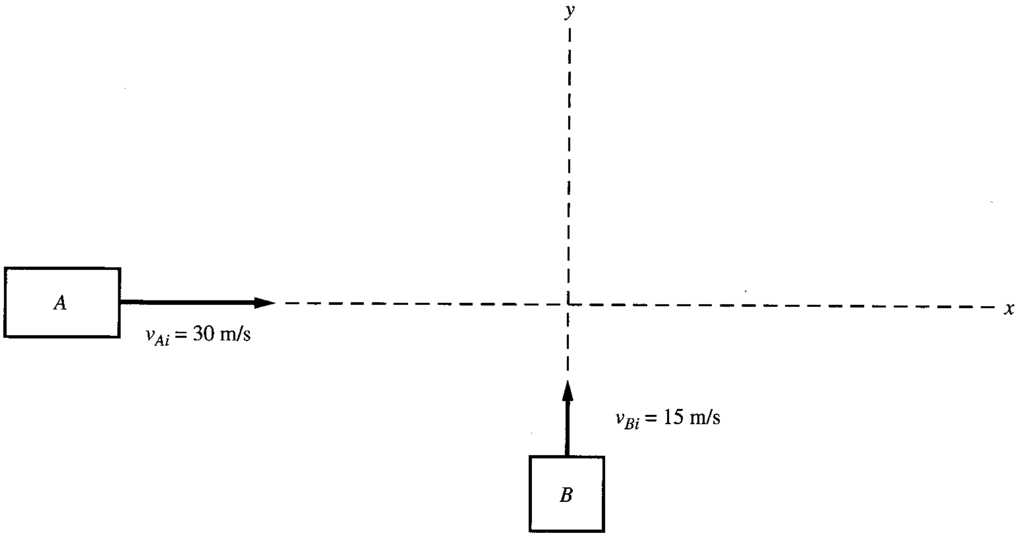


**Fig. 8-11**

**Solution**

(*a*) We solve the $x$ and $y$ components of the momentum conservation equation. For the $x$ direction, we have $m_A v_{Ai} + 0 = (m_A + m_B)v_{fx}$. Substituting, we get

$$5.0(30\text{ m/s}) = 17.0v_{fx} \qquad \text{or} \qquad v_{fx} = 8.82\text{ m/s}$$

For the $y$ direction, $0 + m_B v_{Bi} = (m_A + m_B)v_{fy}$, so

$$12.0(15\text{ m/s}) = 17.0v_{fy} \qquad \text{or} \qquad v_{fy} = 10.6\text{ m/s}$$

$$v_f = [(8.82)^2 + (10.6)^2]^{1/2}\text{ m/s} = 13.8\text{ m/s}$$

Letting $\theta$ equal the angle of $\mathbf{v}_f$ with the $x$ axis, we have $\tan\theta = 10.6/8.82 = 1.20$, from which we get $\theta = 50.2\,°$.

(*b*) $E_{ki} = \frac{1}{2}(5.0\text{ kg})(30\text{ m/s})^2 + \frac{1}{2}(12.0\text{ kg})(15\text{ m/s})^2 = 3600$ J; $E_{kf} = \frac{1}{2}(17.0\text{ kg})(13.8\text{ m/s})^2 = 1619$ J. The thermal energy generated equals $E_{ki} - E_{kf} = 1981$ J.

### *Recoil*

In addition to collision problems, another class of problems that involves impulse and momentum is related to recoil. In recoil problems a system that is initially at rest, or moving as a unit, breaks up into two or more parts moving with different velocities as a consequence of rapid expenditure of some internal energy of the system. One example is a rifle and bullet. Initially both are at rest, with the bullet within the rifle barrel. When the rifle is fired, the gunpowder in the bullet casing explodes and drives the bullet forward. The forward impulse on the bullet is equal and opposite to the impulse imparted to the rifle. If the rifle is resting against the shoulder, the person firing it feels the "kick" of this impulse. In a rocket the burning fuel hurls hot gas out the back, and the reaction impulse pushes the rocket in the opposite direction (forward). If no external forces act on the rocket we have momentum conservation, and the increase in forward momentum of the rocket just balances the backward momentum gained by the ejected gases.

**Problem 8.26.** Two blocks on a frictionless horizontal surface are pressed back to back so that they compress a spring of negligible mass held between them, as shown in Fig. 8-12. They are held in position by a connecting cord. Assume that $m_A = 14$ kg and $m_B = 8.0$ kg and that the stored potential energy in the spring is 1000 J. When the cord is cut, the two blocks move in opposite directions with speeds $v_A$ and $v_B$.

(*a*) Find the relationship between $v_A$ and $v_B$.

(*b*) Find the values of $v_A$ and $v_B$.

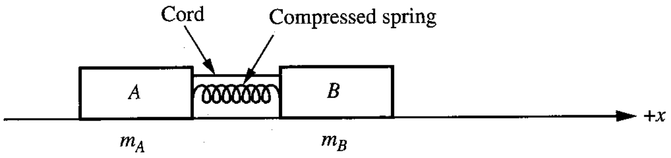


**Fig. 8-12**

**Solution**

(*a*) Considering the spring to be part of the two-block system, we see that there are no external forces acting on the system in the horizontal direction. We thus have momentum conservation in that direction. Choosing positive to the right, we have, since the system starts from rest

$$0 = -m_A v_A + m_B v_B \quad \text{or} \quad m_A v_A = m_B v_B \quad \text{or} \quad 14 v_A = 8.0 v_B \quad \text{or} \quad v_B = 1.75 v_A$$

(*b*) All the spring energy goes into kinetic energy of the two blocks, so $\frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 = 1000$ J, or, using the results of part (*a*) we get

$$\tfrac{1}{2}(14\ \text{kg}) v_A^2 + \tfrac{1}{2}(8.0\ \text{kg})\,(1.75\, v_A)^2 = 1000\ \text{J}$$

or $$(19.25\ \text{kg}) v_A^2 = 1000\ \text{J} \quad \text{or} \quad v_A = 51.9\ \text{m/s} \quad \text{and} \quad v_B = 90.8\ \text{m/s}$$

**Problem 8.27.** A machine gun fires bullets of mass $m = 25$ g at a muzzle velocity of 1200 m/s.

(*a*) What is the recoil impulse on the gun due to each bullet that is fired?

(*b*) If 300 bullets are fired per minute, what is the total recoil impulse imparted to the gun in 1 min?

(*c*) What is the average recoil force acting on the gun?

**Solution**

(*a*) The impulse $I$ on the gun is equal and opposite to that acting on the bullet. The impulse on the bullet equals its change in momentum. Since the bullet starts from rest, we have

$$I = mv - 0 = (0.025\ \text{kg})\,(1200\ \text{m/s}) = 30\ \text{N}\cdot\text{s}$$

(*b*) The total impulse in 1 min is just $300I = 9000\ \text{N}\cdot\text{s}$.

(c) Over the 1 min period, $F_{av}t = 9000\ \text{N} \cdot \text{s}$ so that

$$F_{av}(60\ \text{s}) = 9000\ \text{N} \cdot \text{s} \qquad \text{or} \qquad F_{av} = 150\ \text{N}$$

## 8.4 CENTER OF MASS

The concept of **center of mass** (CM) turns out to be useful in understanding the motion of large extended objects as well as of systems of particles.

### *Systems of Particles*

The CM of a system of particles is defined as the position of the average displacement of the particles, weighted according to mass. The situation is depicted in Fig. 8-13, which shows a system of $N$ particles. The $i$th particle has mass $m_i$ and displacement from the origin $\mathbf{r}_i$. Then, the average displacement weighted according to the mass of each particle is just

$$\mathbf{R}_{\text{CM}} = \frac{\sum m_i \mathbf{r}_i}{\sum m_i} \tag{8.14}$$

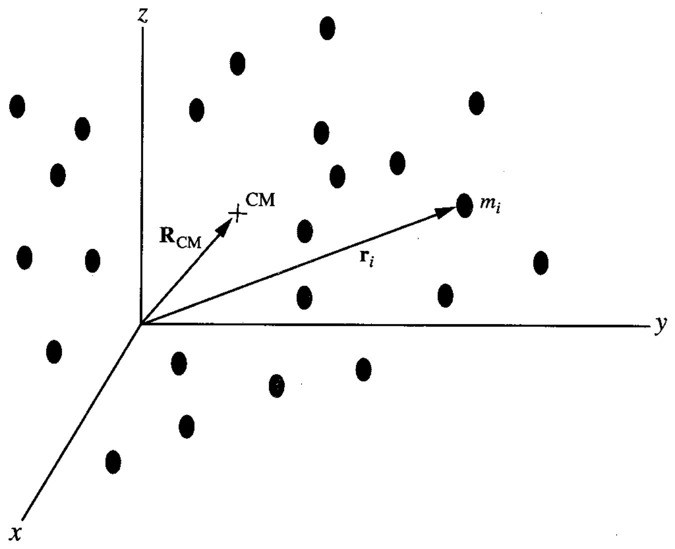


**Fig. 8-13**

where the sums are over all $i$ from 1 to $N$. The sum in the denominator is just the total mass of the system $M$. Thus Eq. (8.14) can be rewritten as

$$M\mathbf{R}_{\text{CM}} = \sum m_i \mathbf{r}_i \tag{8.15}$$

If a (small) amount of time $\Delta t$ elapses, the displacements $\mathbf{r}_i$ will change by (small) amounts $\Delta\mathbf{r}_i$. In other words, $\mathbf{r}_i \to \mathbf{r}_i + \Delta\mathbf{r}_i$ in the time $\Delta t$. The corresponding change $\Delta\mathbf{R}_{\text{CM}}$ in the displacement of the CM is given by $\mathbf{R}_{\text{CM}} \to \mathbf{R}_{\text{CM}} + \Delta\mathbf{R}_{\text{CM}}$. Examining Eq. (8.15) with the new values yields $M\,\Delta\mathbf{R}_{\text{CM}} = \sum m_i\,\Delta\mathbf{r}_i$. Dividing by $\Delta t$ we get

$$M\,\frac{\Delta\mathbf{R}_{\text{CM}}}{\Delta t} = \sum m_i\,\frac{\Delta\mathbf{r}_i}{\Delta t} \tag{8.16a}$$

In the limit of infinitesimal $\Delta t$, the quantity $\Delta\mathbf{r}_i/\Delta t = \mathbf{v}_i$, the velocity of the $i$th particle. Similarly, $\Delta\mathbf{R}_{CM}/\Delta t = \mathbf{V}_{CM}$, the velocity of the CM. Thus

$$M\mathbf{V}_{CM} = \sum m_i\mathbf{v}_i \tag{8.16b}$$

The right side of (*8.16b*) is just the total momentum of the system of particles. Therefore Eq. (*8.16b*) states that the total mass of the system times the velocity of the CM equals the total momentum of the system. Thus, if momentum is conserved, $\mathbf{V}_{CM}$ is constant. If we now consider the small change in the velocity of each particle in a (small) time $\Delta t$, we could redo the steps that lead from (*8.15*) to (*8.16b*), this time starting with Eq. (*8.16b*), to get

$$M\frac{\Delta\mathbf{V}_{CM}}{\Delta t} = \sum m_i\frac{\Delta\mathbf{v}_i}{\Delta t} \tag{8.17a}$$

$$M\mathbf{A}_{CM} = \sum m_i\mathbf{a}_i \tag{8.17b}$$

where the **a**'s are accelerations. From Newton's second law the right side of Eq. (*8.17b*) is just $\sum\mathbf{F}_i$, where $\mathbf{F}_i$ is the resultant force on the $i$th particle and includes the forces due to all the other particles in the system on the $i$th particle, as well as the net external force on the $i$th particle, $(\mathbf{F}_{ext})_i$. In the $\sum\mathbf{F}_i$ all the internal forces cancel in pairs due to Newton's third law, so $\sum\mathbf{F}_i = \sum(\mathbf{F}_{ext})_i = (\mathbf{F}_{ext})_T$, the resultant external force on the system. Equation (*17b*) thus becomes

$$(\mathbf{F}_{ext})_T = M\mathbf{A}_{CM} \tag{8.18}$$

In other words, the CM accelerates as if it were a particle of mass $M$ acted on by a force equal to the resultant of all the external forces acting on the system of particles. We now can deduce our general rule for momentum conservation for an arbitrary system of particles: If $(\mathbf{F}_{ext})_T = 0$, then $\mathbf{A}_{CM} = 0$ and $\mathbf{V}_{CM}$ = constant. But, from Eq. (*8.16b*) this is the same as saying the momentum of the system of particles is constant. Thus it follows that the momentum of a system of particles is conserved as long as the *resultant* external force on the system equals zero.

Note that Eqs. (*8.15*), (*8.16b*), and (*8.18*) are vector equations; they can therefore be broken into component equations. For example, Eq. (*8.15*) can be expressed as

$$MX_{CM} = \sum m_i x_i \qquad MY_{CM} = \sum m_i y_i \qquad MZ_{CM} = \sum m_i z_i \tag{8.19a, b, c}$$

where $x_i$, $y_i$, and $z_i$ are the components of $\mathbf{r}_i$, etc.

### *Rigid Bodies*

In the case of an extended rigid body, Eq. (*8.18*) indicates that the CM of the body moves as if it were a particle having a mass equal to the total mass of the body acted on by the resultant force on the body. Thus, for any body, even for an irregular body, rotating and translating in space, we can describe the motion by studying the motion of its center of mass. To do this, we need to find the location of the CM of rigid bodies. This can be rather difficult when dealing with irregular objects; even for simple objects, such as a uniform cone, one needs calculus to find the CM. We will illustrate how to find the CM in some simple cases.

**Problem 8.28.** Find the center of mass of the three-particle system in the $xy$ plane shown in Fig. 8-14.

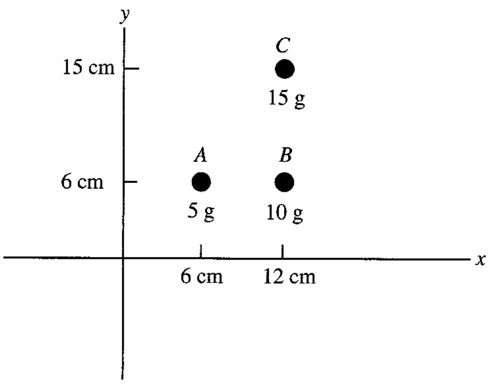


**Fig. 8-14**

**Solution**

We use Eqs. (*8.19*). For the $x$ equation,

$$MX_{\rm CM} = m_A x_A + m_B x_B + m_C x_C$$

$$(30\ {\rm g})X_{\rm CM} = (5.0\ {\rm g})\,(6.0\ {\rm cm}) + (10\ {\rm g})\,(12\ {\rm cm}) + (15\ {\rm g})\,(12\ {\rm cm}) = 330\ {\rm g\cdot cm}$$

or

$$X_{\rm CM} = 11\ {\rm cm}$$

Similarly, for the $y$ equation,

$$MY_{\rm CM} = m_A y_A + m_B y_B + m_C y_C$$

$$(30\ {\rm g})\,Y_{\rm CM} = (5.0\ {\rm g})\,(6.0\ {\rm cm}) + (10\ {\rm g})\,(6.0\ {\rm cm}) + (15\ {\rm g})\,(15\ {\rm cm}) = 315\ {\rm g\cdot cm}$$

or

$$Y_{\rm CM} = 10.5\ {\rm cm}$$

**Problem 8.29.** Find the CM of (*a*) a uniform sphere, (*b*) a uniform cylinder, and (*c*) a uniform donut (torus).

**Solution**

The CM of all three objects appear at their geometric centers by symmetry. To see this we consider the sphere. The CM of the uniform sphere must be at its center since if it were anywhere else it would change location upon rotation of the sphere. Since the mass distribution of the sphere is the same in the rotated position as in the original position, the CM of the same mass distribution would be in two different locations, which is impossible. Thus the CM must be at the center. Similar reasoning can be used for the uniform cylinder and the uniform donut. Note that in the case of the donut the CM is not in the object itself. The CM is a geometric point fixed in relation to a rigid body, but it is not necessarily in the body.

**Problem 8.30.** What can you say about the location of the CM of a uniform cone?

**Solution**

By symmetry, the CM must lie along the central symmetry axis of the cone. It is also clear that it will be closer to the base of the cone than to the apex, since there is more mass toward the base and the CM location is weighted by mass. To obtain the exact location one can use the calculus. The CM turns out to be one-fourth the way from the base to the apex.

## Problems for Review and Mind Stretching

**Problem 8.31.** A rifle bullet of mass $m = 20$ g is fired with a muzzle velocity of 600 m/s. The length of the barrel is 80 cm.

(*a*) What is the total impulse imparted to the bullet?

(*b*) If the rifle were free to "kick" backward, what would its velocity be when the bullet has left the muzzle, assuming the mass of the rifle $M$ was 3.0 kg?

**Solution**

(*a*) $I = mv_f - mv_i = (0.020\ \text{kg})(600\ \text{m/s}) - 0 = 12\ \text{N}\cdot\text{s}$.

(*b*) Choosing the direction of the bullet as positive, from conservation of momentum (noting that the momentum before firing is zero), we have

$$0 = mv + MV = 12\ \text{N}\cdot\text{s} + (3.0\ \text{kg})V \qquad \text{or} \qquad V = -4.0\ \text{m/s}$$

**Problem 8.32.** Show that when two objects $A$ and $B$ of equal mass $m$ have an elastic collision in which one of the objects (say, $B$) is initially at rest, the final velocities of the two objects must be at right angles to each other. (See Problem 8.24 and the accompanying comments.)

**Solution**

The situation is similar to that in Fig. 8-10. From momentum conservation we have $m\mathbf{v}_{Ai} = m\mathbf{v}_{Af} + m\mathbf{v}_{Bf}$ or

$$\mathbf{v}_{Ai} = \mathbf{v}_{Af} + \mathbf{v}_{Bf} \qquad (i)$$

This vector equation implies that the three velocities form a triangle, as shown in Fig. 8-15. Since the collision is elastic we also have $\frac{1}{2}mv_{ai}^2 = \frac{1}{2}mv_{Af}^2 + \frac{1}{2}mv_{Bf}^2$ or

$$v_{Ai}^2 = v_{Af}^2 + v_{Bf}^2 \qquad (ii)$$

Equation (*ii*) implies that $v_{Ai}$ is the hypotenuse of a right triangle whose other sides are $v_{Af}$ and $v_{Bf}$. Thus the triangle in Fig. 8-15 is a right triangle, and $\mathbf{v}_{Af}$ and $\mathbf{v}_{Bf}$ are at right angles.

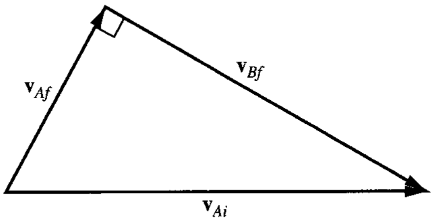


**Fig. 8-15**

**Problem 8.33.** A ball is dropped from rest at a height $h = 20$ ft onto a horizontal concrete floor.

(*a*) If the collision with the floor is perfectly elastic (coefficient of restitution $e = 1$), describe the subsequent events.

(*b*) Repeat parts (*a*) if $e = 0.7$.

**Solution**

(*a*) Assume the ball hits the floor with speed $v_a$. During the very brief collision period between the ball and the earth, we can ignore gravity and assume momentum is conserved in the vertical direction. Since $e = 1$, immediately after the collision the ball has the same magnitude and opposite direction relative to the earth. Since the velocity gained by the earth in the collision is so small, it contributes negligibly to the kinetic energy. Thus, the ball will have the same kinetic energy as before and will rise to the exact height from which it was dropped: $h = 20$ ft. The ball will repeat the bounce again and again.

(*b*) In this situation the ball loses some kinetic energy to thermal energy during the collision so that its rebound velocity is only $v_a' = 0.7v_a$. Therefore, it will rebound only to a height given by $v_a' = (2gh_a')^{1/2}$. Then $e^2 = (v_a'/v_a)^2 = 2gh_a'/2gh_a = h_a'/h_a$ or $(0.70)^2 = h_a'/h_a$. Solving, we get $h_a' = (0.49)h_a = (0.49)(20\text{ ft}) = 9.8$ ft. By the same reasoning, on each succeeding bounce the ball will rise to 0.49 times the previous height.

**Problem 8.34.** A bullet of mass $m = 15$ g is fired into a block of mass $M = 985$ g, which is attached to an uncompressed spring of force constant $k = 1000$ N/m. The spring is anchored to a wall, and the block rests on a horizontal frictionless surface as shown in Fig. 8-16. After the bullet embeds itself in the block, the block compresses the spring a maximum distance of $x = 12$ cm. Find the initial velocity of the bullet.

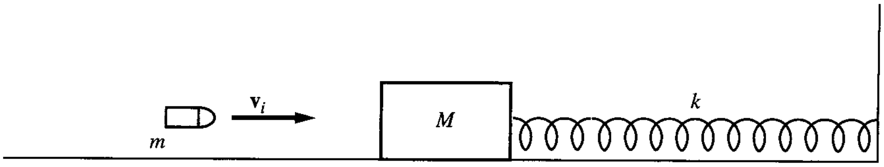


**Fig. 8-16**

**Solution**

This is similar to the earlier ballistic pendulum problems, except that now instead of rising against gravity the block compresses the spring. If we assume the collision of the bullet with the block is very rapid, the spring will not start to compress until after the collision is complete. If $V_f$ is the velocity of the bullet-block combination just after the collision, we have, from conservation of mechanical energy,

$$\tfrac{1}{2}(m+M)V_f{}^2 + 0 = 0 + \tfrac{1}{2}kx^2$$

Substituting the known values we get

$$\tfrac{1}{2}(1.00\text{ kg})V_f{}^2 = \tfrac{1}{2}(1000\text{ N/m})\,(0.12\text{ m})^2 \qquad \text{or} \qquad V_f = 3.79\text{ m/s}$$

Next we note that, during the brief collision process, the spring is negligibly compressed and exerts negligible horizontal impulse. We then have conservation of momentum from just before to just after the collision:

$$mv_i = (m+M)V_f \qquad \text{or} \qquad (0.015\text{ kg})v_i = (1.00\text{ kg})\,(3.79\text{ m/s}) \qquad \text{so that} \qquad v_i = 253\text{ m/s}$$

**Problem 8.35.** A projectile is fired from ground level with a velocity $v_0$ of 600 ft/s, at an angle $\theta_0$ of 30° above the horizontal. At the highest point in the trajectory the projectile suddenly explodes into two equal-mass fragments. One of the fragments continues to move horizontally immediately after the explosion, while the other falls vertically downward. Find the total horizontal distance traveled by the horizontally moving fragment.

**Solution**

The situation is depicted in Fig. 8-17. During the short explosion time, momentum is conserved. Just before the collision, the projectile, being at the highest point, is moving horizontally. Immediately after the collision, the first fragment continues to move horizontally, so the second fragment gains no vertical velocity either. The second fragment falls vertically, so it has lost all its horizontal momentum, and all the original momentum appears in the first fragment, which has half the original mass and thus double the velocity. To find how far this fragment travels horizontally, we first calculate the horizontal distance to the highest point. $v_0 = 600$ ft/s, so

$$v_{0x} = (600\ \text{ft/s})\cos 30° = 520\ \text{ft/s} \qquad v_{0y} = (600\ \text{ft/s})\sin 30° = 300\ \text{ft/s}$$

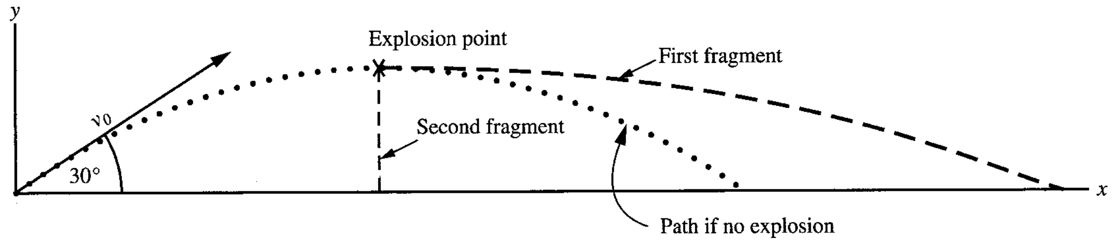


**Fig. 8-17**

The time to reach the highest point is given by $v_y = 0 = v_{0y} - gt \Rightarrow t = (300\ \text{ft/s})/(32\ \text{ft/s}^2) = 9.38$ s. The horizontal distance to this point is just $x_1 = v_{0x}t = (520\ \text{ft/s})(9.38\ \text{s}) = 4880$ ft. After the explosion, the first fragment has the doubled velocity of 1040 ft/s. Since the time of fall depends only on the height, it is the same as that of the original projectile had it not exploded. This is just the time it took to rise to that height, namely, 9.38 s. Then, the horizontal distance traveled by the fragment from the highest point until it reaches ground level is

$$x_2 = (1040\ \text{ft/s})(9.38\ \text{s}) = 9760\ \text{ft}$$

The total distance traveled by the first fragment is then

$$x_T = x_1 + x_2 = 4880\ \text{ft} + 9760\ \text{ft} = 14{,}640\ \text{ft}$$

**Problem 8.36.** Find the center of mass of the object shown in Fig. 8-18, which is made up of a uniform rod glued symmetrically to a rectangular block. Assume $L_R = 3.0$ ft, $L_B = 0.60$ ft, $M_R = 1.5$ slugs, and $M_B = 6.0$ slugs.

**Solution**

By symmetry (as explained in Problem 8.29) the CM lies somewhere along the symmetry axis (dotted line). Assume this is the $x$ axis and measure $x$ from the left end of the rod. To find the $x$ coordinate of the CM, we note that the CM of the rod is located at its midpoint, at $X_A = 1.5$ ft, and the CM of the block is located at its midpoint, at $X_B = 3.0$ ft + 0.30 ft = 3.3 ft. We now show that the CM of the composite object is given by

$$X_{\text{CM}} = \frac{M_A X_A + M_B X_B}{M_A + M_B} \qquad (i)$$

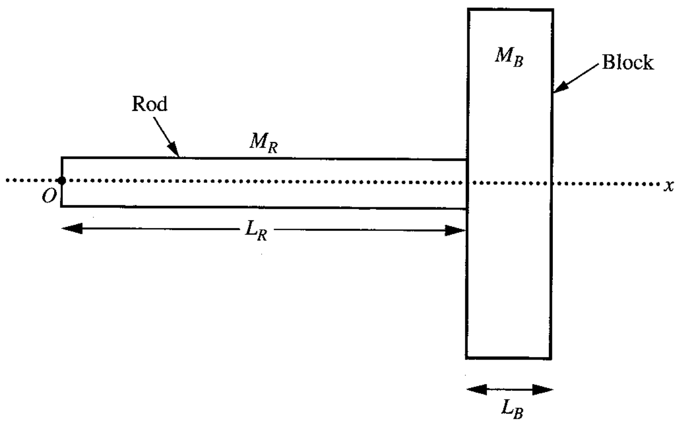


**Fig. 8-18**

In other words, the CM of a composite of different objects is the same as if each object were treated as a particle having the mass of the object, and located at its center of mass. To show this, we apply Eq. (*8.19*) to our two-object composite

$$\left(\sum m_{Ai} + \sum m_{Bi}\right)X_{CM} = \sum m_{Ai}x_{Ai} + \sum m_{Bi}x_{Bi} \qquad (ii)$$

where the sums go over all the respective particles in $A$ and $B$. We know, however, that

$$\sum m_{Ai}x_{Ai} = M_A X_A \quad \text{and} \quad \sum m_{Ai} = M_A \qquad \sum m_{Bi}x_{Bi} = M_B X_B \quad \text{and} \quad \sum m_{Bi} = M_B \qquad (iii)$$

Then substituting from (*iii*) into (*ii*) gives us (*i*). We can now solve for $X_{CM}$ by substituting into (*i*):

$$X_{CM} = \frac{(1.5 \text{ slugs})\,(1.5 \text{ ft}) + (6.0 \text{ slugs})\,(3.3 \text{ ft})}{7.5 \text{ slugs}} = 2.94 \text{ ft}$$

## Supplementary Problems

**Problem 8.37.** A block of mass $m = 3.5$ kg, initially moving at a speed of $v_i = 12$ m/s to the right on a horizontal frictionless surface, is acted on by a variable horizontal force $F$ as follows: For the first 10 s, $F = 10$ N to the right; for the next 5 s, $F = 25$ N to the left; and for the last 8 s, $F = 15$ N to right. Find the velocity of the block at the end of the 23-s interval.

*Ans.* 39 m/s

**Problem 8.38.** A baseball of mass 0.2 kg is pitched at 35 m/s and popped straight up by the batter. The ball rises to a maximum height of 120 m. Find (*a*) the speed with which the ball leaves the bat, and (*b*) the magnitude and direction of the impulse of the bat on the ball.

*Ans.* (*a*) 48.5 m/s; (*b*) 12.0 N · s, 54° above horizontal toward the pitcher

**Problem 8.39.** In Problem 8.38, assume the average force exerted on the baseball by the bat is one-fourth of the peak force. If the peak force is $F = 12{,}000$ N, find the time of contact of the ball with the bat.

*Ans.* 0.0040 s

**Problem 8.40.** Suppose that in Fig. 8-7 the initial velocities are $v_{ai} = -20$ m/s and $v_{bi} = -30$ m/s and the surface is frictionless. If the collision is elastic, find the final velocities of the two blocks.

*Ans.* $v_{af} = -26.7$ m/s and $v_{bf} = -16.7$ m/s

**Problem 8.41.** Assume the same initial velocities as in Problem 8.40, and that the mass of block $B$ is unchanged. For what mass of block $A$ would the final velocity of block $B$ be $-25$ m/s?

*Ans.* 1.67 kg

**Problem 8.42.** Assume that in Problem 8.40 the collision is inelastic and the coefficient of restitution is 0.5. Find (*a*) the final velocities, (*b*) the loss in kinetic energy.

*Ans.* (*a*) $v_{af} = -25$ m/s, $v_{bf} = -20$ m/s; (*b*) 125 J

**Problem 8.43.** Assume that in Fig. 8-7 the surface is frictionless and the velocities are as shown, but the masses are $m_A = 15$ kg; $m_B = 10$ kg. If the collision is totally inelastic, find the final velocity and the loss in kinetic energy.

*Ans.* (*a*) 22 m/s; (*b*) 1200 J

**Problem 8.44.** In Fig. 8-9, assume that the masses are $m = 25$ g and $M = 8.2$ kg and that $v_i = 550$ m/s. If the bullet embeds itself in the block, find (*a*) the velocity of the block just after the collision and (*b*) the height through which the block rises.

*Ans.* (*a*) 1.67 m/s; (*b*) 14.2 cm

**Problem 8.45.** Suppose that in Problem 8.44 the bullet passes through the block and continues in a straight line. If the block rises through half the distance as that in Problem 8.44, what is the final velocity of the bullet?

*Ans.* 161 m/s

**Problem 8.46.** In Fig. 8-10 assume the two pucks are on a frictionless horizontal surface and that $M_A = 50$ g and $M_B = 100$ g. Assume that $\theta = 30°$ and $\phi = 20°$ after the collision. If $v_{af} = 30$ cm/s: (*a*) find $v_{Bf}$, (*b*) find $v_{Ai}$, (*c*) was the collision elastic?

*Ans.* (*a*) 21.9 cm/s; (*b*) 67.1 cm/s; (*c*) no

**Problem 8.47.** Assume that Fig. 8-11 represents a bird's-eye view of a collision between two automobiles at an intersection. Assume $M_A = 2000$ kg, $M_B = 1500$ kg, $v_{Ai} = 20$ m/s, $v_{Bi} = 30$ m/s. The two cars stick together after the collision.

(*a*) Find the magnitude and direction of the velocity immediately after the collision, ignoring the effects of pavement friction over the short collision time.

(*b*) If the coefficient of kinetic friction between the locked wreckage and the pavement is 0.85, how far will it skid?

*Ans.* (*a*) 17.2 m/s, 48.4° with $x$ axis; (*b*) 17.8 m

**Problem 8.48.** A ball of mass 40 g is dropped on a concrete floor from a height of 2.5 m. It is observed to rebound to a height of 1.5 m.

(*a*) Evaluate the coefficient of restitution

(*b*) How much kinetic energy is lost in the brief collision of the ball with the floor?

*Ans.* (*a*) 0.775; (*b*) 0.392 J

**Problem 8.49.** A 50-g cart is sliding without friction on a horizontal air track at a speed of 30 cm/s. A steel ball of mass 15 g is dropped vertically so that it lands in the cart. What is the new speed of the cart after the steel ball comes to rest in it?

*Ans.* 23.1 cm/s

**Problem 8.50.** In Fig. 8-12 the spring is initially compressed 8.0 in and the blocks start at rest on a frictionless surface. The masses are $m_a = 2.5$ slugs and $m_b = 3.5$ slugs, and after the system is released, $v_{bf} = 20$ ft/s. Find $v_{af}$ and the force constant of the spring.

*Ans.* −28 ft/s, 7560 lb/ft

**Problem 8.51.** A boy, of mass 40 kg, is stranded on a frozen pond; the ice is so smooth as to be absolutely frictionless. He is trying to reach the closest bank, 50 m away, but keeps slipping in place. Suddenly he gets an idea: he takes off a boot, of mass 0.50 kg, and hurls it directly away from the bank at a speed of 12 m/s relative to the ice. how long will it take him to reach the bank?

*Ans.* 333 s

**Problem 8.52.** A uranium 238 nucleus is unstable and decays into thorium 234 by emitting an $\alpha$ particle (helium nucleus). The relative masses of the thorium and $\alpha$ particle are 234 and 4, respectively. The thorium nucleus is observed to recoil at a speed of $2.39 \times 10^4$ m/s when the $\alpha$ particle is emitted from a uranium nucleus at rest. What is the speed of the $\alpha$ particle?

*Ans.* $1.4 \times 10^6$ m/s

**Problem 8.53.** A uniform iron rod of length 1 m is bent at its midpoint to make a 90° angle. Find the location of the CM of the bent rod.

*Ans.* 17.7 cm from the midpoint along the symmetry axis (the line at 45° to each half of the rod)

**Problem 8.54.** Two identical rods are rigidly connected to a disk at right angles to each other, as shown in Fig. 8-19. Find the center of mass of the system.

*Ans.* 10.2 cm from the center of the disk and midway between the rods (i.e., along a 45° line to the rods)

**Problem 8.55.** A boy is in a cart which rests on a sheet of frictionless ice. The boy and cart are initially at rest. The boy runs from one end of the cart to the other, hurling himself against that end to get the cart moving. Will he succeed and if not why?

*Ans.* No. The CM of the boy-cart combination is initially at rest, and there is no net external force on the system, so the CM remains at rest. Therefore, when the boy runs across the cart, the cart moves in the opposite direction at a speed that keeps the CM fixed. When he hits the other side, both he and the cart once more come to rest.

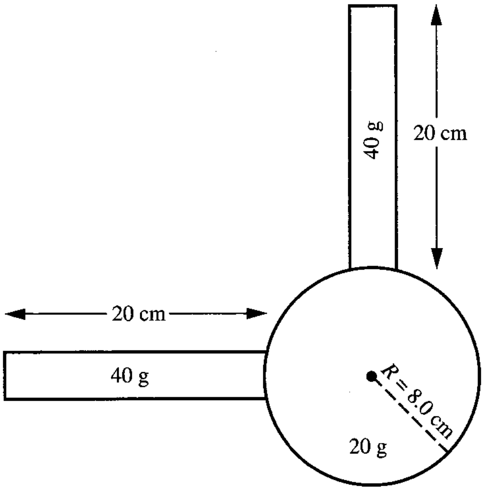


**Fig. 8-19**

**Problem 8.56.** The earth and the moon are separated by a distance (center to center) of $3.8 \times 10^5$ km. The CM of the system is known to be within the earth, at a distance of $4.62 \times 10^3$ km from its center. Find the ratio of the earth's mass to that of the moon.

*Ans.* 81.3