Chapter 7

Energy, Power, and Simple Machines

7.1 GENERALIZATION OF CONSERVATION OF ENERGY

Thus far we have explored a number of conservative forces and the associated potential energies that can be used to represent the work they do. We have also encountered the force of friction and can ask if it too is conservative. Since friction always opposes the motion of an object, the work done by friction is always negative. If we move a block about on a tabletop where there is friction, as shown in Fig. 7-1, the work done by friction in going from a to b along path C_1 is negative, and so is the work done in going from b back to a along path C_2. Thus the total work done by friction in going around the closed loop (i.e., returning to its starting point) is not zero, and the force is not conservative.

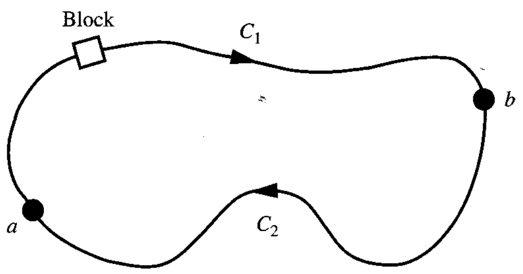

Fig. 7-1

Let us recall our earlier discussion of work being the mechanical transfer of energy between systems. A system doing positive work loses energy, and one doing negative work gains energy. Since friction always does negative work, the system that supplies the force of friction should always gain energy. But what sort of energy would this be? The source of friction is the interaction of the molecules in the surface layers of the two objects that are moving past each other and it is the energy of random jiggling of this vast number of molecules that increases. While such jiggling involves mechanical energy at the atomic level (including both kinetic energy of the molecules and potential energy of the forces between them), it is not mechanical energy on the macroscopic scale. Since the motion is random, it does not manifest itself in an organized group motion of all the molecules, as it does, for example, when a block is moving. When the block moves, all the molecules are also in motion, but in that case they are moving in unison. When the motion of the molecules is of a random nature, (describable, in fact, only by statistical means) we call the associated energy **thermal energy**. Such energy manifests itself macroscopically in various ways, most notably as a rise in temperature, and will be discussed in more detail in the section on heat and thermodynamics.

If we include in our considerations thermal energy, as well as other forms of energy such as electromagnetic radiation (light) and more subtle forms of mechanical energy such as sound, *the law of conservation of energy still holds*. Energy can be transformed from one type to another within a given system, and it can be transferred from one system to another system, but the total amount of energy stays the same.

Problem 7.1. Reexamine Problem 6.12(*b*) from the perspective of general energy conservation.

Solution

Consider the system composed of the block and the incline, as well as the gravitational interaction of the block with the earth. Initially, the system has zero kinetic energy, but the total energy includes the potential energy of the block at the top of the incline and whatever thermal energy the block and incline initially have. When the block reaches the bottom it has lost an amount of potential energy

$$mgh = (5.0\ \text{kg})(9.8\ \text{m/s}^2)(9.0\ \text{m}) = 441\ \text{J}$$

This energy must appear in the form of increased kinetic energy of the block and increased thermal energy of the system. The increase in thermal energy is a consequence of the negative work done by friction:

$$\text{Increase in thermal energy} = |W_f| = f_k L = (\mu_k mg\cos\theta)L = 177\ \text{J}$$

(as previously calculated in Problem 6.6(*b*)). The remainder of the potential energy lost by the block must appear as kinetic energy:

$$E_k = \tfrac{1}{2}mv_2{}^2 = 441\ \text{J} - 177\ \text{J} = 264\ \text{J} \qquad \text{whence} \qquad v_2 = 10.3\ \text{m/s}$$

(as before).

Problem 7.2. A block of mass $m = 6.5$ kg is released from rest at the top of a frictionless incline of height 3.0 m, as shown in Fig. 7-2. Upon reaching the bottom, the block slides a distance $L = 12$ m along a horizontal surface that has friction, until coming to rest.

(*a*) Using energy considerations, find the thermal energy gained by the system.

(*b*) Find the coefficient of friction μ_k between the block and the horizontal surface.

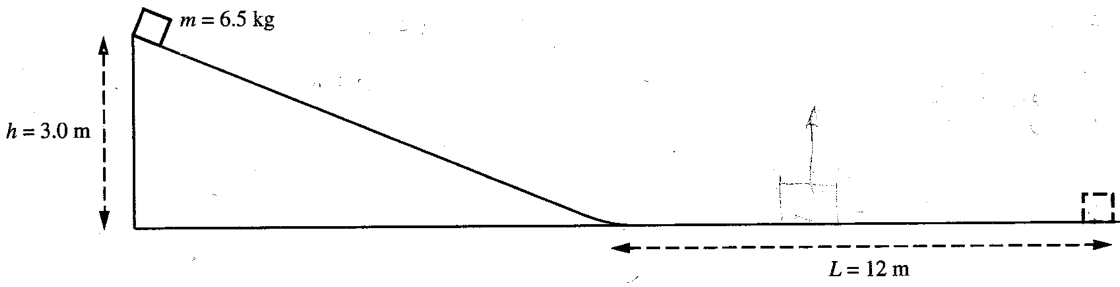

Fig. 7-2

Solution

(*a*) The kinetic energy of the block is zero at the beginning and at the end of the motion. Overall, the potential energy of the block decreases by an amount

$$mgh = (6.5\ \text{kg})(9.8\ \text{m/s}^2)(3.0\ \text{m}) = 191\ \text{J}$$

This energy thus reappears as 191 J of thermal energy gained along the horizontal section.

(*b*) Equating the gain in thermal energy to minus the work done by friction, we get

$$191\ \text{J} = -(-\mu_k mgL) = \mu_k(6.5\ \text{kg})(9.8\ \text{m/s}^2)(12\ \text{m}) = \mu_k(764\ \text{J}) \qquad \text{or} \qquad \mu_k = 0.250$$

Problem 7.3. A block of mass $m = 9.0$ kg is dropped onto a vertical light spring of force constant $k = 300$ N/m, as shown in Fig. 7-3(*a*). The block compresses the spring 1.7 m, as illustrated in Fig. 7-3(*b*). Find the amount of mechanical energy converted to thermal and sound energy due to the collision of the block with the spring.

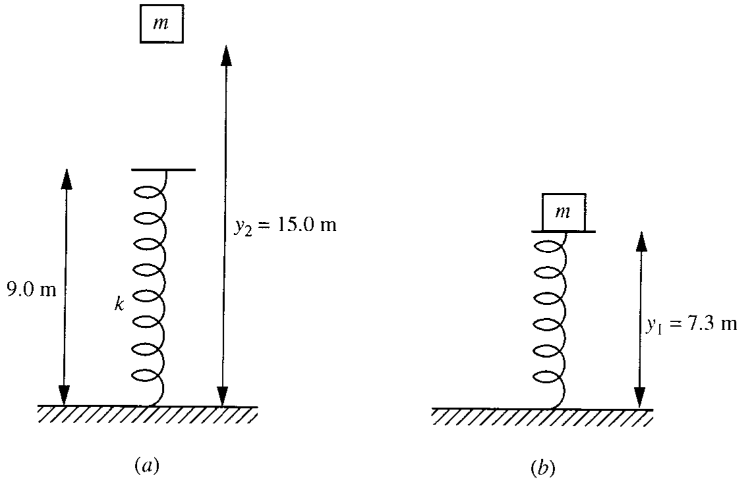

Fig. 7-3

Solution

As in Problem 7.2, the initial and final kinetic energies of the block are zero. The amount of gravitational potential energy lost is

$$mg(y_2 - y_1) = (9.0\ \text{kg})(9.8\ \text{m/s}^2)(15.0\ \text{m} - 7.3\ \text{m}) = 679\ \text{J}$$

Some of this lost energy appears as potential energy of the spring:

$$\tfrac{1}{2}kx^2 = \tfrac{1}{2}(300\ \text{N/m})(1.7\ \text{m})^2 = 434\ \text{J}$$

The remainder must represent energy appearing in some other form. The only possibility is that the collision between the block and spring caused an increased jiggling of their molecules (thermal energy), as well as the molecules of the surrounding air (sound energy). The total amount of such energy generated is 679 J − 434 J = 245 J.

Problem 7.4. A ball of mass 1.5 kg is dropped from a height $h_1 = 3.0$ m above the floor, and the ball bounces straight back up. If 12.0 J of thermal energy is generated in the collision, to what height does the ball rebound?

Solution

The ball starts with no kinetic energy and with potential energy: $mgh_1 = (1.5\ \text{kg})(9.8\ \text{m/s}^2)(3.0\ \text{m}) = 44.1$ J. When the ball rebounds to its maximum height h_2, it again has no kinetic energy, and its potential energy is mgh_2. Since 12.0 J disappears in the collision, we have $mgh_2 + 12.0\ \text{J} = 44.1$ J, or

$$(1.5\ \text{kg})(9.8\ \text{m/s}^2)h_2 = 32.1\ \text{J} \qquad \text{or} \qquad h_2 = 2.18\ \text{m}$$

7.2 POWER

Average Power

We now turn to the question of the *rate* at which work is done, that is, how much work is done per second by a force. The average power P_{av} delivered by a force in a time Δt is defined as the work ΔW done by the force in the time Δt, divided by Δt: $P_{av} = \Delta W/\Delta t$. The SI unit of power is the watt (W), where 1 W = 1 joule/second = 1 J/s. A related unit is the kilowatt, equal to 1000 W. Other units are ergs/s and ft · lb/s. In the English system of units a special unit of power called the horsepower (hp) is defined: 1 hp = 550 ft · lb/s. A calculation shows that 1 hp $\approx$ 0.75 kW.

Problem 7.5. A horizontal force $F = 10$ N pulls a block of mass $m = 2.0$ kg along a frictionless horizontal surface. If the block starts from rest, find the average power delivered by the force (*a*) in the first 5.0 s; (*b*) between $t = 3.0$ s and $t = 5.0$ s. (*c*) How would the answers to (*a*) and (*b*) change if the force made an angle of 30° with the horizontal?

Solution

(*a*) To find the work done in the first 5 s we need the displacement. Since the force is constant, the acceleration is also constant, and we have $x = x_0 + v_0t + \frac{1}{2}at^2$; for our case $x_0 = v_0 = 0$ and $a = F/m = (10\text{ N})/(2.0\text{ kg}) = 5.0\text{ m/s}^2$. Thus $x = (2.5\text{ m/s}^2)t^2$. Substituting $t = 5.0$ s we get $x = 62.5$ m, and $\Delta W = Fx = (10\text{ N})(62.5\text{ m}) = 625$ J. Finally,

$$P_{av} = \frac{\Delta W}{\Delta t} = \frac{625\text{ J}}{5.0\text{ s}} = 125\text{ W}$$

(*b*) Here $P_{av} = \Delta W/\Delta t = F\,\Delta x/\Delta t$, where $\Delta t = 5.0\text{ s} - 3.0\text{ s} = 2.0$ s, and $\Delta x = x_5 - x_3$. We have from (*a*) that $x_5 = 62.5$ m, and we find that $x_3 = (2.5\text{ m/s}^2)(3.0\text{ s})^2 = 22.5$ m. Thus $\Delta x = 40.0$ m. Finally, we get

$$P_{av} = \frac{(10\text{ N})(40.0\text{ m})}{2.0\text{ s}} = 200\text{ W}$$

(*c*) In this case the work is due only to the horizontal component of the force **F**: $F_x = F\cos 30° = 0.866F$. The x displacements in a given time interval are all proportional to the acceleration (since $x = \frac{1}{2}at^2$ for our initial conditions), and the acceleration is now due to the force $F_x = 0.866F$, and is therefore reduced by the factor 0.866 as well. Calling the new acceleration $\mathbf{a}'$ we have $\mathbf{a}' = 0.866\mathbf{a}$. Thus the force doing the work is reduced by a multiplicative factor of 0.866, and the displacements are also down by a multiplicative factor of 0.866. Thus the work in any time interval is down by $(0.866)^2 = 0.75$, and the new average powers will just be 0.75 times the original ones.

Instantaneous Power

In the case of one-dimensional motion with the force along the direction of motion we saw in Problem 7.5 that

$$P_{av} = F\frac{\Delta x}{\Delta t} = F\frac{x_2 - x_1}{t_2 - t_1} \tag{7.1}$$

We define the instantaneous power P at the time t_1 as the limit as $t_2 \to t_1$ (or the limit as $\Delta t \to 0$) of P_{av}. But in this limit, $\Delta x/\Delta t \to v$, the instantaneous velocity. Then Eq. (*7.1*) becomes

$$P = Fv \tag{7.2}$$

This can easily be generalized to the case of an object moving on an arbitrary path and acted on by a force that makes an angle θ with the tangent to the path. The situation is depicted in Fig. 7-4. If Δs is a small incremental arc length moved through in the time interval Δt, then $\Delta W = F \cos\theta\, \Delta s$ and $P_{av} = \Delta W/\Delta t$, or

$$P_{av} = F \cos\theta \frac{\Delta s}{\Delta t} \tag{7.3}$$

In the limit as $\Delta t \to 0$, we have that $\Delta s/\Delta t = v$, the magnitude of the instantaneous velocity. Thus, in general, the instantaneous power is given by

$$P = F \cos\theta\, v \tag{7.4}$$

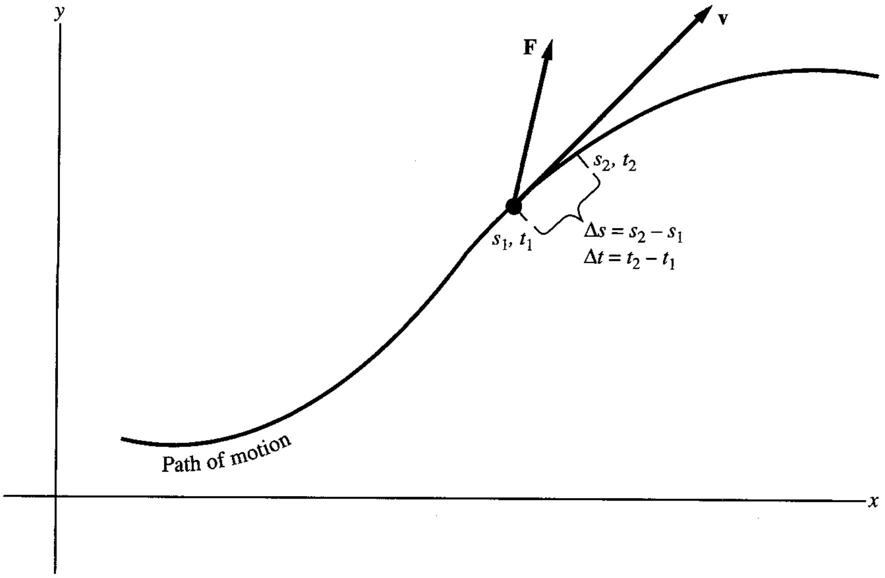

Fig. 7-4

Problem 7.6.

(*a*) Referring back to Problem 7.5, find the instantaneous power generated by the force F at $t = 2.0$, 3.0, 4.0, and 5.0 s.

(*b*) Repeat for the case of Problem 7.5, part (*c*)

Solution

(*a*) For this one-dimensional case we have Eq. (*7.2*): $P = Fv$. We also have that $v = v_0 + at$. Since $v_0 = 0$, $v = at = (5.0 \text{ m/s}^2)t$, from which we get $v_2 = 10.0$ m/s; $v_3 = 15.0$ m/s; $v_4 = 20.0$ m/s; $v_5 = 25.0$ m/s. Recalling that $F = 10$ N, we have $P_2 = 100$ W; $P_3 = 150$ W; $P_4 = 200$ W; $P_5 = 250$ W.

(*b*) In this case the power is $P' = F \cos\theta\, v' = 0.866Fv'$, where v' is the velocity for the new situation. Since the new acceleration is due to F_x, we have $a' = 0.866a \Rightarrow v' = a't = 0.866at = 0.866v$. Thus $P' = (0.866)^2 Fv = 0.75Fv = 0.75P$. Thus all values of power are reduced to 0.75 of their original values, just as was concluded for the average powers of Problem 7.5(*c*).

Problem 7.7. A horse pulls a cart along a level road at 25 ft/s. If the net horizontal force exerted by the horse on the cart is 40 lb, how much horsepower is the horse delivering to the cart?

Solution

$P = Fv = (40 \text{ lb})(25 \text{ ft/s}) = 1000 \text{ ft}\cdot\text{lb/s}$. To convert to horsepower we divide by $(550 \text{ ft}\cdot\text{lb/s})/\text{hp}$ to get $P = 1.82$ hp.

Problem 7.8. A truck travels along a level roadway at 60 mph (88 ft/s). At that speed, air resistance and internal frictional forces are such that an effective forward force of 3000 lb is necessary to keep the truck from slowing down. What power must be delivered by the truck's engine to keep the truck going at constant speed?

Solution

$$P = \frac{(3000 \text{ lb})(88 \text{ ft/s})}{550 \text{ ft}\cdot\text{lb}/(\text{s}\cdot\text{hp})} = 480 \text{ hp.}$$

Note. This is really a much more complex situation than appears at first glance. The actual forward force is supplied by the static frictional force of the ground on the tires. Since the point of contact of the tires with the ground has no horizontal motion if there is no skidding, this force does no work! This is made more obvious by the fact that the ground is just a passive system and not one supplying energy to the truck. The actual work is done by the engine through the action of the drive shaft. This is an example of a system where "internal work" is done by one part of a complicated system on another. An even simpler example of this is the case of a child on ice skates pushing himself off from the ice rink wall. Clearly the force due to the wall accelerates the child, building up kinetic energy, but the wall force does no work since the hands it pushes are at rest against it. The energy is delivered by the internal work of the muscles of the child.

Problem 7.9. In Problem 7.8, assume the truck engine has a maximum power capacity of 600 hp, and the truck weighs 8000 lb. What is the steepest incline that the truck can drive up and maintain its speed of 60 mph?

Solution

It takes 480 hp to just keep the truck moving at the constant speed on level ground. This leaves 120 hp to increase the potential energy E_p as the truck moves up the incline. Thus 120 hp = 66,000 ft·lb/s = $\Delta E_p/\Delta t$, the increase of potential energy per second. The situation is depicted in Fig. 7-5. $\Delta E_p/\Delta t = mg\,\Delta h/\Delta t$, where Δh is the increase in height in an infinitesimal time Δt. But $\Delta h/\Delta t$ is just the vertical component of the truck's velocity v_y, so $\Delta E_p/\Delta t = mgv_y$. If θ is the angle of the incline, then $v_y = v \sin\theta$, and $\Delta E_p/\Delta t = mgv \sin\theta = (8000 \text{ lb})(88 \text{ ft/s}) \sin\theta = 66{,}000 \text{ ft}\cdot\text{lb/s}$ (from before). Solving for $\sin\theta$ we get

$$\sin\theta = 0.0938 \qquad \text{or} \qquad \theta = 5.38^\circ$$

Problem 7.10. An electric light bulb rated at 100 W is left on for 10 days and nights.

(*a*) How much energy was expended?

(*b*) For how long must an electric heater rated at 3500 W be left on to expend the same amount of energy?

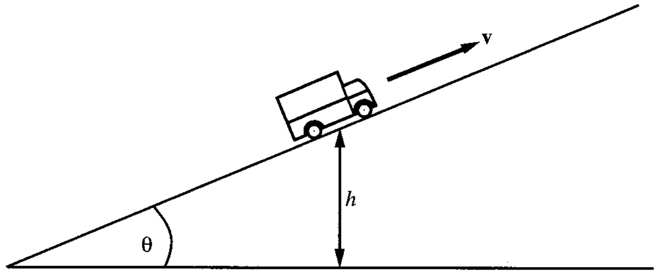

Fig. 7-5

Solution

(*a*) For the case of constant power, from the definition we have $E = Pt$, where E is the total energy expended in joules, P is the constant power generated in watts, and t is the elapsed time in seconds. A period of 10 days and nights corresponds to 240 h. Noting that there are 3600 s/h we get $t = (240\ \text{h})(3600\ \text{s/h}) = 8.64\times10^5$ s. Then

$$E = (100\ \text{W})(8.64 \times 10^5\ \text{s}) = 8.64 \times 10^7\ \text{J} = 86.4\ \text{MJ}$$

(*b*) Now

$$P't' = (3500\ \text{W})t' = 8.64 \times 10^7\ \text{J} \quad \text{or} \quad t' = 24{,}700\ \text{s} = 6.86\ \text{h}$$

Problem 7.11. For purposes of charging customers for energy usage, power companies use a special unit of energy, the kilowatthour (kWh) defined as the energy expended by a 1-kW power source operating for 1 hour.

(*a*) How many joules are there in 1 kWh?

(*b*) How many kilowatthours of energy are expended by the light bulb of Problem 7.10(*a*).

Solution

(*a*) 1 kWh = (1000 J/s)(3600 s) = 3,600,000 J.

(*b*) $E = \dfrac{8.64 \times 10^7\ \text{J}}{3.60 \times 10^6\ \text{J/kWh}} = 24.0\ \text{kWh}.$

7.3 SIMPLE MACHINES

A **simple machine** is any device that allows a small force to move an object against a larger resisting force, or a force in one direction to move an object against a resisting force in another direction. Many simple machines do both. Examples shown in Fig. 7-6 are (*a*) the **lever**, (*b*) the **inclined plane**, and (*c*) a **pulley system**. In all three cases we assume the applied force F_A is the minimum force needed to move the weight, (or "load"), w (that is, to move it with essentially zero acceleration).

Mechanical Advantage

The ratio of the load to the applied force is called the **mechanical advantage** (MA) of the machine. Clearly, the bigger the mechanical advantage the smaller the applied force necessary to accomplish the task. In any simple machine, the applied force F_A necessary when no frictional forces must be overcome will be smaller than F'_A, the applied force necessary when there are frictional forces.

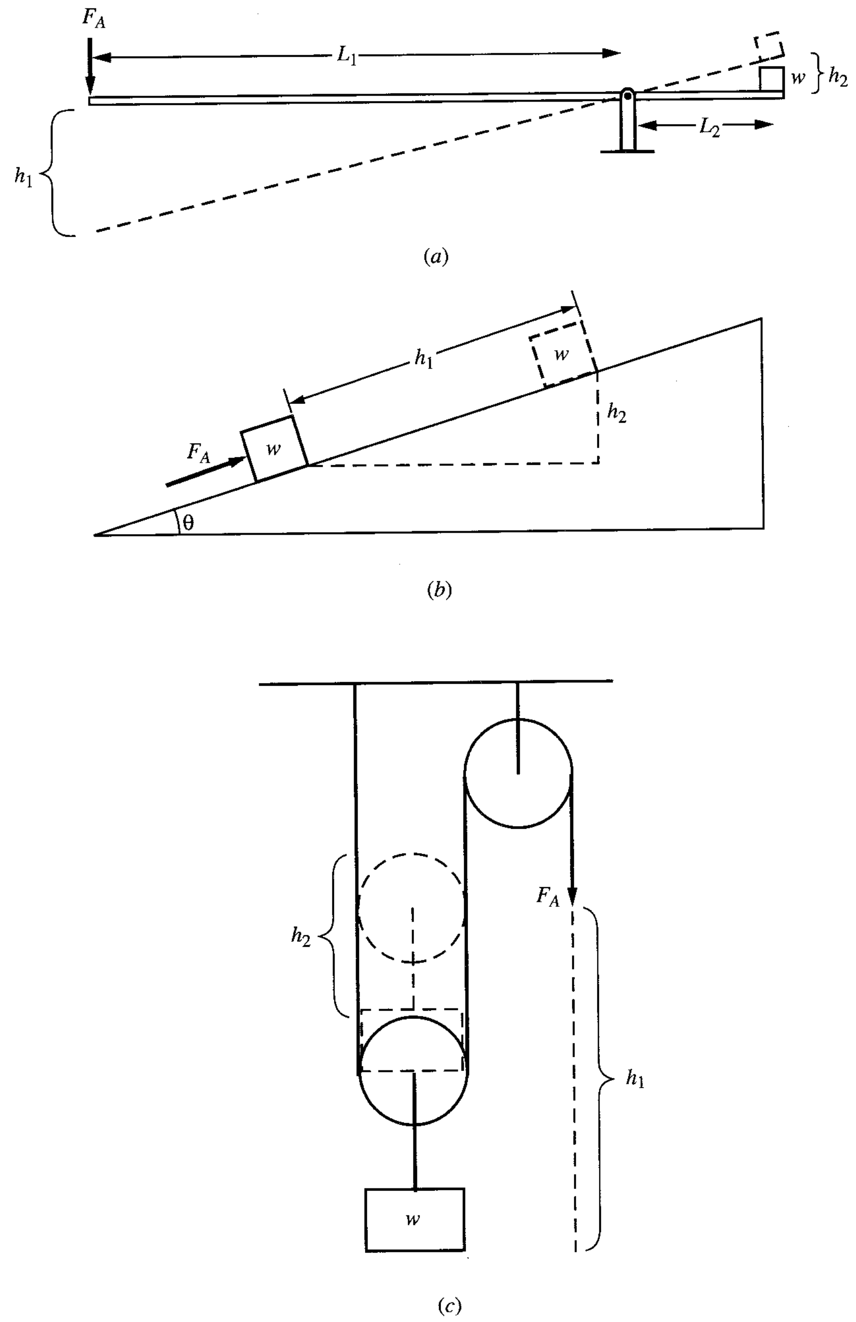

Fig. 7-6

When there is no friction we have the best or the "ideal" MA: (IMA)

$$\text{IMA} = \frac{w}{F_A} \tag{7.5a}$$

The actual MA is given by

$$\text{MA} = \frac{w}{F'_A} \tag{7.5b}$$

and is always smaller than the IMA because there are always frictional losses.

To find the IMA and the MA for a simple machine we can use our basic ideas about work-energy, and general energy conservation. If we assume no friction for the examples in Fig. 7-6, then the work input (work done by F_A in moving the weight) must in each case be equal to the work output (for our frictionless cases, the increase in gravitational energy of the load). This is equivalent to saying that the system supplying force F_A is giving up energy, while the system exerting the load force (gravity in our cases) is gaining a like amount of energy. Thus:

$$\text{Work input} = \text{work output} \tag{7.6}$$

Since in this ideal case all the work output goes into accomplishing the goal of moving the weight, it is also called the *useful work output*. If h_1 and h_2 represent the distances moved through by the applied force and the resisting force, respectively (Fig. 7-6), then Eq. (*7.6*) becomes

$$F_A h_1 = w h_2 \tag{7.7}$$

Then, from Eqs. (*7.5a*) and (*7.7*)

$$\text{IMA} = \frac{w}{F_A} = \frac{h_1}{h_2} \tag{7.8a}$$

In other words

$$\text{IMA} = \frac{\text{input distance}}{\text{output distance}} \tag{7.8b}$$

If, on the other hand, there is friction, the work input (done by $F_A{}'$) must be larger than the useful work output (increase in gravitational energy of the load) by an amount equal to the thermal energy generated by the frictional forces:

$$\text{Work input} = \text{work output} = \text{useful work output} + \text{thermal energy increase} \tag{7.9}$$

Efficiency

The efficiency e of a simple machine is defined as the ratio

$$e = \frac{\text{useful work output}}{\text{work input}} \tag{7.10a}$$

For our examples this is just

$$e = \frac{wh_2}{F'_A h_1} \tag{7.10b}$$

Using Eq. (*7.7*) this becomes

$$e = \frac{F_A h_1}{F'_A h_1} = \frac{F_A}{F'_A} \tag{7.10c}$$

Thus

$$e = \frac{\text{ideal applied force}}{\text{actual applied force}} \tag{7.10d}$$

From Eq. (*7.10c*) we can determine yet another expression for e, in terms of MA and IMA:

$$e = \frac{F_A}{F'_A} = \frac{w/F'_A}{w/F_A} = \frac{\text{MA}}{\text{IMA}} \tag{7.10e}$$

Equations (*7.8b*), (*7.9*), (*7.10a*), (*7.10d*), and (*7.10e*) are general expressions that are true for any simple machines.

Problem 7.12. Assume that for the lever in Fig. 7-6(*a*), $L_1 = 6.0$ m and $L_2 = 1.5$ m, and that $w = 100$ N.

(*a*) If there is no friction in the pivot, find the force F_A necessary to just lift the weight w, and find the IMA of the machine.

(*b*) If the actual force needed is $F_A' = 28$ N, find the efficiency of the machine.

(*c*) For part (*b*), how much thermal energy is generated at the pivot if the weight is lifted 10 cm?

Solution

(*a*) If the weight is lifted through a small distance h_2, the force F_A will act through a corresponding distance h_1. Then from work-energy: $F_A h_1 = w h_2$ or $F_A = w(h_2/h_1)$. But by similar triangles we must have $h_2/h_1 = L_2/L_1 = 0.25$, so $F_A = (100\text{ N})(0.25) = 25$ N. Also, IMA $= w/F_A = h_1/h_2 = L_1/L_2 = 4.0$. (This same result could be obtained by balancing torques about the pivot. Torques will be discussed in Chap. 9.)

(*b*) MA $= w/F_A' = 100/28 = 3.57$; e = MA/IMA $= 3.57/4.0 = 0.893 = 89.3\%$.

(*c*) $h_2 = 0.10\text{ m} \Rightarrow h_1 = 0.40\text{ m} \Rightarrow$ work input $= F_A' h_1 = (28\text{ N})(0.40\text{ m}) = 11.2$ J. Useful work output $= wh_2 = F_A h_1 = (25\text{ N})(0.40\text{ m}) = 10.0$ J. Therefore the thermal energy $= 11.2\text{ J} - 10.0\text{ J} = 1.2$ J. [Or, since we have 89.3% efficiency, 10.7% of the work input is wasted as thermal energy: $0.107(11.2\text{ J}) = 1.2$ J, as before.]

Problem 7.13. For the inclined plane in Fig. 7-6(*b*), assume $\theta = 30°$ and $w = 100$ N.

(*a*) Find an expression for the IMA of this simple machine.

(*b*) If the coefficient of friction between block and incline is $\mu_k = 0.25$, find the true MA and the efficiency of the machine.

(*c*) Using the results of part (*b*), find the thermal heat loss when $h_2 = 35$ cm.

Solution

(*a*) IMA $= w/F_A$. If there is no friction, then, balancing force components along the incline: $F_A = w \sin\theta \Rightarrow$ IMA $= 1.0/\sin\theta = 1.0/\sin 30° = 2.0$. [Or, from Eqs. (*7.8*) and Fig. 7-6(*b*), IMA $= h_1/h_2 = 1.0/\sin\theta$.]

(*b*) MA $= w/F_A'$. Again balancing components of force along the incline, but now including friction, we get $F_A' - w\sin\theta - f_k = 0$ or $F_A' - w\sin\theta - \mu_k W\cos\theta = 0 \Rightarrow F_A' = w(\sin\theta + \mu_k\cos\theta)$. Then

$$\text{MA} = \frac{1.0}{\sin\theta + \mu_k\cos\theta} = \frac{1/0}{0.50 + 0.25\times 0.866} = \frac{1.0}{0.717} = 1.40$$

$$e = \frac{\text{MA}}{\text{IMA}} = \frac{1.40}{2.0} = 0.70$$

(Or, equivalently, $e = F_A/F_A' = 50/71.7 = 0.70$.)

(*c*) Work input = (useful work output)/e $= wh_2/e = (100\text{ N})(0.35\text{ m})/(0.70) = 35\text{ J}/0.70 = 50$ J. Then the thermal loss $= 50\text{ J} - 35\text{ J} = 15$ J. (Or, thermal loss = 30% of work input, while useful work output = 70% of work input $\Rightarrow$ thermal loss $= \frac{3}{7}$ (useful work output) $= \frac{3}{7}(35\text{ J}) = 15$ J.)

Problem 7.14. Consider the pulley system shown in Fig. 7-6(*c*), with weight $w = 100$ N.

(*a*) What is the value of the applied force and the IMA of the machine if both pulleys are massless and frictionless and the cord is light?

(*b*) If the movable pulley from which the weight w is suspended is now assumed to weigh $w_p = 5$ N, but all friction can still be ignored, what is the new value of the applied force, and the MA.

(*c*) What is the efficiency of the machine under the conditions of part (*b*)?

Solution

(*a*) As can be seen from Fig. 7-6(*c*), when the weight moves up a distance h_2, twice that length of cord has been pulled down by the applied force. Thus, $h_1 = 2h_2$. Then, using Eq (*7.8*) we have

$$\text{IMA} = \frac{w}{F_A} = \frac{h_1}{h_2} = 2.0 \qquad \text{or} \qquad F_A = 0.5w = (0.5)(100\text{ N}) = 50\text{ N}$$

(*b*) In this case, the applied force is $F_A' = 0.5(100\text{ N} + 5.0\text{ N}) = 52.5$ N. Even though there is no frictional loss, the useful work output relates only to the lifting of the weight w, so

$$\text{MA} = \frac{w}{F_A'} = \frac{100}{52.5} = 1.90$$

(*c*) The efficiency of the machine e is the ratio of useful work output to work input:

$$e = \frac{F_A h_1}{F_A' h_1} = \frac{F_A}{F_A'} = \frac{\text{MA}}{\text{IMA}} = \frac{1.90}{2.0} = 0.95$$

Note. There are many additional simple machines, such as the wedge, the screw, the jackscrew and more complicated pulley systems. In all cases, the basic work-energy approach discussed above can be used to analyze the IMA, the MA, and the efficiency e of the machine.

Problems for Review and Mind Stretching

Problem 7.15. Reconsider Problem 6.31 if there is friction and the frictional forces do a total of $W_f = -15$ J of work. To what maximum vertical height would the block rise on the incline?

Solution

We approach the problem from an energy balance point of view. As in the Problem 6.31 the kinetic energy is zero at the beginning and at the end. Now, however, the energy given up by the spring goes partly into building up the thermal energy of the surfaces so that less is available to build up gravitational potential energy. Thus $\frac{1}{2}(1000\text{ N/m})(0.20\text{ m})^2 = 15\text{ J} + (98\text{ N})y$ or $20\text{ J} = 15\text{ J} + (98\text{ N})y \Rightarrow y = 5.1$ cm.

Problem 7.16. A 25,000-kg airplane starts from rest on the runway, takes off, and reaches an altitude of 5000 m and a speed of 250 m/s, all in a time of 8.0 min.

(*a*) Assuming no thermal losses, what is the average power generated by the airplane engines during this period?

(*b*) Assuming that the average engine output was 1.3 times the power output of part (*a*), how much thermal energy loss was there during that period?

Solution

(*a*) $P_{av} = \Delta W/\Delta t$. From the work-energy theorem, $\Delta W = \Delta E_k + \Delta E_p$. Since the airplane starts from rest, ΔE_k = final kinetic energy = $\frac{1}{2}$(25,000 kg)(250 m/s)^2 = 7.81×10^8 J. Choosing ground level as the zero of potential energy, ΔE_p = final potential energy = (25,000 kg)(9.8 m/s^2)(5000 m) = 1.23×10^9 J. Then $\Delta W = 0.78\times10^9$ J + 1.23×10^9 J = 2.01×10^9 J. Noting that Δt = (8 min)(60 s/min) = 480 s, we get

$$P_{av} = \frac{2.01 \times 10^9 \text{ J}}{480 \text{ s}} = 4.19 \times 10^6 \text{ W} = 4190 \text{ kW}$$

(*b*) The average power lost to thermal sources is $0.3(4.190\times10^6$ W) = 1.26×10^6 W. The total thermal energy loss in 8 min is thus (1.26×10^6 W)(480 s) = 6.05×10^8 J.

Problem 7.17. A tugboat pulls a barge directly behind it at a speed of 4.0 m/s. The tugboat's engines generate 700 kW of power, of which 60% is needed just to keep the tugboat alone moving at that speed.

(*a*) Find the tension in the rope connecting the tug and barge.

(*b*) If the tugboat was in front but slightly to the side of the barge so that the connecting rope made a 20 ° angle with the direction of motion, what would the tension now be?

Solution

(*a*) 40% of the tugboat's engine power goes to moving the barge. So, calling this amount of power P_{barge}, we get P_{barge} = 0.40(700 kW) = 280 kW. Let T be the tension in the rope.

$$P_{barge} = Tv \qquad \text{or} \qquad T = \frac{P_{barge}}{v} = \frac{280 \text{ kW}}{4.0 \text{ m/s}} = 70.0 \text{ kN}$$

(*b*) Now $T \cos 20° = 70.0$ kN, or $T = 74.5$ kN.

Problem 7.18.

(*a*) In Fig. 7-7 assume all the pulleys are massless and frictionless. Use energy considerations to determine the force F necessary to just lift a weight w = 50 lb.

(*b*) If there were some friction in the pulleys and their mass were not negligible, the force F would have to be larger than in part (*a*). If the force F was in fact 20% larger than in part (*a*), what is the efficiency of this simple machine, and what fraction of the work done by F is converted into potential energy of the weight w?

Solution

(*a*) If the weight and the two massless pulleys to which it is attached move up a distance h_2, the amount of slack let out in the cords is $2h_2$ about *each* moved pulley. Thus, $h_1 = 4.0h_2$ and

$$F = 0.25w = (0.25)(50 \text{ lb}) = 12.5 \text{ lb}$$

(*b*) $F' = 1.2F$, so $e = F/F' = 0.833$. This means that 83.3% of the work input is used to lift the weight and hence is converted to potential energy of the weight.

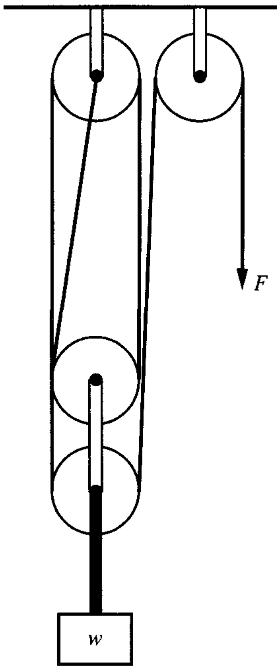

Fig. 7-7

Problem 7.19. A jackscrew that can be used to lift a weight w is shown in Fig. 7-8. The **pitch** p of the screw is defined as the vertical height through which the screw moves when it is turned through 360°. The lever arm L is measured from the axis of the screw. The turning force F_A is applied horizontally, and perpendicular to the lever arm.

(*a*) Find F_A in the ideal case of no friction, and find the IMA.

(*b*) If the jackscrew has an efficiency of 15.0%, find the MA and the actual force F'_A necessary to lift the weight.

Solution

(*a*) Work input = work output $\Rightarrow$ (for one complete turn) $F_A(2\pi L) = W(p)$ or $F_A(6.28)(0.30\text{ m}) = (1200\text{ kg})(9.8\text{ m/s}^2)(0.0035\text{ m}) \Rightarrow F_A = 21.8$ N. IMA $= W/F_A = 2\pi L/p = 539$.

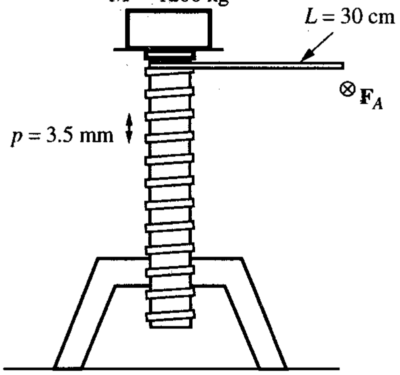

Fig. 7-8

(*b*) $e = F_A/F_A' \Rightarrow F_A' = F_A/e = 21.8\ \text{N}/0.15 = 145\ \text{N}$. Also, $e = \text{MA}/\text{IMA} \Rightarrow \text{MA} = e(\text{IMA}) = 0.15(539) = 81$. [Or, $\text{MA} = W/F_A' = (1200\ \text{kg})(9.8\ \text{m/s}^2)/145\ \text{N} = 81$.]

Supplementary Problems

Problem 7.20. Refer to Problem 6.47 and to Fig. 6-28. If the straightaway following the loop has friction ($\mu_k = 0.40$) and the rest of the track is smooth (*a*) how far along the straightaway will the bead slide? (*b*) Repeat (*a*) for a starting height of 11.0 R.

Ans. (*a*) 2.48 m; (*b*) 4.95 m

Problem 7.21. A bullet of mass 150 g is fired with velocity 300 m/s into the trunk of a tree and penetrates to a depth of 8.0 cm. Find (*a*) the thermal energy generated; (*b*) the average resistive force of the tree over the 8.0-cm distance.

Ans. (*a*) 6750 J; (*b*) 84,400 N

Problem 7.22. In Problem 6.48, how much work is done by a person moving the bob to the 50° position?

Ans. 9.24 J

Problem 7.23. A 5.0-kg projectile is fired vertically upward from the ground level with initial velocity $v_0 = 100$ m/s. By the time it reaches the highest point, 4 kJ of thermal energy has been generated due to air resistance. How high does the projectile rise?

Ans. 429 m

Problem 7.24. A rifle fires a 20-g bullet with a muzzle velocity of 3000 m/s. The barrel of the gun is 0.80 m long.

(*a*) If a thermal energy of 40,000 J is generated when the gun is fired, what is the *total* energy released in the gunpowder explosion that projects the bullet out of the barrel?

(*b*) What is the average force exerted on the bullet as it moves down the gun barrel?

Ans. (*a*) 130,000 J; (*b*) 112,500 N

Problem 7.25. Assume that the net force acting on the bullet in Problem 7.24 while in the barrel is constant.

(*a*) What is the time of flight down the barrel?

(*b*) What is the time-average power exerted on the bullet?

Ans. (*a*) 533 μs; (*b*) $P_{av} = \text{KE}/t = 169$ MW

Problem 7.26. A water-skier is pulled by a motorboat in such a way that the towline makes a 40° angle with the skier's direction of motion. The skier is moving at a constant speed of 18 m/s, and the tension in the line is 40 N. How much power is needed?

Ans. 552 W

Problem 7.27. When an object falls from a great height, the air resistance exerts an upward force that increases with the speed. Eventually the air resistance just balances the force of gravity, and from that moment on the object descends at a constant speed called the **terminal velocity**. Assume that an object weighing 80 lb has reached a terminal velocity of 110 ft/s when it is 1000 ft above the ground.

(*a*) Find the power, in hp, exerted on the object by the force of gravity as it descends with terminal velocity.

(*b*) Find the total thermal energy generated by the air resistance in the last 1000 ft before the object reaches the ground.

Ans. (*a*) 16.0 hp; (*b*) 80,000 ft · lb

Problem 7.28. An engine delivers a power of 30 kW in pulling a block horizontally along a level surface at a constant speed of 12 m/s. If the block has a mass of 500 kg, find (*a*) the frictional force between the block and the surface, and (*b*) the coefficient of friction.

Ans. (*a*) 2500 N; (*b*) 0.51

Problem 7.29. A 2000-lb car accelerates from 0 to 60 mph (88 ft/s) in 7.0 s on a straight, level track. Assuming no frictional losses, what minimum power must the engine have to accomplish this?

Ans. 62.5 hp

Problem 7.30.

(*a*) A 5000-kg truck coasts down a steep (22°) hill in low gear without using the brakes and reaches a constant speed of 12.0 m/s. Find the thermal power generated due to friction in the drivetrain. (Ignore all other sources of friction, including air resistance). What is the source supplying that power to the truck?

(*b*) If the truck were "in neutral" (with the drivetrain disconnected from the wheels) and the brakes were used instead, how much thermal power would have to be generated in the brakes to keep the truck moving at 12.0 m/s?

Ans. (*a*) 220 kW, the force of gravity; (*b*) 220 kW

Problem 7.31. A student uses a pulley system to hoist physics lecture notes weighing 200 lb to a height of 20 ft. To accomplish this the student exerts a force of 30 lb and pulls a total of 200 ft of rope.

(*a*) Find the IMA, the MA, and the efficiency of the pulley system.

(*b*) If the student takes 25 s, what average power did he exert?

Ans. (*a*) 10.0, 6.67, 66.7%; (*b*) 240 ft · lb/s

Problem 7.32.

(*a*) For the lever system in Fig. 7-9(*a*), find the IMA and the force F needed to lift the weight in the ideal case.

(*b*) If the efficiency is 85%, find the MA and the actual force F' necessary to lift the weight.

Ans. (*a*) 3.0, 98 N; (*b*) 2.55, 115 N

Problem 7.33. Repeat Problem 7.32 for the machine of Fig. 7-9(*b*).

Ans. (*a*) 4.0, 12.5 lb; (*b*) 3.4, 14.7 lb

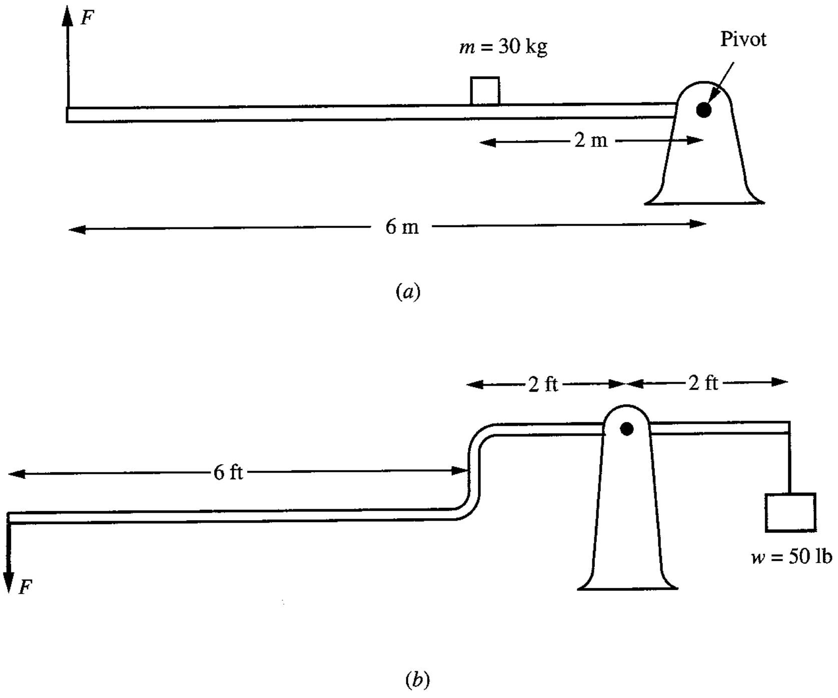

Fig. 7-9

Problem 7.34. Water is pulled up from a well by means of a bucket connected to the handle-and-axle system shown in Fig. 7-10. The rope is wound on the axle as the handle is turned. If a force $F = 28$ lb is necessary to turn the handle while raising a 100-lb bucket, what is the efficiency of the simple machine?

Ans. 89%

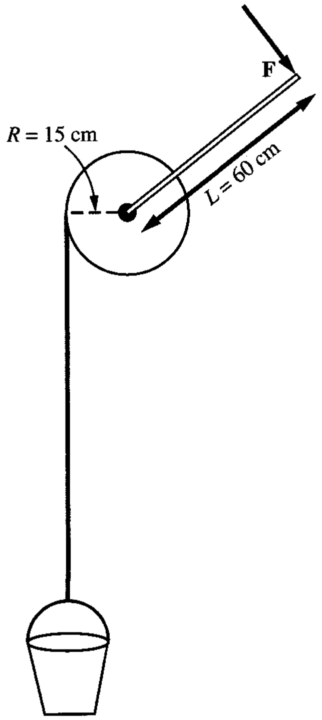

Fig. 7-10

Problem 7.35. A trunk is pushed up a 20° inclined plane from ground level to a platform 1.2 m above the ground by applying a force of 100 N parallel to the incline.

(*a*) Find the IMA of this simple machine.

(*b*) If this machine has an efficiency of 80%, find the MA.

(*c*) Assuming 80% efficiency, find the weight of the trunk.

Ans. (*a*) 2.92; (*b*) 2.34; (*c*) 234 N

Problem 7.36. A simple machine consists of a meshed gear system as shown in Fig. 7-11. The handle on gear 1 sweeps out a circle of radius $R_1 = 40$ cm, while the axle on gear 2 (on which the rope winds up) has radius $R_2 = 10$ cm.

(*a*) If there are 10 teeth on gear 1 and 50 teeth on gear 2, find the IMA of the machine and the turning force F needed on the handle to lift a 500-N weight in the ideal case.

(*b*) If in general the ratio of the number of teeth on gear 2 to gear 1 is labeled N, find a general expression for the IMA for arbitrary R_1, R_2 and N.

Ans. (*a*) 20, 25 N; (*b*) IMA $= NR_1/R_2$

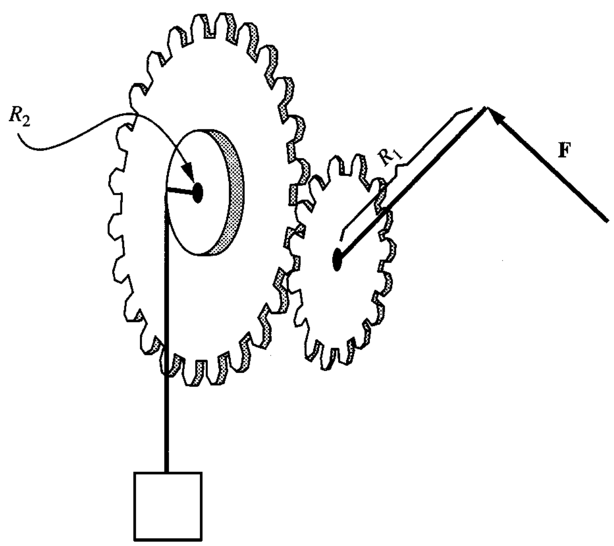

Fig. 7-11