Chapter 4

Forces and Equilibrium

Note. In introductory mechanics it is usually assumed that all forces act in the same plane (usually called the xy plane). Such forces are said to be *coplanar*. This assumption simplifies the mathematics considerably but still allows for a substantial understanding of the underlying physics involved.

4.1 FORCES

A **force** is a mechanical effect of the environment on an object. It is either a push or a pull on an object, and has both a magnitude (in appropriate units such as newtons, dynes, or pounds—units of force are discussed in detail in Chap. 5) and a direction. It can thus be represented by a vector. A force has two basic effects on an object. (1) It can change the motion of the object, which is the subject of Newton's famous second law (Chap. 5). (2) It can distort the shape of an object such as by stretching, compressing, or twisting the object.

Types of Forces

A force can be either due to direct contact (**contact force**) such as a hand pushing a block or a rope dragging a box or due to influence from afar (**action at a distance**) such as the gravitational pull of the earth on a satellite or the push of one magnet on another not in contact with it. On the human scale there are many different forces of either type. But on the atomic scale there are only four fundamental forces: gravitational, electromagnetic, weak nuclear, and strong nuclear—all of them actions at a distance.

The Resultant of a System of Forces

The vector sum of the forces acting on an object is called the **resultant** force on the object. The laws of nature are such that when two or more forces are acting at the same point in an object, they can be replaced by their resultant acting at the same point, which will have the same exact effect on the object as the original set of forces.

Problem 4.1. In Fig. 4-1(a) two forces are shown acting at a point in an object. Find the magnitude and direction of the single force that can replace those two forces and have the exact same effect.

Solution

In Fig. 4-1(b) the resultant $\mathbf{R}$ and the replaced forces $\mathbf{F}_1$ and $\mathbf{F}_2$ (in dashed form), as well as $\mathbf{F}_2$ shifted parallel to itself so that it is tail to head with $\mathbf{F}_1$ (see Sec. 3.1). Since the two original forces are at right angles to each other, we can use the pythagorean theorem to obtain the magnitude of the resultant force: $R^2 = F_1{}^2 + F_2{}^2 = (30\ \text{lb})^2 + (40\ \text{lb})^2 = 2500\ \text{lb}^2$. Taking the square root, we obtain $R = 50$ lb. To get the direction of $\mathbf{R}$ we determine its angle θ with the horizontal. We have $\tan\theta$ = opposite/adjacent = 40/30 = 1.33 or $\theta = 53°$. Thus $\mathbf{R}$ has magnitude 50 lb and acts at an angle 53° above the horizontal.

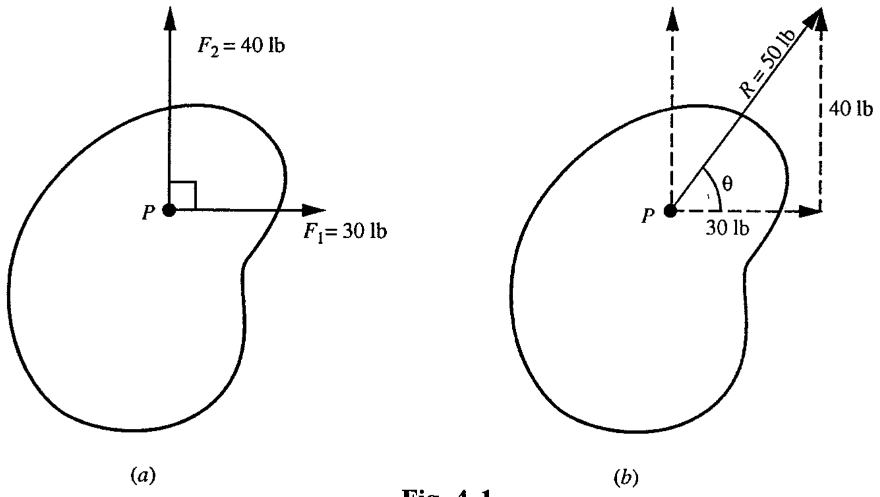


Fig. 4-1

Line of Action

When a force acts at a point in an object, one can draw an imaginary line through that point and parallel to the force. This is called the **line of action** of the force.

A rigid body refers to an object that doesn't change its shape when forces act on it. No real object is truly rigid, but the concept is a good approximation for stiff objects. In studying the relation of force and motion we will usually assume that we have rigid bodies. While in general the effect of a force on a rigid body depends on where it acts, a force acting on a rigid body can be applied anywhere along its line of action and still have exactly the same effect.

Problem 4.2. In Fig. 4-2(a) we have the same two forces acting on a rigid body as in Fig. 4-1(a), but now they are acting at different points B and C. Can one still replace these two forces by a single resultant force that has exactly the same effect on the motion of the rigid body and, if so, give an example of such a resultant force?

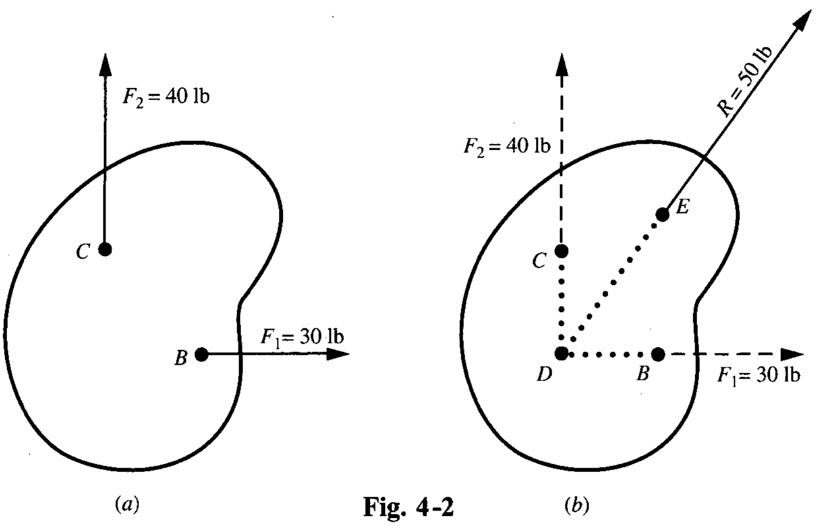


Fig. 4-2

Solution

The answer is *yes*. Since $\mathbf{F}_1$ and $\mathbf{F}_2$ can be moved anywhere along their lines of action without changing their effects, we can imagine moving them so that they both act at point D, the intersection of their lines of action (Fig. 4-21(b)). They can then be replaced by their resultant $\mathbf{R}$, acting at the same point D. As already calculated in Problem 4.1, $\mathbf{R}$ is 50 lb acting 53° above the horizontal. Furthermore, this resultant force can be moved or slid anywhere along its own line of action without change in effect. Figure 4-2(b) shows the resultant $\mathbf{R}$ acting at point E, where it still has exactly the same effect as the original two forces (shown in dashed form) that it has replaced.

4.2 EQUILIBRIUM

Translational motion is the motion of the object *as a whole* through space, without regard to how it spins on itself. The translational motion of a very small object, idealized as a particle, is just the motion of the particle along its path. For a large, irregular body it is less clear what is meant by the motion of the object as a whole or the path of the object through space. Fortunately, the idea can still be defined precisely as the motion of a special point of the object, called the **center of mass**. For simple uniform symmetric objects, such as a disk, a sphere, a rod, or a rectangular solid, the center of mass is at the geometric center of the object (see Sec. 8.4).

Problem 4.3. Describe the translational motion of the board eraser in Fig. 4-3.

Solution

The dashed parabolic line represents the path followed by the center of mass; it thus represents the translational motion of the eraser.

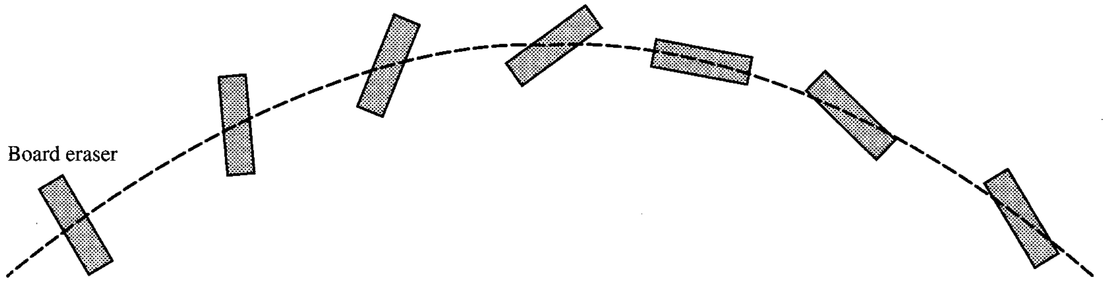


Fig. 4-3

Rotational motion is the spinning motion of an object, without regard to the motion of the object as a whole. Often rotational motion refers to the spinning of an object about a fixed axis, such as the spinning of a wheel on a shaft, but it can also refer to the spinning of an object on itself as the object as a whole moves through space.

Problem 4.4. How does one describe the rotational motion of the board eraser from left to right in Fig. 4-3?

Solution

The change in the angular orientation of the eraser represents its rotational motion. Note that the eraser has rotated clockwise through 180°.

Problem 4.5. Describe the translational and rotational motion of the cratered moon around the planet in Fig. 4-4.

Solution

The circular dashed line represents the translational motion of the moon. This moon has no rotational motion since its orientation does not change. The moon, in effect, stays parallel to itself throughout the motion.

Translational equilibrium means that the object as a whole, aside from rotation, has *uniform* translational motion, that is, its center of mass is either at rest or moving at constant speed in a straight line.

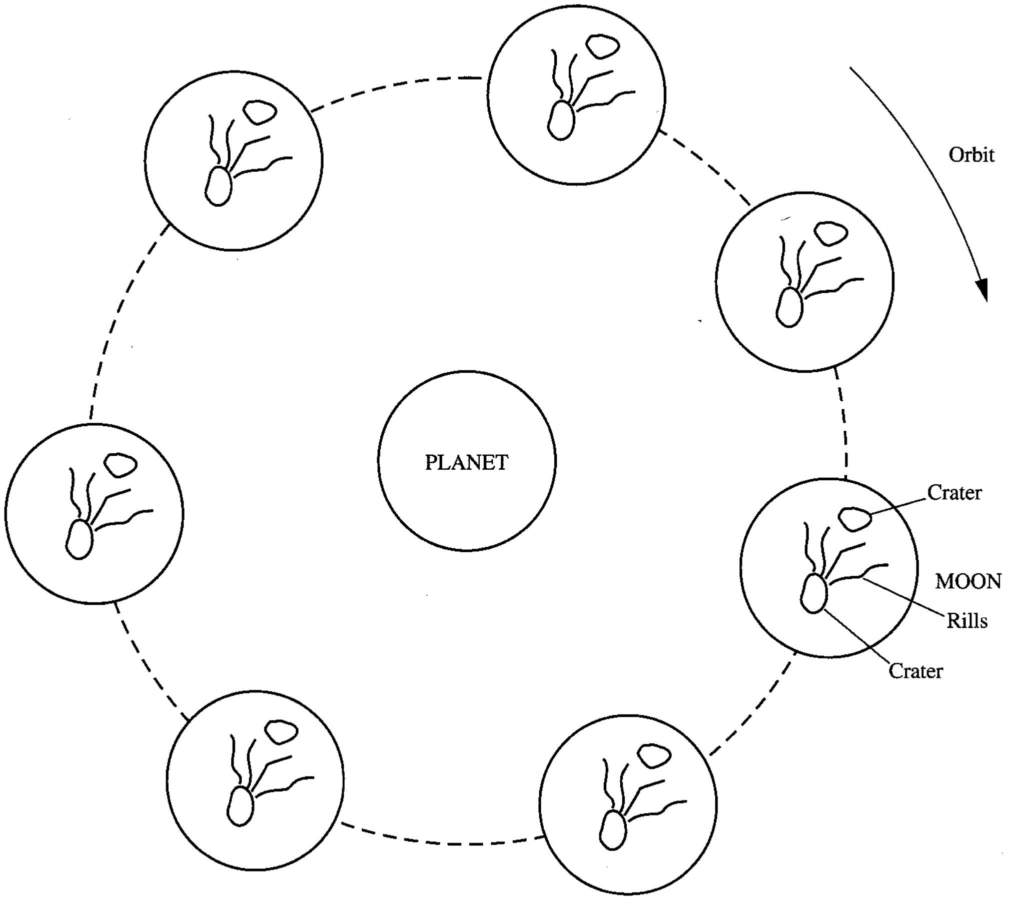


Fig. 4-4

Problem 4.6. Does the motion of the eraser in Fig. 4-3 or of the moon in Fig. 4-4 correspond to translational equilibrium?

Solution

No. The translational motion of the eraser is a parabolic arc and that of the moon is a circle, whereas for translational equilibrium the motion must be in a straight line. An example of *approximate* translational equilibrium would be a block sliding on an ice-covered lake; the block would move in a straight line without slowing down.

Rotational equilibrium means that the object—whether it is undergoing translational motion or not—is either not spinning or it is spinning in a uniform fashion. For simple symmetric objects this means spinning at a constant rate about a fixed direction.

Problem 4.7. Does the motion of the eraser in Fig. 4-3 and of the moon in Fig. 4-4 correspond to rotational equilibrium?

Solution

If the eraser were tumbling at a uniform rate, it would indeed be in rotational equilibrium; that, in fact, is a good approximation to what happens if air resistance is not an important factor. The moon is certainly in rotational equilibrium, since we are shown that the moon does not rotate at all.

A Frame of Reference refers to the "framework" that defines the coordinate system in which one's measurements and observations are made. If a coordinate system is fixed to the earth and another one is fixed to a rotating merry-go-round, one is going to observe things differently in each. Each of these coordinate systems is fixed in a different *frame of reference*.

An inertial frame of reference, by definition, is a frame of reference in which a completely isolated object (no forces) will appear to be in both translational and rotational equilibrium. For most purposes the earth can be considered an *inertial frame*; that is only an approximation, however, because the earth spins on its axis—although it is a very slow spin—once every 24 h. The importance of inertial frames is that Newton's laws hold only in such frames, and most of the other laws of physics take on simpler form when described in such frames. We will always assume that we are describing things in an inertial frame of reference unless otherwise indicated.

4.3 NEWTON'S FIRST LAW

A totally isolated object (no forces) is in both translational and rotational equilibrium in an inertial reference frame. However, even rigid bodies that *do* have forces acting on them can be in either translational or rotational equilibrium, or both, under suitable conditions. The condition for translational equilibrium is the statement of *Newton's first law*, also known as the *law of equilibrium*. We give here some simple cases.

Equilibrium with Only Two Forces Acting

If the two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ (see Fig. 4-5) are equal in magnitude and opposite in direction (that is, $\mathbf{F}_1 + \mathbf{F}_2 = 0$), then the object is in translational equilibrium. If in addition the two forces act along a common line of action (*collinear forces*), as in Fig. 4-5(*b*), then the object is also in rotational equilibrium.

Note. It is also possible to have rotational equilibrium without translational equilibrium, a situation that will be discussed in a later chapter.

Problem 4.8. A uniform rod is connected to two cords that exert the only forces on the rod, as depicted in Fig. 4-6; (i.e., we assume there is no pull of gravity on the rod). For each case determine whether the rod is in translational equilibrium. If so, can it also be in rotational equilibrium?

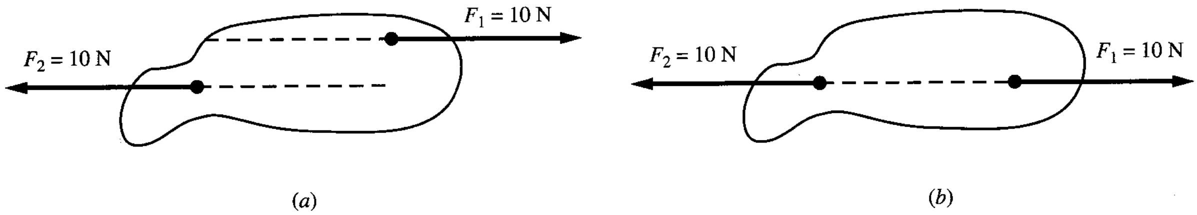


Fig. 4-5

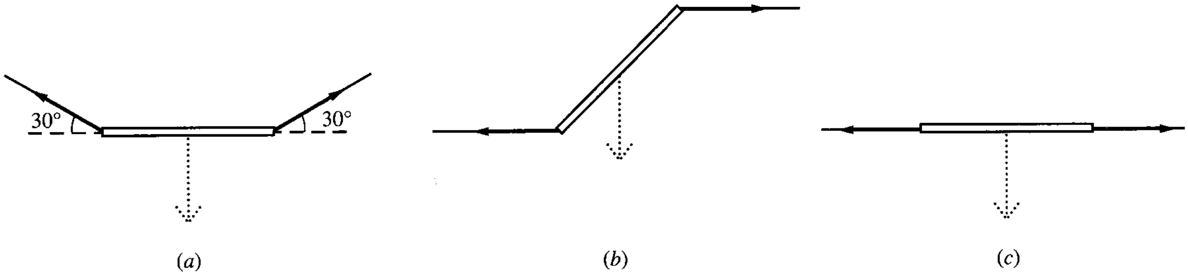


Fig. 4-6

Solution

Since the cords are flexible and exert a force only when they are taut, they can only pull along their length, as is depicted by arrows. Case (*a*) cannot correspond to translational equilibrium because the two forces are not equal and opposite ($\mathbf{F}_1 + \mathbf{F}_2 \neq 0$), Case (*b*) can correspond to translational equilibrium, *if* the two forces have equal magnitude, but it cannot represent rotational equilibrium because the two forces don't have a common line of action. Case (*c*) corresponds to both translational and rotational equilibrium if the two cords pull with forces of equal magnitude.

Equilibrium with Three Forces Acting

If the vector sum of the three forces is zero ($\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 0$), then the object is in translational equilibrium. If in addition the lines of action of the three forces pass through a common point, then the object is in rotational equilibrium as well. Such a system of forces is called *concurrent.*

Problem 4.9. Consider the same cases as in Problem 4.8, except now take into account the weight of the rod. Which of the cases can now correspond to equilibrium?

Solution

Since the rod is uniform, we can assume the weight is a single force acting downward at its center (dotted arrows in Fig. 4-6). Now only case (*a*) can correspond to translational equilibrium since only in that case could the vector sum of the three forces add up to zero if the magnitudes were suitable (see Problem 4.10). The rod would also be in rotational equilibrium, because, by symmetry, the three forces are concurrent. In neither case (*b*) nor (*c*) could the three vector forces add up to zero since the weight is perpendicular to the vector sum of the two other forces and could never be balanced by them.

Problem 4.10. For case (*a*) of Problem 4.9, if the weight is 100 N, find the force exerted on the rod by each of the two cords if the rod is in equilibrium (*a*) by geometric means; (*b*) by the component method.

Solution

(*a*) Newton's first law tells us that the resultant of the three forces acting on the rod must be zero. In Fig. 4-7(*a*) we redraw the rod as an isolated object and include only the forces acting on it (body diagram). The condition $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 0$ implies that the three vectors, drawn head to tail, form a closed triangle. As can be seen in Fig. 4-7(*b*), the triangle is equilateral for our case, so $F_1 = F_2 = F_3 = 100$ N.

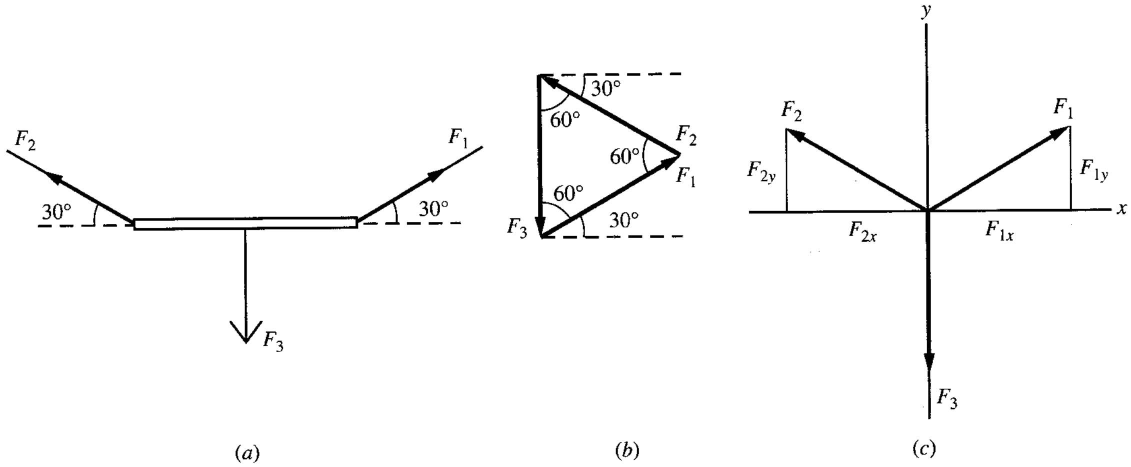


Fig. 4-7

(*b*) We now solve the problem algebraically. Choose the x axis along the rod and the y axis perpendicular to the rod at its center. Now slide the vectors parallel to themselves to the origin, for easier visualization Fig. 4-7(*c*). Since the vector sum of the three forces equals zero, we must have for the components

$$F_{1x} + F_{2x} + F_{3x} = 0 \qquad \text{and} \qquad F_{1y} + F_{2y} + F_{3y} = 0$$

From Fig. 4-7(*c*), we have

$$F_{1x} = F_1 \cos 30^\circ \qquad F_{2x} = -F_2 \cos 30^\circ \qquad F_{3x} = 0$$
$$F_{1y} = F_1 \sin 30^\circ \qquad F_{2y} = F_2 \sin 30^\circ \qquad F_{3y} = -100\,\text{N}$$

Substituting into the x-component equation,

$$F_1 \cos 30^\circ - F_2 \cos 30^\circ + 0 = 0 \qquad \text{or} \qquad F_1 = F_2$$

Similarly, the y-component equation gives

$$F_1 \sin 30^\circ + F_2 \sin 30^\circ - 100\,\text{N} = 0 \qquad \text{or} \qquad 0.5F_1 + 0.5F_2 = 100\,\text{N}$$

Using $F_1 = F_2$ in the y-component equation gives

$$0.5F_1 + 0.5F_1 = 100\,\text{N} \qquad \text{or} \qquad F_1 = 100\,\text{N} = F_2$$

While this method of solving a vector equation seems more cumbersome than the geometric method, it can be applied to more general cases where the geometric approach is too difficult to use.

Equilibrium with Any Number of Forces

For the general case of any number n of forces, we again have two conditions for equilibrium. The first is the condition for translational equilibrium, or *Newton's first law,* which says that the vector sum of all the forces is zero: $\Sigma \mathbf{F}_i = 0$. For small objects or particles, where rotation can be ignored, it is the only condition of equilibrium. For extended objects, the second condition, for rotational equilibrium, is again needed. The general case of rotational equilibrium will be discussed in a later chapter. The rest of this chapter is concerned only with translational equilibrium.

4.4 NEWTON'S THIRD LAW

This law, otherwise known as the *law of action and reaction,* states that if some object A exerts a force $\mathbf{F}_{ab}$ on object B, then object B exerts a force $\mathbf{F}_{ba}$ on object A that is equal in magnitude and opposite in direction: $\mathbf{F}_{ba} = -\mathbf{F}_{ab}$. The law holds both for contact forces and for action-at-a-distance forces.

Problem 4.11. Consider a book lying at rest on a horizontal table.

(*a*) What are the forces on the book?

(*b*) What is the reaction force to each of these forces?

(*c*) What effect do the reaction forces have on the book?

Solution

(*a*) There are two forces acting on the book: its weight (the downward pull of gravity toward the center of the earth) and the force exerted upward on the book by the tabletop.

(*b*) The reaction to the weight is an upward pull of equal magnitude exerted on the earth by the book. The reaction to the table's force is a downward push of equal magnitude on the table by the book.

(*c*) The reaction forces have no effect on the book! By definition, any effect on the book is represented by a force *on the book.* The reaction forces act on the earth and on the table—not on the book.

Problem 4.12. An elephant and a teenager are having a tug-of-war, as shown in Fig. 4-8(*a*). Does Newton's third law imply a draw?

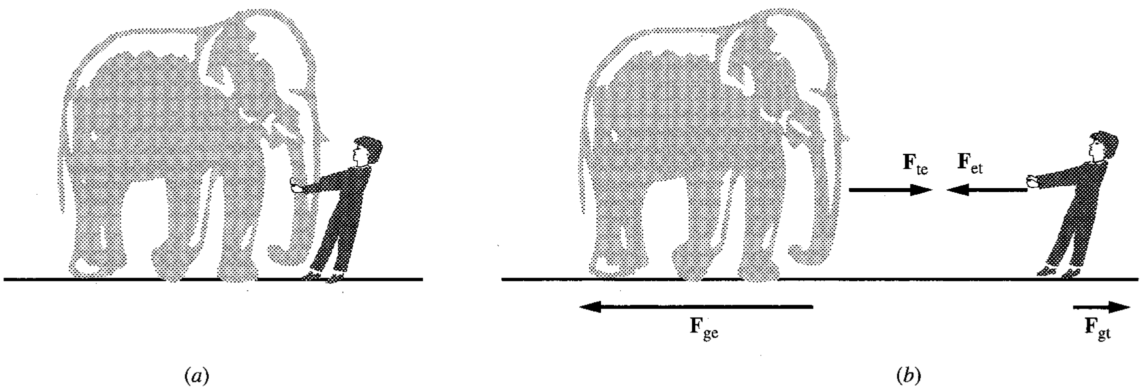


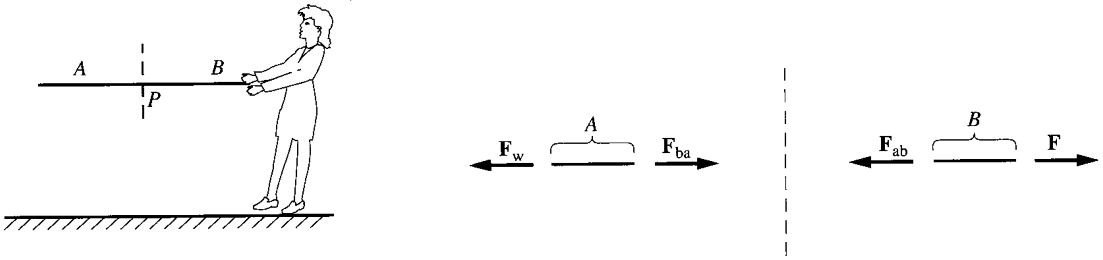
Fig. 4-8

Solution

No. Unless the elephant is very weak, the teenager will definitely lose. It is true that the force the elephant exerts on the teenager $\mathbf{F}_{et}$ is equal and opposite to the force the teenager exerts on the elephant $\mathbf{F}_{te}$, but the motion of either "object" depends on the resultant of *all* the forces acting on it. Both the teenager and the elephant are pushing the ground forward with their feet, and in each case the ground exerts an opposite reaction force. The situation is depicted in Fig. 4-8(*b*), where $\mathbf{F}_{gt}$ and $\mathbf{F}_{ge}$ represent the horizontal forces exerted by the ground on the teenager and on the elephant, respectively. Thus, for example, suppose that $F_{et} = F_{te} = 250$ lb. We might have $F_{gt} = 100$ lb and $F_{ge} = 650$ lb. Then a net force of 150 lb acts on the teenager to the left, and he moves leftward. Similarly, a net force of 400 lb acts on the elephant to the left, and the elephant also moves leftward. The next section deals with friction and shows why it is reasonable to assume that $F_{ge} > F_{gt}$.

Tension

At any given point in a taut rope (or cord, string, thread, or cable) we can ask: With what force does the segment of rope on one side of the point pull on the segment of rope on the other side? Consider the situation in Fig. 4-9(*a*), where a girl pulls on one end of a horizontal rope with a force **F**, while the other end is attached to the wall. We consider an arbitrary point p of the rope that divides it into two segments A and B, as shown. Figure 4-9(*b*) shows the segments as separate bodies, with the horizontal forces on each drawn in. By Newton's third law, the forces with which the two segments pull on each other $\mathbf{F}_{ab}$ and $\mathbf{F}_{ba}$ are equal in magnitude and opposite in direction. The **tension** T at the point p is the magnitude of either of these forces: $T = F_{ab} = F_{ba}$. Since each rope segment is in equilibrium, we also have $F_{ab} = F$, and $F_w = F_{ba}$, where $\mathbf{F}_w$ is the force of the wall on the rope. Thus all these forces have the same magnitude T. Furthermore, since point p was chosen arbitrarily we conclude that the tension is the same everywhere in the rope.



(*a*) (*b*)

Fig. 4-9

"Weightless" Ropes

In general these results are true only for a horizontal rope in equilibrium. If the rope were vertical, with one end attached to the ceiling and the other end pulled down by the girl, then the weight of each segment of the rope would have to be taken into account, and the tension at a point p of the rope would equal neither the force with which the girl pulled down nor the force with which the ceiling pulled up. Indeed, the tension would vary from point to point in the rope. The same would be true if we had a horizontal rope that was not in equilibrium, because the forces applied to either end would not balance out.

There is, however, one circumstance where there is a common tension throughout the rope, and this tension always equals the magnitude of the forces acting at the ends of the rope—whether the rope is horizontal or vertical, whether the rope is in equilibrium or not. This is the circumstance where the rope is weightless. In most problems one characterizes such a rope as a cord, string, or thread to indicate its "lightness." Obviously no cord is completely weightless, but if it is very light in comparison to the other objects in the problem, it can be assumed weightless without much error.

Problem 4.13. A block of weight $w = 15$ N hangs at the end of a (weightless) cord suspended from the ceiling. What is the tension in the cord, and with what force does the cord pull down on the ceiling?

Solution

The tension is the same at all points of the cord and is equal to the magnitude of the force pulling at either end. Since the block is in equilibrium under the action of two vertical forces (the weight downward and the pull of the cord upward), these two forces must have the same magnitude. Hence the upward pull of the cord = 15 N. By Newton's third law the magnitude of the pull of the block downward on the cord is also 15 N, so $T = w = 15$ N. The tension T also equals the magnitude of the pull of the ceiling on the cord, which by Newton's third law equals the pull of the cord downward on the ceiling. Thus the downward pull of the top of the cord on the ceiling is the same as the downward pull of the block on the bottom of the cord. Thus we see that a weightless rope transmits an applied force from one end to the other.

4.5 FRICTION

Friction is the rubbing force between two objects whose surfaces are in contact. The force of friction always acts parallel to the touching surfaces. By Newton's third law each surface exerts a frictional force that is equal in magnitude and opposite in direction to that exerted by the other. The magnitude of the frictional force exerted by each surface on the other depends on how tightly the two surfaces are pressed together.

Normal Force

The force responsible for this "pressing together" is called the **normal force** because it acts perpendicular to the two surfaces. By Newton's third law each surface exerts a normal force that is equal in magnitude and opposite in direction to that exerted by the other. Figure 4-10 indicates the frictional and normal force on each object when a block is in contact with an inclined plane. The frictional force (parallel to the surface) and the normal force (perpendicular to the surface) acting on a surface can always be thought of as the components of the overall force acting on that surface due to the other surface in contact with it.

Static Friction

When two surfaces are at rest with respect to one another, the frictional force each exerts on the other always opposes any tendency to relative motion. The frictional force on an object adjusts itself in magnitude and direction to oppose and counterbalance any other forces on the object that would tend to make the object start to slide. It varies, as needed, from zero magnitude up to some maximum value to stop such slippage. Such a frictional force is called a **static friction** force ($\mathbf{f}_s$). The maximum

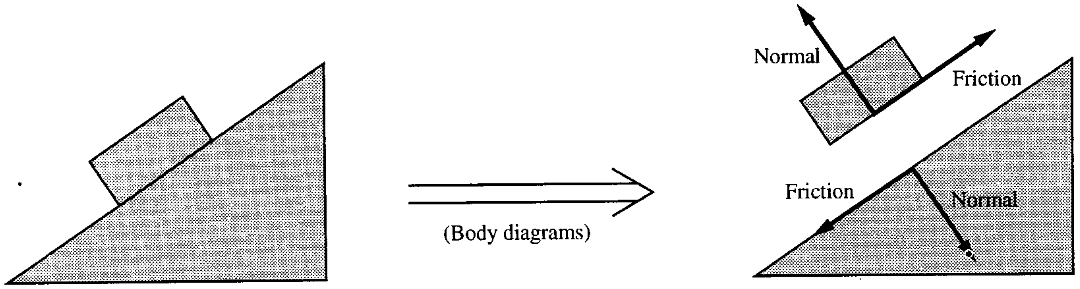


Fig. 4-10

static friction force $f_{s,\max}$ that one surface can exert on another is proportional to the normal force **N** between the surfaces: $f_{s,\max} = \mu_s N$, where N is the magnitude of the normal force, and μ_s is a proportionally constant, called the **coefficient of static friction**, that depends on the nature of the two surfaces. It is possible to force one object to slide over the other by applying a parallel force to one of the objects that is larger than $\mu_s N$, the maximum possible static friction force.

Problem 4.14. A book of weight $w = 10$ N rests on a horizontal table top, as shown in Fig. 4-11(*a*), and a horizontal force **F** is applied to it. If the coefficient of static friction μ_s between the book and the tabletop is 0.25, calculate (*a*) the normal force exerted by the tabletop on the book, and (*b*) the maximum value of the static friction force.

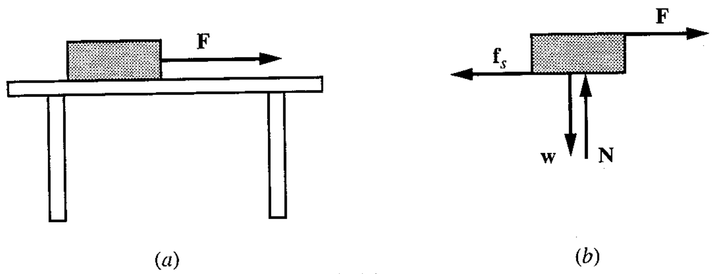


Fig. 4-11

Solution

(*a*) Since the book is in equilibrium, the sum of the forces acting on it must equal zero. Figure 4-11(*b*) shows the body diagram for the book with all the forces acting on it. The frictional force is $\mathbf{f}_s$, and the normal force is **N**. Noting that $\mathbf{f}_s$ and **F** have no y components, from the condition that the sum of the y components equals zero we have $N - 10\text{ N} = 0$, or $N = 10$ N.

(*b*) The maximum value attainable by the static friction force is

$$f_{s,\max} = \mu_s N = (0.25)(10\,\text{N}) = 2.5\,\text{N}$$

Problem 4.15.

(*a*) In Problem 4.14, if the magnitude of the applied force is $F = 2.0$ N, what is the magnitude and direction of the frictional force on the book?

(*b*) What if $F = 1.0$ N; 0 N?

(*c*) What is the biggest value that F can be before the book starts to slide?

Solution

(*a*) The frictional force opposes the tendency to motion, so it is in the direction opposite to **F**, as shown in Fig. 4-11(*b*). The magnitude of the frictional force adjusts itself to keep the book at rest, which in this case means $f_s = F = 2.0$ N. This value is possible, since it is smaller than the maximum found in Problem 4.14(*b*).

(*b*) If $F = 1.0$ N, then, by the same reasoning as in part (*a*), we have $f_s = 1.0$ N in the direction *opposite to* **F**. If $F = 0$, then $f_s = 0$, and there is no frictional force at all.

(*c*) If F is bigger than $f_{s,\max}$, then the frictional force cannot rise to match F and maintain equilibrium. Thus $F = 2.5$ N is the limiting value; beyond this value equilibrium cannot be maintained, and the book starts to move.

Kinetic Friction

Once two surfaces are in motion relative to one another, the frictional force, now called **kinetic friction** ($\mathbf{f}_k$), acting on a surface is always in a direction opposed to the velocity of that surface. To a good approximation, its magnitude is independent of the magnitude of the velocity and is again proportional to the normal force between the two surfaces. Thus it can be expressed as $f_k = \mu_k N$, where μ_k, the **coefficient of kinetic friction**, depends only on the nature of the two surfaces. For any given pair of surfaces, $\mu_k \leq \mu_s$.

Problem 4.16. Assume the book in Fig. 4-11(*a*) is moving to the right with speed v.

(*a*) Now what are the magnitude and direction of the force of friction exerted by the tabletop on the book?

(*b*) Does f_k depend on the magnitude of the applied force **F**?

(*c*) If the book instead moves to the left with speed v, with **F** still to the right, what are the magnitude and direction of the force of friction? Assume that $\mu_k = 0.2$.

Solution

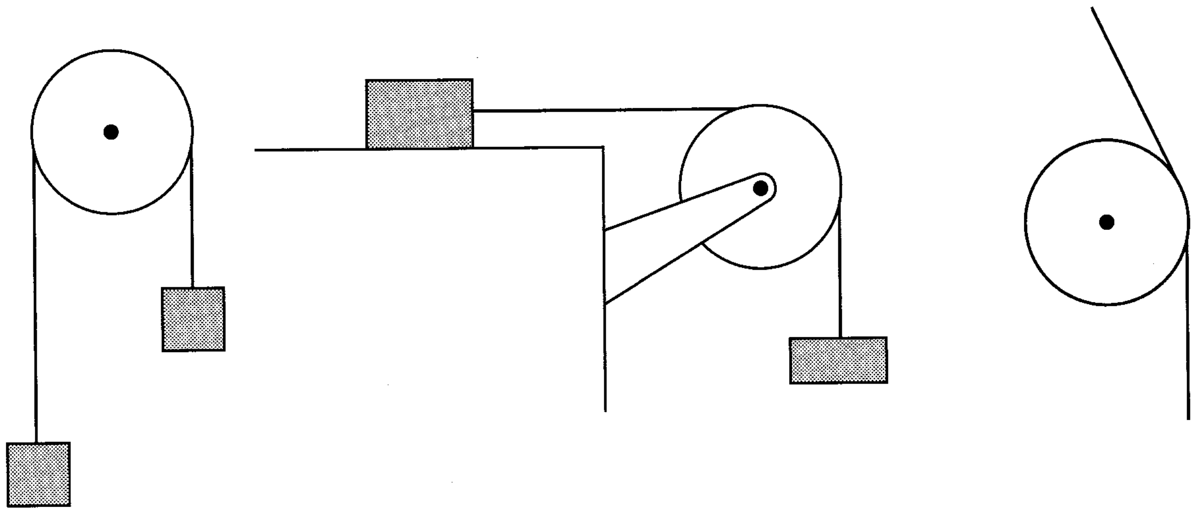
(*a*) Once the book is moving the (kinetic) friction is of fixed magnitude, $f_k = \mu_k N$. Since we still have equilibrium in the y direction, we still have the same normal force; Thus $f_k = (0.2)(10 \text{ N}) = 2.0$ N. The direction of the kinetic friction force is always opposite to the direction of motion, so it would be to the left.

(*b*) No.

(*c*) Since the normal force is still the same, the value of f_k is still 2.0 N. The direction of $\mathbf{f}_k$ is now to the right. Note that the direction of $\mathbf{f}_k$ depends only on the direction of motion and not on the direction of **F**.

4.6 CORDS AND PULLEYS

If a (weightless) cord is bent over a pulley, as in Fig. 4-12, there are two idealized situations in which the tension in the part of the cord on one side of the pulley will be the same as the tension in the part of the cord on the other side of the pulley.



(a) (b) (c)

Fig. 4-12

1. The surface of the pulley is frictionless so that the cord slides effortlessly over it (frictionless pulley).
2. The surface of the pulley has friction, *but* the pulley has no weight *and* there is no friction between the pulley and the axle on which it rotates (**weightless pulley**).

In a problem, being told that the pulley is frictionless and/or weightless (massless) is generally shorthand for case 1 or case 2, and you can assume as much unless told otherwise.

Problem 4.17. In Fig. 4-13(*a*), the two blocks are connected by a light rope over a frictionless, weightless pulley. If the system is initially at rest, will it stay at rest? If so, what is the frictional force exerted by the table on block A?

Solution

Figure 4-13(*b*) gives the body diagrams for the two blocks. For block B, assuming equilibrium, the y-component equation gives $T - W_b = 0$ or $T = W_b = 10$ N. Since we have a rope and a frictionless, weightless pulley, the tension is the same on the block-A side of the pulley, and $T = 10$ N for block A as well.

Vertical equilibrium of block A requires that $N - W_a = 0$, or $N = W_a = 30$ N. Then the maximum possible static frictional force is

$$f_{s,\max} = \mu_s N = (0.5)(30\,\mathrm{N}) = 15\,\mathrm{N}$$

Since $T < f_{s,\max}$, the frictional force can balance T and the system remains at rest. The actual frictional force can be obtained from the equilibrium of block A:

$$T - f_s = 0 \qquad \text{or} \qquad f_s = T = 10\,\mathrm{N}$$

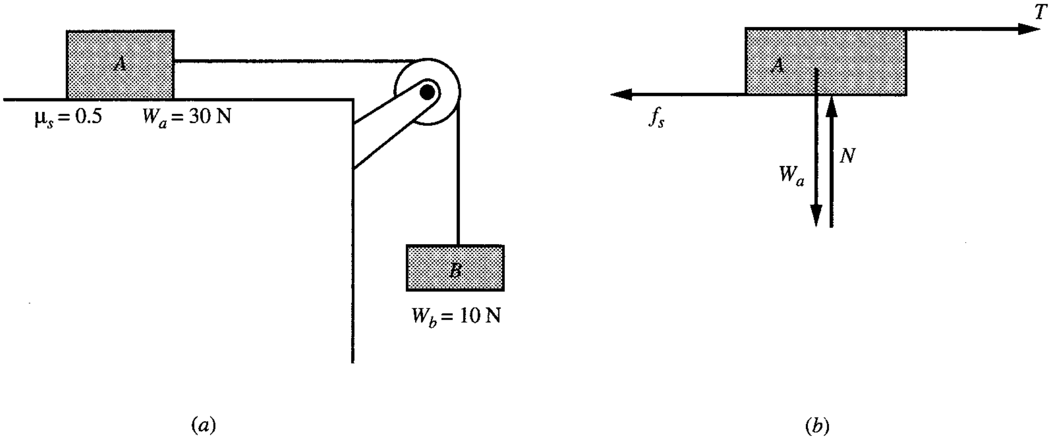


Fig. 4-13

Problems for Review and Mind Stretching

Problem 4.18. Find the resultant **R** of the two forces shown in Fig. 4-14.

Solution

R = **F**$_1$ + **F**$_2$. We choose x and y axes as shown in the figure and use the component method of addition.

$$F_{1x} = 0 \qquad F_{1y} = 20\,\text{N} \qquad F_{2x} = -(60\,\text{N})\cos 37^\circ \qquad F_{2y} = -(60\,\text{N})\sin 37^\circ$$

$$R_x = F_{1x} + F_{2x} = 0 - (60\,\text{N})\,(0.8) = -48\,\text{N}$$
$$R_y = F_{1y} + F_{2y} = (20\,\text{N}) - (60\,\text{N})\,(0.6) = -16\,\text{N}$$
$$R = [(-48)^2 + (-16)^2]^{1/2} = 50.6\,\text{N}$$

From the signs of its components, **R** is in the third quadrant. If θ is the acute angle that **R** makes with the negative x axis,

$$\tan\theta = \left|\frac{R_y}{R_x}\right| = \frac{16}{48} = \frac{1}{3} \qquad \text{or} \qquad \theta = 18.4^\circ$$

Thus **R** has magnitude 50.6 N and points away from the origin at 18.4° below the negative x axis.

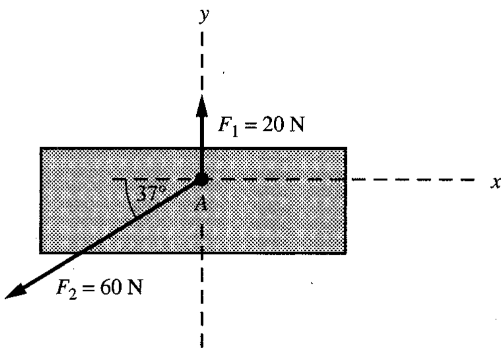


Fig. 4-14

Problem 4.19. Three forces act on a rigid body, as shown in Fig. 4-15, with their lines of action passing through the common point B. Find their resultant and its point of application for equilibrium.

Solution

$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$. Choose the x and y axes as shown. Then

$$R_x = F_{1x} + F_{2x} + F_{3x} = (-50\,\text{N})\cos 30° + (40\,\text{N})\cos 45° + (0\,\text{N})$$
$$= (-50\,\text{N})(0.866) + (40\,\text{N})(0.707) = -15.0\,\text{N}$$
$$R_y = F_{1y} + F_{2y} + F_{3y} = (50\,\text{N})\sin 30° + (40\,\text{N})\sin 45° + (-30\,\text{N})$$
$$= (50\,\text{N})(0.5) + (40\,\text{N})(0.707) + (-30\,\text{N}) = 23.3\,\text{N}$$
$$R = [(-15)^2 + (23.3)^2]^{1/2} = 27.7\,\text{N}$$

R is in the second quadrant, with

$$\tan\theta = |R_y/R_x| = \frac{23.3}{15.0} \quad \text{or} \quad \theta = 57.2° \text{ above the negative } x \text{ axis}$$

R can act anywhere along a line of action through B.

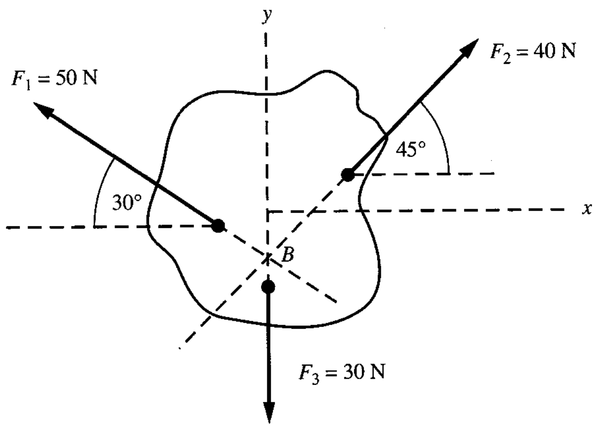


Fig. 4-15

Problem 4.20. Refer to Problem 4.18.

(*a*) What third force **E**, must be exerted on the body for it to be in translational equilibrium?

(*b*) Where must **E** be applied to give rotational equilibrium as well?

Solution

(*a*) For translational equilibrium, $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{E} = 0$, or $\mathbf{E} = -(\mathbf{F}_1 + \mathbf{F}_2) = -\mathbf{R}$. Hence $E = 50.6$ N, and **E** points 18.4° above the positive x axis (see Fig. 4-16).

(*b*) **E** must have the same line of action as **R**; that is, its line of action must also pass through point A.

Note. The force which, when added to an existing set of forces on an object, will cause the object to be in equilibrium is called the *equilibrant* of the set. (The force **E** in the previous problem is thus an equilibrant.)

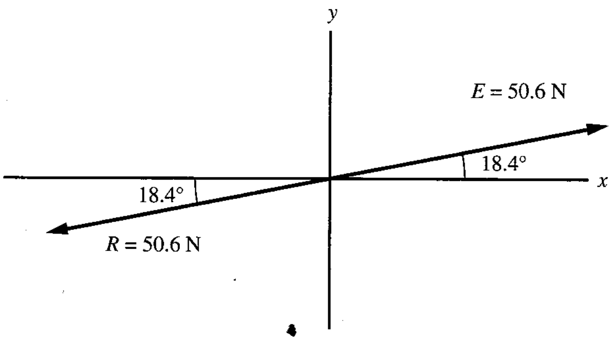


Fig. 4-16

Problem 4.21. Find the equilibrant of the forces in Problem 4.19.

Solution

Here we have the concurrent forces $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ which can be replaced by the single resultant force $\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$ with line of action through point B, as obtained in Problem 4.19. Clearly, to have equilibrium, the added fourth force, the equilibrant $\mathbf{E}$, must obey $\mathbf{E} = -\mathbf{R}$. Thus $E = 27.7$ N pointing $57.2°$ below the positive x axis, with a line of action that must also pass through point B.

Problem 4.22. A block of weight $w_1 = 400$ N hangs from a uniform heavy rope of length 3 m and weight $w_2 = 300$ N, as shown in Fig. 4-17(*a*). Find (*a*) the force with which the rope pulls on the block; (*b*) the tension in the rope 1 m above the contact point with the block; (*c*) the force with which the ceiling pulls on the rope.

Solution

In Fig. 4-17(*b*) we have the body diagrams for the block, the lower third of the rope, and the full rope, respectively. Each is in equilibrium, and the vector sum of the forces on each equals zero. Since the forces are all in the y direction, only the equilibrium condition in that direction need be applied.

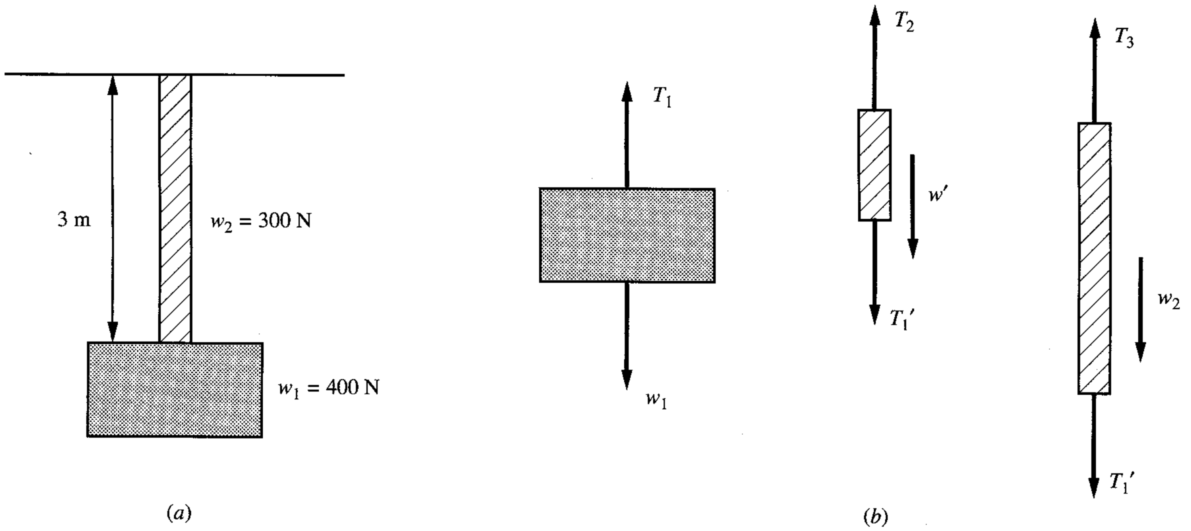


Fig. 4-17

(*a*) For the block, $T_1 - w_1 = 0$, or $T_1 = 400$ N equals the force of the rope on the block.

(*b*) For the lower third of the rope, $T_2 - T_1' - w' = 0$, where T_2 is the contact force of the upper two-thirds of the rope on the lower third and is the tension in the rope at that point; T_1' is the force of the block on the rope, given by Newton's third law as $T_1' = T_1 = 400$ N; w' is the weight of the lower third of the rope, or $w' = 100$ N. Thus $T_2 = T_1' + w' = 400\text{ N} + 100\text{ N} = 500$ N.

(*c*) For the rope as a whole, $T_3 - T_1' - w_2 = 0$, or $T_3 = T_1' + w_2 = 400\text{ N} + 300\text{ N} = 700$ N, equals the force of the ceiling on the rope.

Problem 4.23. For the weight-and-strings setup of Fig. 4-18(*a*), find the tensions T_1, T_2, and T_3.

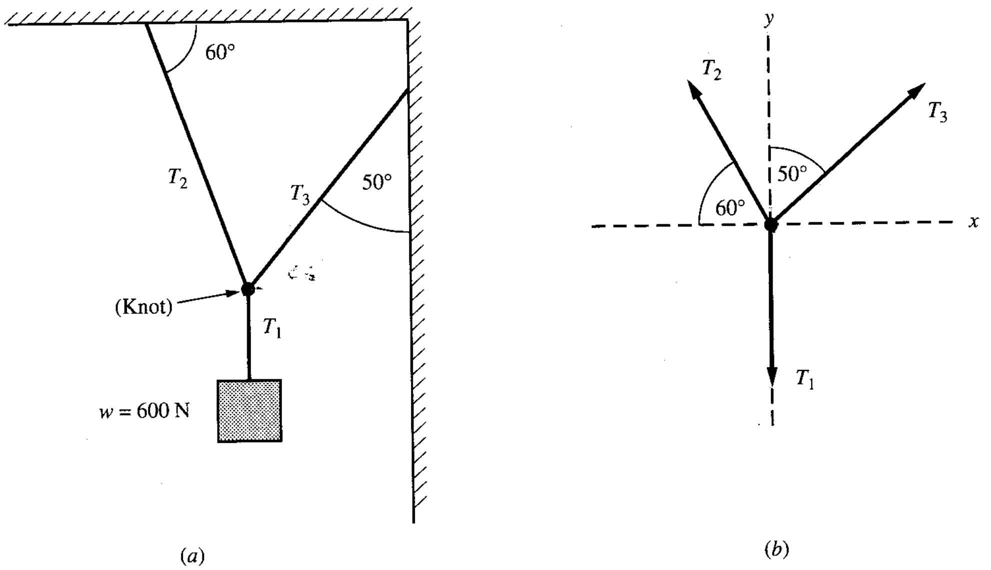


Fig. 4-18

Solution

From the equilibrium of the block, $T_1 = 600$ N. Since the knot is in equilibrium, the body diagram, Fig. 4-18(*b*), gives $\mathbf{T}_1 + \mathbf{T}_2 + \mathbf{T}_3 = \mathbf{0}$. Using the component method, we get

$$T_{1x} + T_{2x} + T_{3x} = 0 - T_2 \cos 60^\circ + T_3 \sin 50^\circ = 0 \qquad \text{or} \qquad 0.5T_2 = 0.766T_3$$

or $T_2 = 1.532\ T_3$. (A sine appears in the x-component equation because the angle of $\mathbf{T}_3$ is given relative to the y axis). Similarly,

$$T_{1y} + T_{2y} + T_{3y} = -T_1 + T_2 \sin 60^\circ + T_3 \cos 50^\circ = 0 \qquad \text{or} \qquad 0.866T_2 + 0.643T_3 = 600\,\text{N}$$

Substituting for T_2,

$$(0.866)\,(1.532T_3) + 0.643T_3 = 600\,\text{N} \qquad \text{or} \qquad 1.970T_3 = 600\,\text{N} \qquad \text{or} \qquad T_3 = 305\,\text{N}$$

Finally, $T_2 = 1.532T_3 = 467$ N.

Problem 4.24. A block of weight $w = 200$ N is pulled along a horizontal surface at constant speed by a force $F = 80$ N acting at an angle of 30° above the horizontal, as shown in Fig. 4-19.

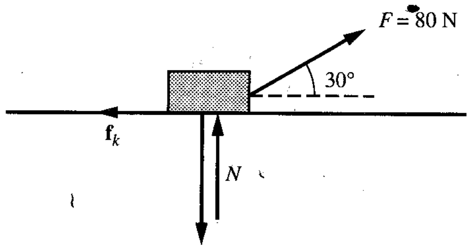


Fig. 4-19

(*a*) Find the frictional force **f** exerted on the block by the surface.

(*b*) Find the normal force **N** exerted on the block by the surface.

(*c*) Find the coefficient of kinetic friction, μ_k, between the block and the surface.

Solution

(*a*) The four vector forces acting on the block are shown in Fig. 4-19. Since the block is in equilibrium, their sum equals zero. For the x components we thus have

$$F\cos 30^\circ - f_k = 0 \quad \text{or} \quad f_k = (80\,\text{N})\,(0.866) = 69.3\,\text{N}$$

(*b*) Similarly, for the y components,

$$F\sin 30^\circ + N - w = 0 \quad \text{or} \quad N = 200\,\text{N} - (80\,\text{N})\,(0.5) = 160\,\text{N}$$

Note that the normal force is not equal to the weight even though the block is on a horizontal surface, because the force **F** has a vertical component.

(*c*) $\mu_k = f_k/N = 69.3/160 = 0.433$.

Problem 4.25. A hanging weight w_1 is connected by a light cord over a frictionless pulley to a block on a frictionless incline of weight $w_2 = 500$ N, as shown in Fig. 4-20. If the block on the incline moves down at constant speed, what is the weight of the hanging block? How would your answer change if it were moving up the incline at constant speed?

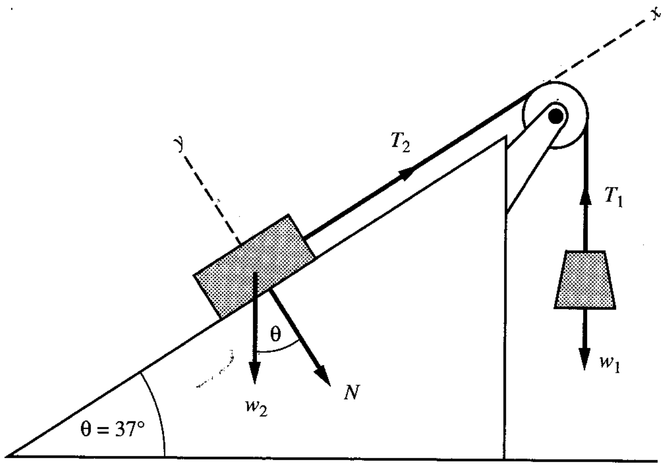


Fig. 4-20

Solution

In Fig. 4-20, all the forces on the respective blocks are shown right on the diagram for the system as a whole. Since both blocks move in straight lines at constant speed, they are each in equilibrium. For the hanging block, using y components, we have $T_1 - w_1 = 0$, or $w_1 = T_1$. To find T_1 we turn to the block on the incline. We choose x and y axes along the incline and perpendicular to it, respectively. We also note that the force of the cord on each block has the same magnitude, so $T_2 = T_1 = T$, since the cord is light and the pulley is frictionless. Then, for the x-component equilibrium equation we get

$$T - w_2 \sin\theta = 0 \qquad \text{or} \qquad T = (500\,\text{N})(\sin 37°) = 300\,\text{N}$$

Then from our earlier result $w_1 = T = 300$ N. Note that we did not need to solve the y-component equilibrium equation for the block on the incline to solve for T and w_1. This is because the y-component equation gives us the normal force N, which does not affect the x-component equation when there is no friction. If the block were moving up the incline, the blocks would still be in equilibrium under the action of the same forces, so the answer would remain the same.

Problem 4.26. Suppose that in Problem 4.25 there was friction between the block and the incline and that the coefficient of sliding friction was $\mu_k = 0.3$, but all the other data in the problem remained unchanged. Find the weight of the hanging block, w_1, if the other block moves at constant speed (*a*) down the incline; (*b*) up the incline.

Solution

(*a*) We can use Fig. 4-20 with the modification that there is an additional force on the block on the incline, a frictional force of magnitude f_k opposing the motion of the block and hence pointing parallel to the incline in the upward direction. From the rules for friction we have $f_k = \mu_k N$, where N is the normal force exerted on the block by the incline. Following the reasoning of Problem 4.25 we now have for the x components

$$T + \mu_k N - w_2 \sin 37° = 0 \qquad \text{or} \qquad T = (500\,\text{N})(0.6) - 0.3\,N$$

For the y components

$$N - w_2 \cos 37° = 0 \qquad \text{or} \qquad N = (500\,\text{N})(0.8) = 400\,\text{N}$$

Substituting into the previous equation we have

$$T = (500\,\text{N})(0.6) - 0.3(400\,\text{N}) = 300\,\text{N} - 120\,\text{N} = 180\,\text{N}$$

Since the hanging block obeys $w_1 = T$, we have our result, $w_1 = 180$ N.

(*b*) If the block is moving up the incline at constant speed, we proceed as before, noting that the frictional force is now directed down the incline although it still has the same magnitude $f_k = \mu_k N$. Furthermore the y-component equation for the block on the incline is unchanged, so we still have $N = 400$ N and $f_k = 0.3(400\text{ N}) = 120$ N. The x-component equation changes only in that the sign of the x-component of the frictional force changes, and we get

$$T - \mu_k N - w_2 \sin 37° = 0 \qquad \text{and} \qquad T = 300\,\text{N} + 120\,\text{N} = 420\,\text{N}$$

Finally, $w_1 = T = 420$ N.

Problem 4.27. For the setup in Fig. 4-18(*a*)—first discussed in Problem 4.23—the breaking point of the two cords attached to the wall and ceiling is 1500 N. How heavy can the block be without one of the cords snapping? Assume the cord attached to the block can handle any weight.

Solution

We first determine which of the two cords will reach a tension of 1500 N first. To do this we recall from Problem 4.23 that equilibrium in the x direction requires

$$T_3 \sin 50^\circ = T_2 \cos 60^\circ \quad \text{or} \quad 0.766T_3 = 0.50T_2 \quad \text{or} \quad T_3 = 0.653T_2 < T_2$$

Clearly T_3 is always less than T_2, and hence T_2 will reach 1500 N first. We now set $T_2 = 1500$ N; from above, this immediately yields $T_3 = 0.653$ (1500 N) = 980 N. We can now determine the corresponding weight w of the block using the equilibrium equation in the y direction:

$$w = T_1 = T_2 \sin 60^\circ + T_3 \cos 50^\circ = (1500\,\text{N})\,(0.866) + (980\,\text{N})\,(0.643) = 1929\,\text{N}$$

Supplementary Problems

Problem 4.28.

(*a*) The earth's moon revolves about the earth once a month and always keeps the same side facing the earth. Describe the translational and rotational motion of the moon.

(*b*) Is the moon in translational and/or rotational equilibrium?

Ans. (*a*) The moon as a whole translates in a circular orbit about the earth; it rotates on its axis once a month.

(*b*) The moon is not in translational equilibrium; if the moon's monthly rotation on its axis is uniform (it is, approximately), then the moon is in rotational equilibrium.

Problem 4.29. An automobile travels in a straight line with no skidding.

(*a*) If the automobile travels at constant speed, are its wheels in translational and/or rotational equilibrium?

(*b*) If the automobile accelerates from 0 to 60 mph, are its wheels in translational and/or rotational equilibrium?

(*c*) Is a bit of chewing gum on the rim of a wheel of the automobile in translational and/or rotational equilibrium for the case of part (*a*) or part (*b*)?

Ans. (*a*) In both; (*b*) in neither; (*c*) not in translational equilibrium for either case; the bit of gum goes through one rotation every time the wheel makes one complete turn. For part (*a*) it is in rotational equilibrium, while for part (*b*) it is not.

√ **Problem 4.30.** A uniform rod of weight 100 N is acted on by a force $\mathbf{F}_1$ as shown in Fig. 4-21. What force $\mathbf{F}_2$ must be added to the rod to ensure translational equilibrium?

Ans. 56.6 N at an angle of 58.0° above the negative x axis

√ **Problem 4.31.** In Fig. 4-22(*a*), assume the somewhat artificial condition that the strut is weightless and the wall is frictionless. The cord makes an angle $\theta = 37^\circ$ with the strut.

(*a*) What are the conditions imposed on T and N if the strut is to be in translational equilibrium?

(*b*) Can the strut be in rotational equilibrium under the circumstances shown? Give your justification.

Ans. (*a*) $T = 83$ N, $N = 66$ N; (*b*) No. The three forces cannot possibly be concurrent.

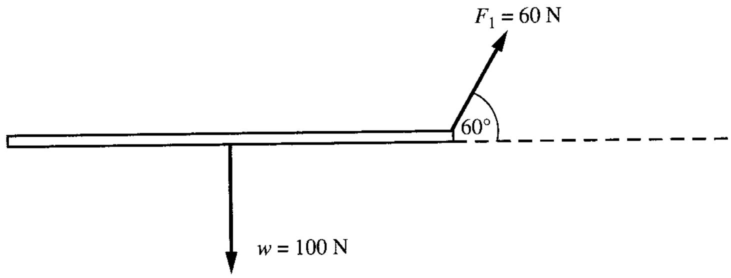


Fig. 4-21

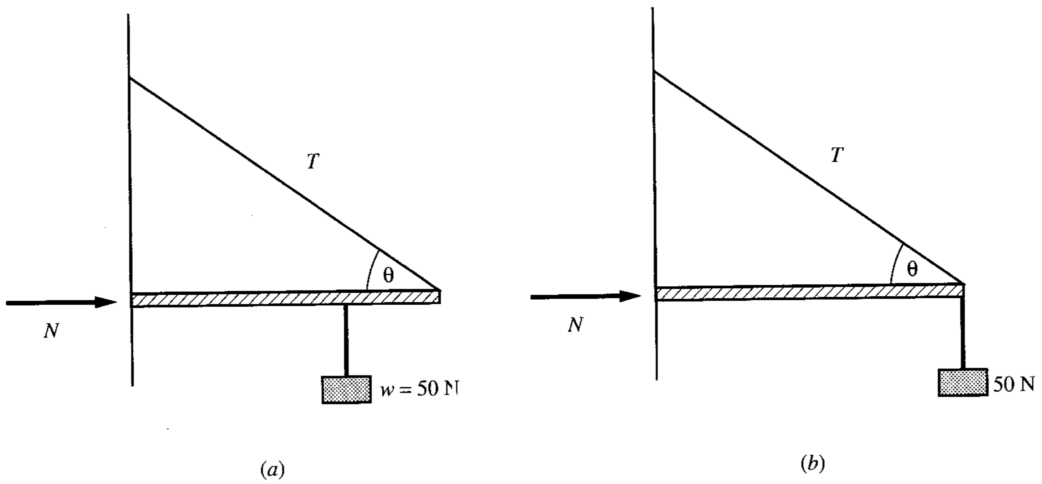


Fig. 4-22

Problem 4.32. Assume the same situation as in Problem 4.31, except that the weight now hangs from the end as shown in Fig. 4-22(*b*).

(*a*) Find the values of T and N for translational equilibrium.

(*b*) Is the strut now in rotational equilibrium and if so why?

Ans. (*a*) The forces are as before: $T = 83$ N, $N = 66$ N.

(*b*) Yes. If the forces are as in part (*a*), then the strut is also in rotational equilibrium since the three forces are concurrent.

√ **Problem 4.33.** A block weighing 200 N is suspended from the ceiling by means of three light cords joined in a knot (Fig. 4-23). Find the tensions in the cords and the forces the cords exert on the ceiling.

Ans. $T_1 = 200$ N, $T_2 = 104$ N, $T_3 = 146$ N; 104 N and 146 N, downward along the cord directions

√ **Problem 4.34.** A block slides down a 30° incline at constant speed. Find the coefficient of kinetic friction.

Ans. $\mu_k = 0.58$

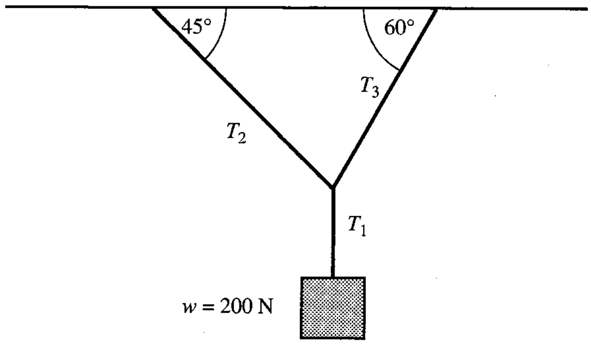


Fig. 4-23

Problem 4.35. The same block as in Problem 4.34, when placed at rest on the incline, does not move. When the angle of inclination is increased by 10°, the block starts to slide. What is the coefficient of static friction?

Ans. $\mu_s = 0.84$

Problem 4.36. The block in Problem 4.34 is now connected to a hanging block by means of a light cord over a frictionless pulley (Fig. 4-24). If the block on the incline weighs 30 N, what must be the weight of the hanging block if it falls at constant speed?

Ans. 30 N

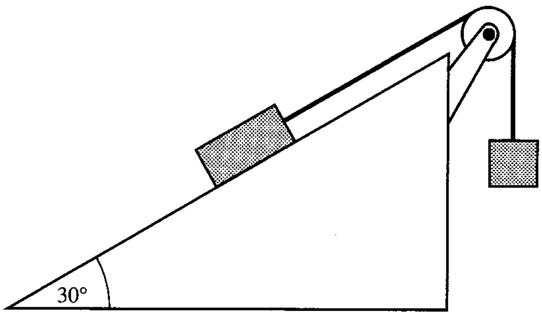


Fig. 4-24

Problem 4.37. Suppose that for the situation of Problem 4.26 (Fig. 4-20) the block is initially at rest and the coefficient of static friction is $\mu_s = 0.4$. For what range of weights w_1 will the block remain at rest?

Ans. 140 to 460 N

Problem 4.38. If the block in Problem 4.24 was initially at rest and $\mu_s = 0.6$, how big would the applied force have to be to just get the block moving?

Ans. $F = 103$ N

Problem 4.39. Suppose the rope in Problem 4.22 has a weak spot at its midpoint so that it will break if the tension at that point reaches 2000 N. What is the heaviest block that can be suspended by the rope?

Ans. 1850 N

Problem 4.40. A block is pushed along a tabletop at constant speed by a force acting 20° below the horizontal as in Fig. 4-25(*a*). If the weight of the block is 100 N and the coefficient of kinetic friction is $\mu_k = 0.30$, find the magnitudes of (*a*) the pushing force, (*b*) the normal force due to the table.

Ans. (*a*) 35.8 N; (*b*) 112 N

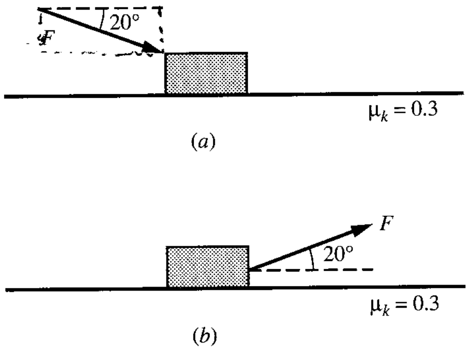


Fig. 4-25

Problem 4.41. Repeat Problem 4.40 if the block is being pulled at constant speed by a force acting at an angle of 20° above the horizontal [Fig. 4-25(*b*)].

Ans. (*a*) 28.7 N; (*b*) 90 N

Problem 4.42. Find the tensions T_1 and T_2 in the two cords for the equilibrium situation depicted in Fig. 4-26(*a*).

Ans. $T_1 = 80$ N; $T_2 = 41$ N

Problem 4.43. Repeat Problem 4.42 for Fig. 4-26(*b*).

Ans. $T_1 = 139$ lb; $T_2 = 160$ lb

Problem 4.44. Repeat Problem 4.42 for Fig. 4-26(*c*).

Ans. $T_1 = 253$ N; $T_2 = 288$ N

Problem 4.45. What is the minimum coefficient of static friction between table and block for which the blocks in Fig. 4-27 will remain in equilibrium? What is the tension *T*?

Ans. 0.29; 115 N

Problem 4.46. A 50-N weight is hung symmetrically from the ceiling by two light cords, as shown in Fig. 4-28. The breaking strength of the cords is 1200 N. What is the minimum angle θ at which the weight can be hung without the cords breaking? (Assume the vertical cord is very strong.)

Ans. 1.19°

Problem 4.47. In Fig. 4-26(*b*) the breaking point of the horizontal cord is 1000 lb, while that of the cord attached to the ceiling is 1200 lb.

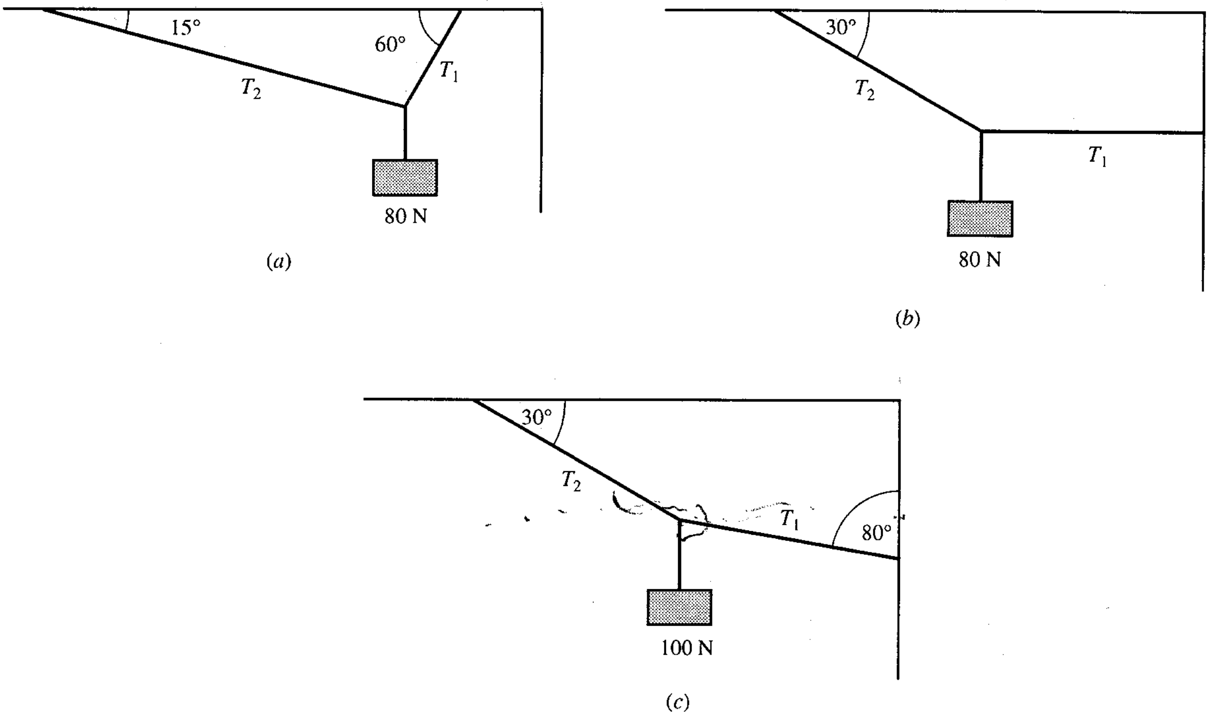


Fig. 4-26

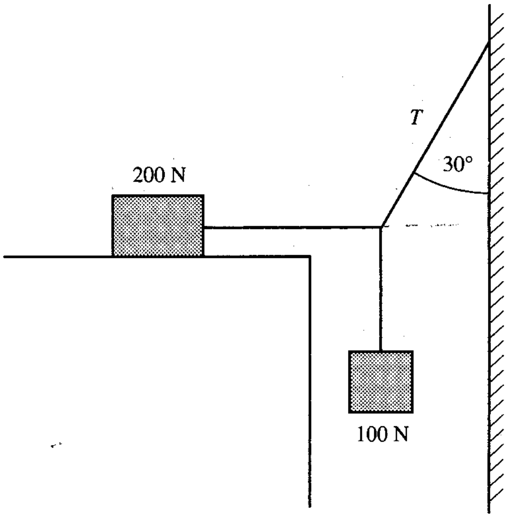


Fig. 4-27

(*a*) If the weight of the hanging block is steadily increased, which cord will snap first?

(*b*) What is the maximum weight that can be supported by the cords?

Ans. (*a*) The horizontal cord; (*b*) 577 lb

Problem 4.48. Referring to Fig. 4-20, suppose $\mu_k = 0.50$ and $w_1 = 900$ N. Find the weight w_2 such that the block just slides (*a*) up the incline at constant speed; (*b*) down the incline at constant speed.

Ans. (*a*) 900 N; (*b*) 4500 N

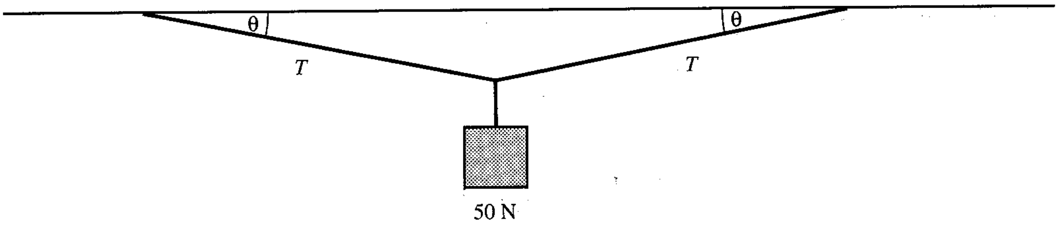


Fig. 4-28

√ **Problem 4.49.** A child pushes a block of weight $w = 300$ N against a wall with a force **F** acting upward at 45° to the horizontal to stop it from falling. The situation is shown in Fig. 4-29. $\mu_s = 0.6$ between the block and the wall.

(*a*) What is the minimum value of F for which the block will not fall?

(*b*) Would the child have an easier time of it by instead exerting a force in the horizontal direction?

Ans. (*a*) 265 N; (*b*) no. The minimum force would now be 500 N.

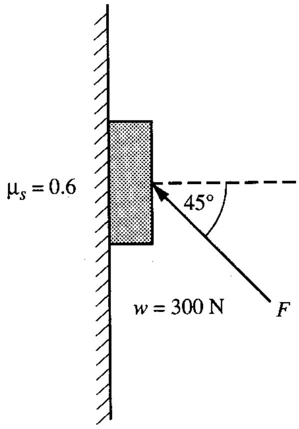


Fig. 4-29