Chapter 11

Deformation of Materials and Elasticity

11.1 DEFORMATION OF OBJECTS—STRETCHING AND COMPRESSING

So far we have investigated the motion of objects under the action of forces. Therefore we have ignored the deformation of extended objects—the distortion of their shapes. In previous chapters we assumed that extended objects were rigid bodies, but even the stiffest materials have some give in them. In this section we briefly explore the relationship between the forces acting on an object and the deformations they cause.

Horizontal Rods

Consider a horizontal rod of unstretched length L and cross-sectional area A, with one end firmly attached to a very strong wall. A horizontal force of magnitude F is applied to the other end, as shown in Fig. 11-1(*a*). (Since the rod remains attached to the wall, it is in equilibrium, and the wall exerts an equal and opposite force on the rod.) One would expect that under these circumstances the rod would stretch somewhat, with the amount of stretch depending on the strength of the force F. Let ΔL represent the amount of stretch. For a given force F, the stretch ΔL will depend on the intrinsic nature of the rod, i.e., the material of which it is made. It also will depend on the particular length and cross-sectional area of the rod in question. We would like to describe the stretching effect of a force on a rod in a way that doesn't depend on the particular cross section of the rod or on its length but only on the material of which it is made. We do this by defining the *stress* and the *strain*.

It is easy to see that the greater the area of our rod the greater the force necessary to get the same stretch. Indeed if we were to put another identical rod alongside the first and consider them as a unit, as shown in Fig. 11-1(*b*), a force F on the end of each would be needed to have the combination stretch the same ΔL as the single rod. If we had a single rod of the same length and cross-sectional area $2A$, as shown in Fig. 11-1(*c*), we would need the force $2F$ to stretch it by ΔL. Thus the force needed for a certain stretch is proportional to the cross-sectional area of the rod. If we define the **stress** as the ratio of the force to the cross-sectional area, we have a quantity that measures the effectiveness of a force in accomplishing a given stretch, independent of the cross-sectional area of the rod. To illustrate, we note that for the case of Fig. 11-1(*a*), we have stress = F/A. For the case of Fig. 11-1(*c*), we have stress = $2F/2A = F/A$. As we see, the stress is the same in both cases and gives rise to the same amount of stretch ΔL. The stress thus measures the effect of a force in stretching a rod without regard to how "thick" the particular rod is. The dimensions of stress are force per area (the same as for pressure). The SI unit of stress is the newton per square meter (N/m^2). This is given the special name the *pascal*: 1 Pa = 1 N/m^2. Other common units are dyn/cm^2, lb/ft^2, and lb/in^2.

Problem 11.1.

(*a*) If the cross-sectional area of the rod in Fig. 11-1(*a*) is $A = 2.50 \times 10^{-4}\ m^2$ and the force is $F = 300$ N, what is the stress?

(*b*) If the cross-sectional area of the same-length rod were three times as great, what force F' would be necessary to get the same amount of stretch?

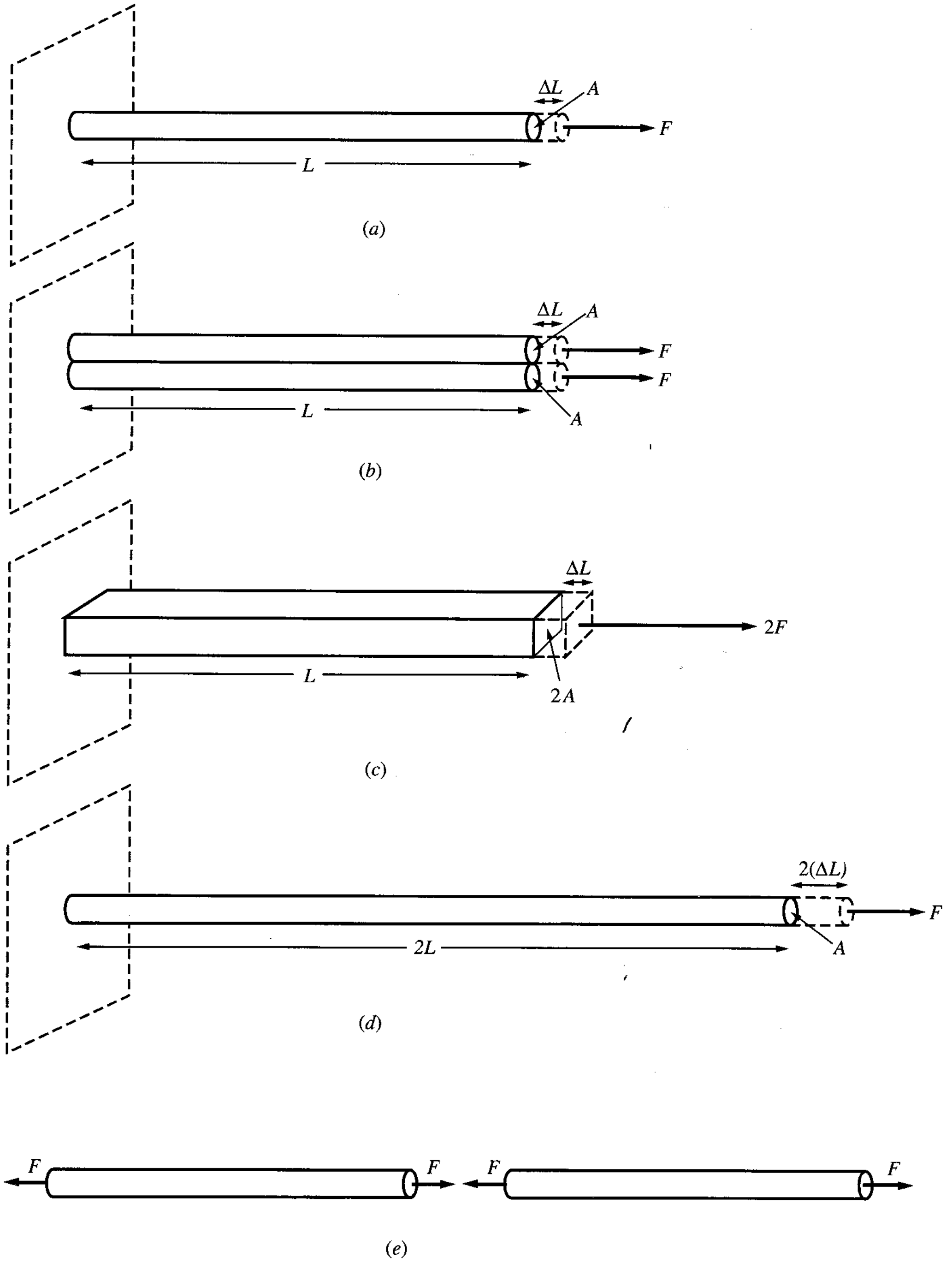

Fig. 11-1

Solution

(a) Stress = F/A = 300 N/(2.50 × 10^{-4} m^2) = 1.20 × 10^6 Pa.

(b) To get the same stretch the stress must be the same. Therefore, if we triple the area, we must also triple the force, so F' = 900 N.

We now turn to the effect of the length of the rod. If we had a rod of the same cross section as in Fig. 11-1(a) but of twice the unstretched length, the force F would stretch it twice as much; i.e., the extension would be $2\Delta L$, as shown in Fig. 11-1(*d*). To understand this, we consider the unstretched rod of length $2L$ to be made up of two equal sections, each of length L, as shown in Fig. 11-1(*e*). When the force F is applied, each half is under the action of equal and opposite forces at its two ends, because each is in equilibrium. By Newton's third law the magnitude of the forces on one half must be the same as those on the other half. Thus each half undergoes the same stretch ΔL, and the complete rod undergoes the stretch $2\Delta L$, as claimed. A given force will, therefore, cause a stretch that is proportional to the length of the unstretched rod. We define the **strain** as the ratio of the change in length of the rod to the unstretched length of the rod; for example, $\Delta L/L$ for the case of Fig. 11-1(*a*). As we have just seen, the strain due to a given force will be the same for any length rod of the same material and cross section. This is illustrated by comparing Fig. 11-1(*a*) and (*d*), where the force F is the same, and the strain is $\Delta L/L$ in the first case and $2\Delta L/2L = \Delta L/L$ in the second case. The strain is thus a measure of the stretch of a rod that is independent of the length of the rod. Note that strain is dimensionless, since it is the ratio of two lengths.

Problem 11.2.

(*a*) If the rod in Problem 11.1 has length $L = 2.00$ m, and $\Delta L = 2.50 \times 10^{-5}$ m, find the strain.

(*b*) What would the amount of stretch $\Delta L'$ be if the initial length of the rod were $L' = 1.50$ m, all else being the same?

Solution

(*a*) Strain $= \Delta L/L = (2.50 \times 10^{-5}\ \text{m})/2.00\ \text{m} = 1.25 \times 10^{-5}$.

(*b*) Since the stress is unchanged, the strain will be the same, and

$$\frac{\Delta L'}{L'} = \frac{\Delta L}{L} = 1.25 \times 10^{-5} \qquad \text{or} \qquad \Delta L' = (1.25 \times 10^{-5})(1.5\ \text{m}) = 1.88 \times 10^{-5}\ \text{m}$$

Combining the ideas of stress and strain we conclude that:

> A given stress will give rise to a definite strain in a rod of a certain material irrespective of either the thickness or the length of the rod.

Hooke's Law, Elastic Limit, and Ultimate Strength

We are now in a position to ask: What relation exists between the stress and the strain for a rod of a given material? It turns out that for many materials, as long as the stresses on the rod are not too large, upon removal of the stress the material returns to its original length. Any material that returns to its original shape after the distorting forces are removed is said to be **elastic**. For a rod of any given material there is a stress beyond which the material will no longer return to its original length. This boundary stress is called the **elastic limit**. For stresses below this elastic limit it is found that, to a good approximation, the strain is proportional to the stress; for example, if we double the stress, the strain will double. This result is called **Hooke's law**. Thus, in this "elastic" region, stress/strain = constant. The constant is called the **Young's modulus** Y, and its value depends on the material. Mathematically, we have

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{F/A}{\Delta L/L} \tag{11.1}$$

Young's modulus has dimensions of stress and can be measured in pascals.

Note. If a force tends to compress a rod rather than stretch it, Eq. (*11.1*) still holds with the same Young's modulus. In that case, ΔL represents a compression rather than a stretch.

If one applies a stress to a rod beyond the elastic limit, the rod will retain some permanent strain when the stress is removed. If the stress gets too great, the rod will break. The stress necessary to just reach the breaking point is called the **ultimate strength** of the material.

Problem 11.3.

(*a*) Assume that the rod in Fig. 11-1(*a*) obeys the conditions of Problems 11.1(*a*) and 11.2. Find the Young's modulus for the material of which the rod is made.

(*b*) Show that for a rod of definite cross section A and length L, the applied force F is proportional to the elongation ΔL and can therefore be expressed as $F = kx$, where k is the **force constant** of the system.

Solution

(*a*)
$$Y = \frac{F/A}{\Delta L/L} = \frac{1.20 \times 10^6\ \text{Pa}}{1.25 \times 10^{-5}} = 9.6 \times 10^{10}\ \text{Pa} = 96\ \text{GPa}$$

(*b*) $F = (YA/L)\ \Delta L$, or $F = kx$, with $k = YA/L$ and distance x measured from the undeformed position.

Problem 11.4. For a certain steel beam of length 10 m and cross-sectional area 25 cm^2, the elastic limit is 400 MPa and the ultimate strength is 800 MPa; Young's modulus is 196 GPa.

(*a*) Find the maximum elongation in the elastic region.

(*b*) What stretching force is required to break the beam?

Solution

(*a*) $Y = (F/A)/(\Delta L/L)$ so that $\Delta L = (F/A)(L/Y)$. We get

$$\Delta L = \frac{(4.0 \times 10^8\ \text{Pa})(10.0\ \text{m})}{1.96 \times 10^{11}\ \text{Pa}} = 0.0204\ \text{m}$$

(*b*) The ultimate strength $= F/A$ so that

$$F = (8.0 \times 10^8\ \text{Pa})(25 \times 10^{-4}\ \text{m}^2) = 2.0 \times 10^6\ \text{N}$$

Problem 11.5. A copper wire of length 3.0 m and diameter 4.0 mm has $Y = 1.18 \times 10^{11}$ Pa and elastic limit $= 158 \times 10^6$ Pa.

(*a*) Find the force constant k [as defined in Problem 11.3(*b*)] of the copper wire.

(*b*) What tension T is necessary to stretch the wire 1.50 mm?

(*c*) Does this exceed the elastic limit?

Solution

(*a*) From Problem 11.3(*b*), we have $T = kx$, where $k = YA/L$. We have $A = \pi d^2/4 = 3.14(4.0 \times 10^{-3}\ \text{m})^2/4 = 1.26 \times 10^{-5}\ \text{m}^2$. Then

$$k = \frac{(1.18 \times 10^{11}\ \text{Pa})(1.26 \times 10^{-5}\ \text{m}^2)}{3.0\ \text{m}} = 4.96 \times 10^5\ \text{N/m}$$

(*b*) Since $T = kx$, we get $T = (4.96 \times 10^5)(1.50 \times 10^{-3}) = 743$ N.

(*c*) From part (*b*), $T/A = (743\ \text{N})/(1.26 \times 10^{-5}\ \text{m}^2) = 59 \times 10^6$ Pa, so the answer is no.

11.2 SHEAR DEFORMATION AND SHEAR MODULUS

In addition to stretching or compressing, objects can be deformed in other ways, such as by shear deformation. A prototypical shear is shown in Fig. 11-2, where we have a rectangular solid whose base is held in place while parallel forces acting along one edge of the upper surface are applied. We assume the forces are distributed uniformly along the edge, their resultant being the *shear* force F. The base and top surfaces are each of area A, and the height of the solid is L. The deformation produced by F is a shifting of the top surface relative to the bottom surface by the amount Δx and the corresponding tilting of two of the vertical sides by the angle ϕ. We assume the distortion is very small compared to the original dimensions of the solid. Let us define the **shear stress** as F/A and the **shear strain** as $(\Delta x)/L = \tan\phi \approx \phi$ (rad). Then, by reasoning similar to that employed in the case for stretching, one can show that a given shear stress will give rise to a unique shear strain, irrespective of the dimensions of the rectangle. We find that for sufficiently small stresses the deformation is elastic and is governed by Hooke's law in the form

$$\frac{\text{Shear stress}}{\text{Shear strain}} = \frac{F/A}{\phi} = S \qquad (11.2)$$

where the **shear modulus** S is a constant for any given material and has the same dimensions as Y.

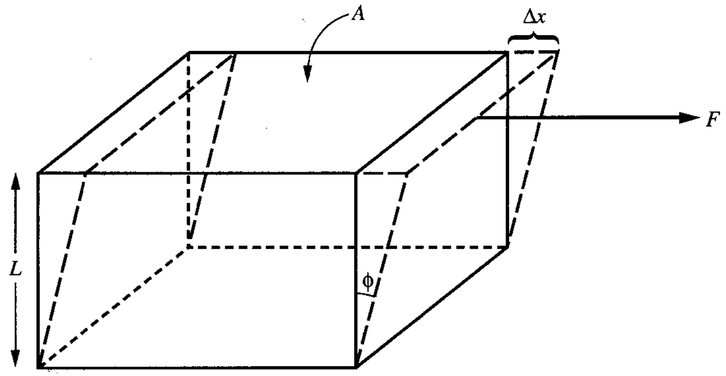

Fig. 11-2

Problem 11.6. Two equal and opposite parallel forces each of magnitude 6 kN, are applied to a cubical metal block which is 40 cm on a side, as shown in Fig. 11-3. The shear angle is found to be $\phi = 0.00036°$. Find (*a*) the shear stress and the shear strain on the block, and (*b*) the shear modulus for the metal. (*c*) What would the shear angle be if the same forces were applied to a cubical block of the same metal that was 120 cm on a side?

Solution

(*a*) The case at hand, with equal and opposite forces, is identical to the situation in Fig. 11-2, where the bottom is held fixed. Indeed, in Fig. 11-2 the surface in contact with the bottom supplies the equal

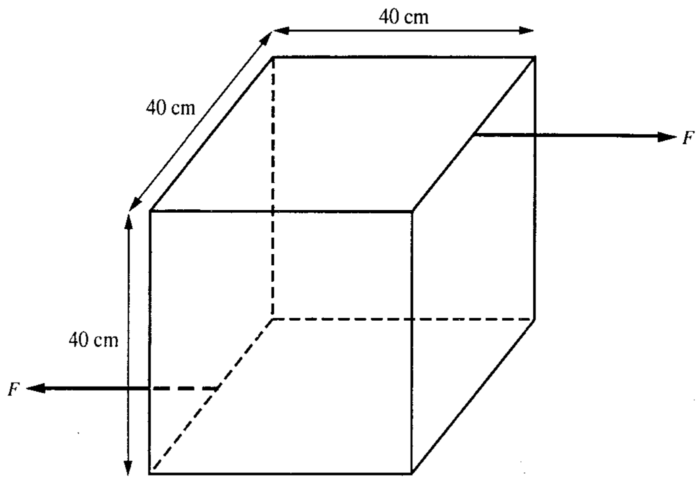

Fig. 11-3

and opposite force, since the block remains in equilibrium. Then,

$$\text{Shear stress} = \frac{6\ \text{kN}}{(0.40\ \text{m})^2} = 37.5\ \text{kPa}$$

$$\text{Shear strain} = 0.00036\,^\circ \left(\frac{\pi\ \text{rad}}{180\,^\circ}\right) = 6.28 \times 10^{-6}$$

(*b*)
$$S = \frac{37{,}500\ \text{Pa}}{6.28 \times 10^{-6}} = 5.97\ \text{GPa}$$

(*c*) If the force is the same but the area increases ninefold, as it does for our case, the shear stress decreases by a factor of 9. Since S stays the same, the shear strain decreases by the same factor, and $\phi' = \phi/9 = 0.00004\,^\circ$.

Twisting Deformation

Twisting a rod or a wire preserves its volume and indeed is a form of shear stress. Consider the situation in Fig. 11-4. When a torque Γ about the axis of the wire is applied at one end, and the other end is kept fixed (i.e., has an equal and opposite torque exerted on it), it can be shown that the angle of twist θ of one end of the wire relative to the other end is proportional to Γ. In fact, $\Gamma = \delta\theta$, where θ is in radians and the **torsion constant** δ is given by

$$\delta = \frac{\pi S R^4}{2L} \tag{11.3}$$

Here, S is the shear modulus for the material from which the wire is made. The units of δ are just those of torque, N · m.

By Newton's third law, the torque, Γ_w exerted by the wire on the external system is equal and opposite to the torque Γ exerted by the external system on the wire:

$$\Gamma_w = -\delta\theta \tag{11.4}$$

Problem 11.7. The steel driveshaft of an engine is 5.0 cm in diameter and 3.0 m long. The shear modulus for steel is 84 GPa.

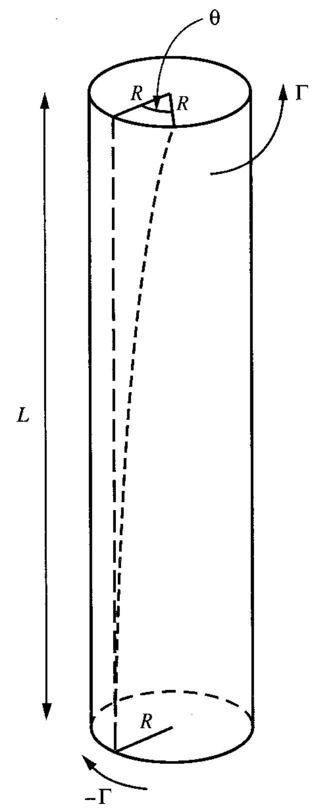

Fig. 11-4

(*a*) Find the torsion constant for the driveshaft.

(*b*) If one end is held fixed, what torque at the other end will give a twist angle of 22 °?

Solution

(*a*) From (*11.3*),

$$\delta = \frac{(3.14)(84 \times 10^9\ \text{Pa})(0.025\ \text{m})^4}{6.0\ \text{m}} = 17.2\ \text{kN} \cdot \text{m}$$

(*b*) $\Gamma = \delta\theta$. We must convert θ to radians, so $\theta = 22\ ° (\pi/180°) = 0.384$ rad. Then, $\Gamma = (17.2\ \text{kN} \cdot \text{m})$ $(0.384) = 6.61\ \text{kN} \cdot \text{m}$.)

Problem 11.8.

(*a*) How would the results of parts (*a*) and (*b*) of Problem 11.7 change if the length were doubled and all else remained the same?

(*b*) What if the radius doubled instead and all else remained the same?

Solution

(*a*) From Eq. (*11.3*), we see that the torsion constant δ varies inversely with the length. Thus doubling the length halves δ, and we have $\delta' = 8.6\ \text{kN} \cdot \text{m}$. Similarly, for the same angle of twist we have that the new torque $\Gamma' = 3.3\ \text{kN} \cdot \text{m}$, half the old torque Γ.

(b) The torsion constant δ varies as the fourth power of R. Doubling R increases δ 16-fold, so $\delta' = 275.2\ \text{kN}\cdot\text{m}$. Then, for a 22° twist, $\Gamma' = 105.7\ \text{kN}\cdot\text{m}$.

Problem 11.9.

(a) A thin wire is hung vertically from the ceiling, and the lower end is tied to the middle of a horizontal crossbar of length 10 cm. When equal and opposite horizontal forces of magnitude 0.50 N are applied, and maintained, at right angles to the ends of the crossbar, the system reaches equilibrium when the bar has rotated through 65°. Find the torsion constant of the wire.

(b) If the wire is replaced by one of the same material and cross section, but twice as long, what is now the angle of rotation?

Solution

(a)
$$\delta = \frac{\Gamma}{\theta} = \frac{2(0.50\ \text{N})(0.05\ \text{m})}{65°(\pi/180°)} = 0.0441\ \text{N}\cdot\text{m}$$

(b) The torsion would be halved, and so for the same applied torque the angle of rotation would be doubled, to 130°.

Spring Deformation

A more complicated deformation is undergone by a stiff wire shaped into a spiral (in other words, a *coil spring*) that is pulled at one end, as shown in Fig. 11-5. Here the elongation x of the spring is primarily due to shear strain throughout the coiled wire and is therefore proportional to the applied force F. Thus we again have Hooke's law, $F = kx$, where k, the effective force constant, depends on the shear modulus and the geometry of the spring, and where x is the amount by which the length of the spring is changed from the unstretched position. Often one expresses Hooke's law not in terms of the external force F giving rise to the strain, but rather in terms of the force F_{sp} that the stretched spring exerts on the system pulling it. Since by Newton's third law $F_{sp} = -F$, we have

$$F_{sp} = -kx \tag{11.5}$$

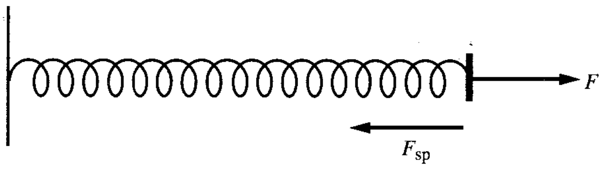

Fig. 11-5

in evident analogy to (*11.4*). Equation (*11.5*) shows F_{sp} to be a *restoring force*: If x is positive (a stretch), the force is in the negative direction, trying to pull the spring back to the unstretched position. If x is negative (a compression), the force is in the positive direction, now trying to push back to the uncompressed position.

Problem 11.10. A light spring hangs vertically from the ceiling. A block of mass 40 kg is attached to the lower end of the spring, and the system is slowly let down until it comes to rest in the equilibrium position. (Fig. 11-6).

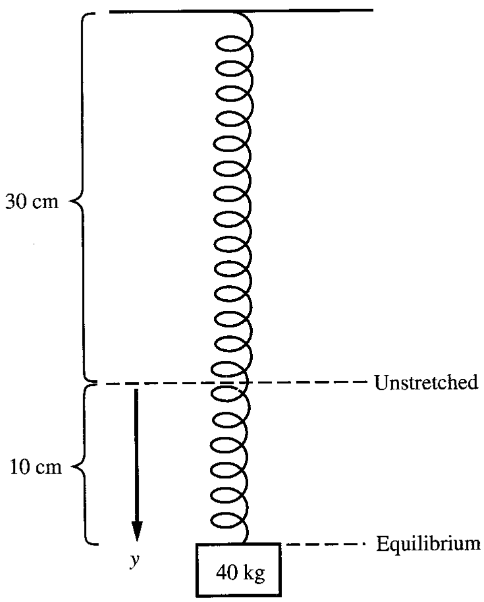

Fig. 11-6

(*a*) Determine the force constant of the spring.

(*b*) What is the force exerted by the spring on the block if the block is (i) pulled down 10 cm from the equilibrium position? (ii) pushed up 15 cm from the equilibrium position?

Solution

(*a*) We measure positive y downward from the unstretched position, and $F_{sp} = -ky$. For equilibrium of the block, we have $mg - ky = 0$, so that

$$k = \frac{mg}{y} = \frac{(40\,\text{kg})(9.8\,\text{m/s}^2)}{0.10\,\text{m}} = 3920\,\text{N/m}$$

(*b*) $F_{sp} = -ky$. In the first case $y = 20$ cm, so

(i) $$F_{sp} = -(3920\,\text{N/m})(0.20\,\text{m}) = -784\,\text{N} \quad \text{(i.e., pulling upward)}$$

In the second case, $y = -5.0$ cm, and

(ii) $$F_{sp} = -(3920\,\text{N/m})(-0.050\,\text{m}) = 196\,\text{N} \quad \text{(i.e., pushing downward)}$$

Note. In the last case the spring is actually compressed 5.0 cm and hence pushes downward.

11.3 OVERALL COMPRESSION UNDER UNIFORM PRESSURE

Definitions

By definition, the **pressure** p at a point Q on the surface of an object is given by $\Delta F_n/\Delta A$, where ΔA is the area of a small patch of the surface centered on Q and ΔF_n is the *normal* force on the patch (see Fig. 11-7). The units of pressure are therefore those of stress.

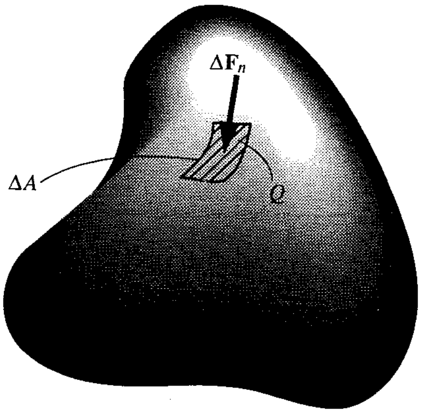

Fig. 11-7

Consider a distribution of normal forces over the surface of the object such that the pressure has the same value at each point of the surface. Let the pressure now increase everywhere by the same amount Δp, and let the corresponding change in volume of the object be ΔV. The change in pressure Δp is called the **volume stress**; the **volume strain** is then defined as $-(\Delta V)/V$, where V is the volume of the object before the change in pressure was applied. (The minus sign appears in the definition because ΔV is negative when Δp is positive, and we want a positive strain to be associated with a positive stress). With these definitions, the volume strain caused by a given volume stress will be independent of the size and shape of the particular object and will depend only on the material of which the object is made. It also turns out that, to a good approximation, Hooke's law again holds:

$$\frac{\text{Stress}}{\text{Strain}} = -\frac{\Delta p}{\Delta V/V} = B \tag{11.6}$$

in which the constant B, the **bulk modulus** of the material, has the dimensions of pressure or stress.

Compressibility of Liquids

Liquids cannot be stretched or sheared like solids, but like solids they are subject to volume compression. In fact, liquids are generally more easily compressed than solids, so a smaller Δp will produce the same fractional change in volume. In other words, bulk moduli will usually be smaller for liquids than for solids. It is often convenient to use the reciprocal of the bulk modulus, which is called the **compressibility** of the liquid:

$$\kappa \equiv \frac{1}{B} = -\frac{\Delta V/V}{\Delta p} \tag{11.7}$$

The various moduli are somewhat temperature-dependent. Table 11.1 gives room-temperature values.

Problem 11.11. Express (*a*) Young's modulus of brass in lb/in^2; (*b*) the compressibility of mercury in ft^2/lb.

Table 11.1

Substance	Y, GPa	S, GPa	B, GPa
Aluminum	70	24	70
Brass	91	36	61
Copper	118	42	140
Glass	55	23	27
Iron	91	70	100
Lead	16	5.6	7.7
Steel	196	84	160
Water	...	...	2.0
Glycerine	...	...	4.5
Carbon disulfide	...	...	1.5
Mercury	...	...	26
Ethyl alcohol	...	...	0.89

Note: 1 GPa = 10^9 Pa.

Solution

(*a*) Using the conversion factors from newtons to pounds and from meters to inches, we obtain from Table 11.1:

$$Y = (91 \times 10^9\ \mathrm{N/m^2})\ \frac{0.225\ \mathrm{lb/N}}{(39.37\ \mathrm{in/m})^2} = 1.32 \times 10^7\ \mathrm{lb/in^2}$$

(*b*)

$$B = (26 \times 10^9\ \mathrm{N/m^2})\ \frac{0.225\ \mathrm{lb/N}}{(3.28\ \mathrm{ft/m})^2} = 5.43 \times 10^8\ \mathrm{lb/ft^2}$$

$$\kappa = \frac{1}{B} = 1.84 \times 10^{-9}\ \mathrm{ft^2/lb}$$

Problem 11.12. A certain copper cube under ordinary atmospheric pressure (about 100 kPa) is 60 mm on a side. What would be the volume change in the cube if (*a*) it were subjected to a pressure increase of 50 MPa? (*b*) The atmospheric pressure were removed?

Solution

(*a*) From (*11.6*) and Table 11.1,

$$\Delta V = -\frac{V\,\Delta p}{B} = -\frac{(0.060\ \mathrm{m})^3\,(50 \times 10^6\ \mathrm{Pa})}{140 \times 10^9\ \mathrm{Pa}} = -77.1\ \mathrm{mm^3}$$

(*b*)

$$\Delta V = -\frac{(0.060\ \mathrm{m})^3\,(-100 \times 10^3\ \mathrm{Pa})}{140 \times 10^9\ \mathrm{Pa}} = +0.154\ \mathrm{mm^3}$$

Problem 11.13. Compute the compressibility of glycerine if a pressure change of 2.0 MPa causes 128 mL of glycerine to change in volume by 0.0563 mL. [The *liter* (L) is the common unit for liquid and gaseous volumes: 1 L = 10^{-3} $\mathrm{m^3}$.]

Solution

From (*11.7*),

$$\kappa = -\frac{(-0.0563)/128}{2.0 \times 10^6 \text{ Pa}} = 2.2 \times 10^{-10} \text{ Pa}^{-1}$$

which is consistent with Table 11.1.

Problems for Review and Mind Stretching

Problem 11.14. An aluminum and a steel wire, each of length 3.0 m and diameter 2.8 mm, are fused together at one end to form a wire of overall length 6.0 m which is hung from a ceiling.

(*a*) Compute the force constants for each wire.

(*b*) If a 40-kg mass is hung from the end of the combined wire, what would the overall elongation be?

(*c*) What is the "effective" force constant for the wire as a whole (i.e., the force constant which a single wire would need so that the same force would cause the same overall stretch)?

Solution

(*a*) We note that the cross-sectional area A of each wire is $A = \pi d^2/4 = (3.14)(2.8 \times 10^{-3})^2/4 = 6.15 \times 10^{-6}$ m^2. Then, recalling the relationship between the force constant and the Young's modulus [from Problem *11.3(b)*)], we have

$$k_{\text{al}} = \frac{Y_{\text{al}}A}{L} = \frac{(0.70 \times 10^{11} \text{ Pa})(6.15 \times 10^{-6} \text{ m}^2)}{3.0 \text{ m}} = 1.43 \times 10^5 \text{ N/m}$$

$$k_{\text{st}} = \frac{Y_{\text{st}}A}{L} = \frac{(1.96 \times 10^{11} \text{ Pa})(6.15 \times 10^{-6} \text{ m}^2)}{3.0 \text{ m}} = 4.02 \times 10^5 \text{ N/m}$$

(*b*) When the weight is hung, each of the two wires will be in equilibrium under the stretching action of equal and opposite forces at its end. These forces all have the same magnitude F, which is equal to the weight of the block: $F = W = Mg = (40 \text{ kg})(9.8 \text{ m/s}^2) = 392$ N. Then, using the results of part (*a*), we can calculate the stretch of each wire:

$$\Delta y_{\text{al}} = \frac{F}{k_{\text{al}}} = \frac{392 \text{ N}}{1.43 \times 10^5 \text{ N/m}} = 2.74 \times 10^{-3} \text{ m}$$

$$\Delta y_{\text{st}} = \frac{F}{k_{\text{st}}} = \frac{392 \text{ N}}{4.02 \times 10^5 \text{ N/m}} = 0.98 \times 10^{-3} \text{ m}$$

The overall stretch is $\Delta y_T = \Delta y_{\text{al}} + \Delta y_{\text{st}} = 3.72 \times 10^{-3}$ m.

(*c*) $k_T = F/\Delta y_T = 392 \text{ N}/(3.72 \times 10^{-3} \text{ m}) = 1.05 \times 10^5$ N/m.

Problem 11.15. When any two wires (or rods or coil springs), each of arbitrary length and cross section, are attached end to end to form a single unit, as in Problem 11.14, the effective (or total) force constant k_T is defined by $F = k_T \Delta L_T$, where F is the applied force and ΔL_T is the overall elongation of the system.

(*a*) Find a general formula for the effective force constant in terms of the force constants of the individual wires (rods, or coil springs).

(*b*) Apply this result to obtain the answer to Problem 11.14(*c*).

Solution

(*a*) By definition, $\Delta y_T = \Delta y_1 + \Delta y_2$ so that $F/k_T = F/k_1 + F/k_2$, where 1 and 2 refer to the individual components. Dividing both sides by F we get

$$\frac{1}{k_T} = \frac{1}{k_1} + \frac{1}{k_2} \tag{i}$$

(*b*) From Problem 11.14(*a*) we have $k_{al} = 1.43 \times 10^5$ N/m; $k_{st} = 4.02 \times 10^5$ N/m. Then $1/k_T = 1/(1.43 \times 10^5 \text{ N/m}) + 1/(4.02 \times 10^5 \text{ N/m}) \Rightarrow k_T = 1.05 \times 10^5$ N/m.

Problem 11.16. Assume the torsion constant for the driveshaft of a truck is $\delta = 20{,}000$ N · m. If the truck's engine delivers power $P = 105$ kW to the driveshaft which is rotating at a constant frequency of $f = 600$ r/min, find the angle of twist between the two ends of the driveshaft.

Solution

First we need to determine the torque exerted on the driveshaft by the engine. We recall from Chap. 10 that $P = \Gamma\omega$. We have that $\omega = 2\pi f = 2(3.14)$ (600 r/min) (1/60 s/min) = 62.8 rad/s. Then $\Gamma = P/\omega = (1.05 \times 10^5 \text{ W})/(62.8 \text{ rad/s}) = 1670$ N · m. Next we recall that $\Gamma = \delta\theta$, from which we get

$$\theta = \frac{1670 \text{ N} \cdot \text{m}}{20{,}000 \text{ N} \cdot \text{m}} = 0.0835 \text{ rad} = 4.78\,^\circ$$

Problem 11.17. Two containers, labeled 1 and 2, contain equal volumes of glycerine and ethyl alcohol, respectively. Both liquids are under an atmospheric pressure of 100 kPa. When the pressure on both liquids is increased by the same amount, Δp, the difference in volume of the liquids becomes 0.25 L. If the original volume of each liquid was 2000 L, find Δp.

Solution

For a given Δp, we have $\Delta V = -(V\,\Delta p)(1/B)$. We know the values of B and V for each liquid, but we are given only the difference in the ΔV's for the two liquids: $|\Delta V_2 - \Delta V_1| = 0.25$ L. Thus, by difference,

$$|\Delta V_2 - \Delta V_1| = (V\,\Delta p)\left|\frac{1}{B_2} - \frac{1}{B_1}\right|$$

$$0.25 \text{ L} = (2000 \text{ L})(\Delta p)\left|\frac{1}{0.89 \text{ GPa}} - \frac{1}{4.5 \text{ GPa}}\right|$$

$$\Delta p = 0.000139 \text{ GPa} = 139 \text{ kPa}$$

Problem 11.18. A printer stand having four solid aluminum legs is firmly epoxied to the top of a rigid table. Each leg has cross-sectional area $A = 5.0$ cm^2 and height $L = 10.0$ cm. A student, not realizing it is epoxied, tries to move the stand by pulling horizontally at the middle of one edge, as shown in Fig. 11-8. The student pulls harder and harder until he exerts a force of 160 N. Assuming the epoxy holds, how far will the top of the stand move?

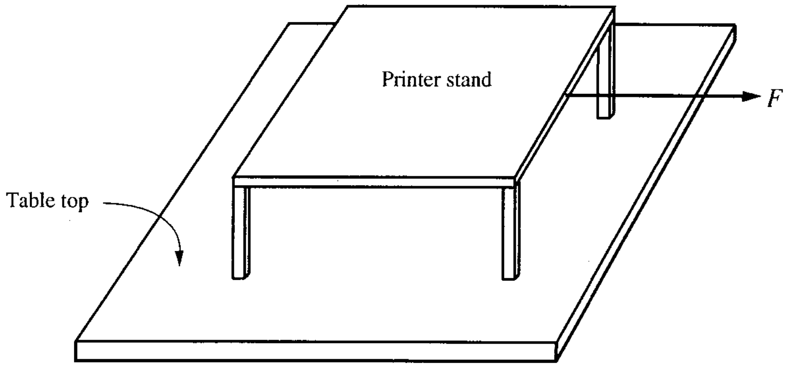

Fig. 11-8

Solution

Assume that the horizontal shear force on the top of each leg is $F = 40$ N. The top surface of each leg then moves by an amount Δx relative to the bottom surface (cf. Fig. 11-2). For each leg, (*11-2*) gives

$$\phi = \frac{\Delta x}{L} = \frac{F}{AS} \qquad \text{or} \qquad \Delta x = \frac{FL}{AS}$$

Substituting in the values of F, L, and A given above and the value of S for aluminum from Table 11.1, we get

$$\Delta x = \frac{(40\ \mathrm{N})(0.10\ \mathrm{m})}{(5.0 \times 10^{-4}\ \mathrm{m}^2)(24 \times 10^9\ \mathrm{N/m}^2)} = 0.333\ \mu\mathrm{m}$$

or about one-hundredth the thickness of a human hair!

Problem 11.19. A steel wire of length 1.0 m and cross-sectional area 1.0 mm^2 is tied horizontally between two rigid hooks. A heavy weight W is suspended from the middle of the wire, causing it to sag as shown in Fig. 11-9. If $\theta = 10°$, find the weight W.

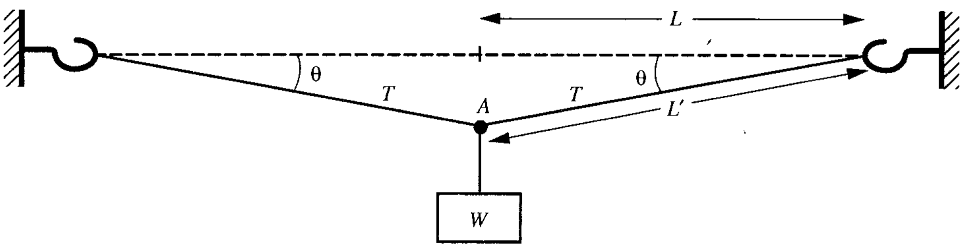

Fig. 11-9

Solution

Consider one of the halves of the wire, of unstretched length $L = 0.50$ m and under tension T. Then its stretched length is $L' = L/(\cos\theta)$, so

$$\Delta L = L' - L = L\left(\frac{1}{\cos\theta} - 1\right) = 0.0077\ \mathrm{m}$$

From Hooke's law,

$$T = YA\frac{\Delta L}{L} = (196 \times 10^9 \text{ Pa})(1.0 \times 10^{-6} \text{ m}^2)\frac{0.0077 \text{ m}}{0.50 \text{ m}} = 3020 \text{ N}$$

Finally, since the junction point A is in equilibrium, $W = 2T \sin\theta = 1050$ N.

Supplementary Problems

Problem 11.20.

(*a*) An iron rod of length 5.0 m hangs vertically, with its top end firmly attached to a rigid ceiling beam. A 10-kN weight is hung from the lower end, and the rod is observed to stretch by 6.0 mm. Find the cross-sectional area of the rod.

(*b*) How would the answer change if the rod were steel?

(*c*) What is the ultimate strength of the iron rod if it breaks under a weight of 25 kN?

Ans. (*a*) 91.6 mm^2; (*b*) 42.5 mm^2; (*c*) 0.273 GPa

Problem 11.21.

(*a*) Find the force constants for the iron and steel rods of Problem 11.20.

(*b*) If the steel rod had the same cross section as the iron rod, what would its force constant be?

Ans. (*a*) $k_{ir} = k_{st} = 1670$ kN/m; (*b*) 3590 kN/m

Problem 11.22.

(*a*) If the iron rod of Problem 11.20 and the steel rod of Problem 11.21(*b*) were attached end to end to form a single rod of length 10.0 m, what would the effective force constant be?

(*b*) What force would increase the length of this combined rod by 8.0 mm?

Ans. (*a*) 1140 kN/m; (*b*) 9.12 kN

Problem 11.23. A light, rigid, horizontal bar is suspended from its ends by two equally long copper wires of cross-sectional areas A_1 and A_2, as shown in Fig. 11-10. A weight of 1200 N is then hung one-third of the way from the left end.

(*a*) If $A_1 = 2.0$ mm^2, what must A_2 be for the two wires to stretch by equal amounts?

(*b*) If each wire had an original length of 0.80 m, by what distance was the bar lowered when the weight was hung?

Ans. (*a*) 1.0 mm^2; (*b*) 2.7 mm

Problem 11.24. A steel rod of cross-sectional area 2.5 in^2 and length 3.0 ft is compressed by a force of 3000 lb. Find the distance of compression.

Ans. 0.0015 in

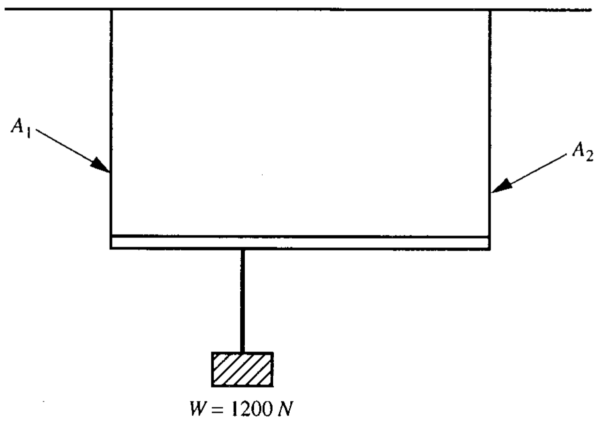

Fig. 11-10

Problem 11.25. A 20-mm-thick brass disk of radius 50 mm has one face rigidly attached to a vertical wall, while the other face is attached to a 20,000-kg lead plate that is otherwise unsupported. By what vertical distance will one face of the disk be displaced relative to the other?

Ans. 20.8 μm

Problem 11.26. The torsion constant of a cable is 300 lb · ft.

(*a*) What torque at one end of the cable is necessary to twist it through 360°?

(*b*) What torque would be necessary if the cable were twice as thick and twice as long?

Ans. (*a*) 1885 lb · ft; (*b*) 15,080 lb · ft

Problem 11.27. Suppose the cable in Problem 11.26(*a*) hangs from the ceiling, with its lower end attached to the center of a horizontal uniform disk of moment of inertia $I = 20$ slug · ft^2. If the disk is rotated through 360° and then released, find its angular acceleration (*a*) at the instant of release, (*b*) after it had rotated back through 180°

Ans. (*a*) 94 rad/s^2; (*b*) 47 rad/s^2

Problem 11.28. A coil spring, of force constant $k = 1000$ N/m, is at rest on a horizontal frictionless surface. One end is connected to a wall and the other end is connected to a block of mass $M = 30$ kg. Suppose the block is pulled back 20 cm and released.

(*a*) Find the acceleration of the block at the moment of release.

(*b*) How far from the unstretched position will the block be when the acceleration is 0.20 m/s^2?

Ans. (*a*) 6.67 m/s^2; (*b*) 0.60 cm

Problem 11.29. Two metal plates are connected by aluminum rivets, each of cross-sectional area 180 mm^2. The plates are expected to experience a shearing force of 120 kN. If each rivet can withstand a maximum shear stress of 70 MPa, and the rivets share the stress equally, what is the minimum number of rivets necessary?

Ans. 10

Problem 11.30.

(*a*) What increase in pressure is necessary to reduce a 100-mL volume of water by 0.002%?

(*b*) What initial volume of glycerine would give the same absolute change in volume under the same pressure increase?

Ans. (*a*) 40 kPa; (*b*) 225 mL

Problem 11.31. An iron rod of length 3.0 m and cross-sectional area 1200 mm^2 is brought from the earth's surface (where atmospheric pressure is 100 kPa) to a location on the ocean floor where the pressure is 10 MPa.

(*a*) All else being the same, what is the decrease in the volume of the bar?

(*b*) What is the decrease in the length of the bar?

[*Hint*: The fractional change in any linear dimension of the bar is one-third the fractional change in the volume.]

(*c*) What stretching force would be necessary to return the bar to its original length?

Ans. (*a*) 356 mm^3; (*b*) 0.100 mm; (*c*) 3.60 kN