BOOK IV

AREAS OF POLYGONS

313. Unit of Surface. A square the side of which is a unit of length is called a *unit of surface.*

Thus a square that is 1 inch long is 1 square inch, and a square that is 1 mile long is 1 square mile. If we are measuring the dimensions of a room in feet, we measure the surface of the floor in square feet. In the same way we may measure the page of this book in square inches and the area of a state in square miles.

314. Area of a Surface. The measure of a surface, expressed in units of surface, is called its *area.*

If a room is 20 feet long and 15 feet wide, the floor contains 300 square feet. Therefore the area of the floor is 300 square feet. Usually the two sides of a rectangle are not commensurable, although by means of fractions we may measure them to any required degree of approximation. The incommensurable cases in theorems like Prop. I of this Book may be omitted without interfering with the sequence of the course.

315. Equivalent Figures. Plane figures that have equal areas are said to be *equivalent.*

In propositions relating to areas the words *rectangle*, *triangle*, etc., are often used for *area of rectangle*, *area of triangle*, etc.

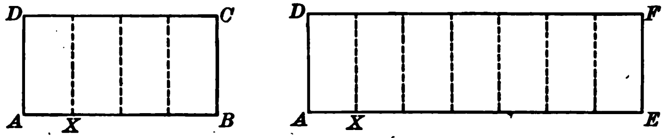
Since congruent figures may be made to coincide, congruent figures are manifestly equivalent.

Because their areas are equal, equivalent figures are frequently spoken of as equal figures. The symbol $=$ is used both for "equivalent" and for "congruent," the sense determining which meaning is to be assigned to it. Occasionally these symbols are used: $\triangleq$, $\cong$, or $\equiv$ for congruent, $=$ for equal, and $\backsimeq$ for equivalent.

Since the word *congruent* means "identically equal," the word *equal* is often used to mean "equivalent."

Proposition I. Theorem

316. *Two rectangles having equal altitudes are to each other as their bases.*



Given the rectangles *AC* and *AF*, having equal altitudes *AD*.

To prove that $\square AC : \square AF =$ *base* AB : *base* AE.

Case 1. *When AB and AE are commensurable.*

Proof. Suppose AB and AE have a common measure, as AX. Suppose AX is contained m times in AB and n times in AE.

Then $$AB : AE = m : n.$$

(*For m and n are the numerical measures of AB and AE.*)

Apply AX as a unit of measure to AB and AE, and at the several points of division erect ⊥s.

These ⊥s are all ⊥ to the upper bases, § 97

and these ⊥s are all equal. § 128

Since to each base equal to AX there is one rectangle,

$\therefore \square AC$ is divided into m rectangles,

and $\square AF$ is divided into n rectangles. § 119

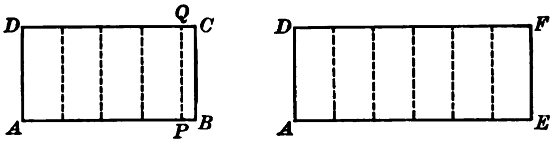
These rectangles are all congruent. § 133

$\therefore \square AC : \square AF = m : n.$

$\therefore \square AC : \square AF = AB : AE$, by Ax. 8. Q.E.D.

In this proposition we again meet the incommensurable case, as on pages 116 and 157. This case is considered on page 193 and may be omitted without destroying the sequence of the propositions.

CASE 2. *When AB and AE are incommensurable.*



Proof. Divide AE into any number of equal parts, and apply one of these parts to AB as many times as AB will contain it.

Since AB and AE are incommensurable, a certain number of these parts will extend from A to some point P, leaving a remainder PB less than one of them. Draw $PQ \perp$ to AB.

Then $$\frac{\square AQ}{\square AF} = \frac{AP}{AE}.$$ Case 1

By increasing the *number* of equal parts into which AE is divided we can diminish the *length* of each, and therefore can make PB less than any assigned positive value, however small.

Hence PB approaches zero as a limit, as the number of parts of AE is indefinitely increased, and at the same time the corresponding $\square PC$ approaches zero as a limit. § 204

Therefore AP approaches AB as a limit, and $\square AQ$ approaches $\square AC$ as a limit.

$\therefore$ the variable $\frac{AP}{AE}$ approaches $\frac{AB}{AE}$ as a limit,

and the variable $\frac{\square AQ}{\square AF}$ approaches $\frac{\square AC}{\square AF}$ as a limit.

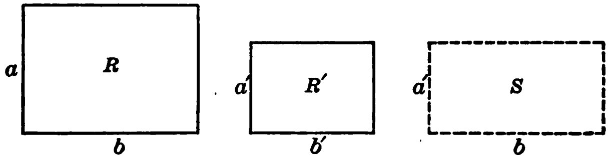
But $\frac{AP}{AE}$ is always equal to $\frac{\square AQ}{\square AF}$, as AP varies in value and approaches AB as a limit. Case 1

$$\therefore \frac{\square AC}{\square AF} = \frac{AB}{AE}, \text{ by § 207.}$$ Q.E.D.

317. COROLLARY. *Two rectangles having equal bases are to each other as their altitudes.*

Proposition II. Theorem

318. *Two rectangles are to each other as the products of their bases by their altitudes.*



Given the rectangles R and R', having for the numerical measure of their bases b and b', and of their altitudes a and a' respectively.

To prove that $$\frac{R}{R'}=\frac{ab}{a'b'}.$$

Proof. Construct the rectangle S, with its base equal to that of R, and its altitude equal to that of R'.

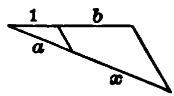
Then $$\frac{R}{S}=\frac{a}{a'},$$ § 317

and $$\frac{S}{R'}=\frac{b}{b'}.$$ § 316

Since we are considering areas, we may treat R, R', and S as numbers and take the products of the corresponding members of these equations. § 272

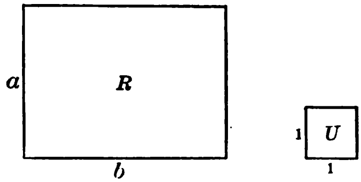
We therefore have $\frac{R}{R'}=\frac{ab}{a'b'}$, by Ax. 3. Q. E. D.

319. Products of Lines. When we speak of the product of a and b we mean the product of their numerical values. It is possible, however, to think of a line as the product of two lines, by changing the definition of multiplication. Thus in this figure in which two parallels are cut by two intersecting transversals, we have $1:a=b:x$. Therefore $x=ab$. In the same way we may find xc, or abc, the product of three lines.



Proposition III. Theorem

320. *The area of a rectangle is equal to the product of its base by its altitude.*



Given the rectangle R, having for the numerical measure of its base and altitude b and a respectively.

To prove that *the area of* $R = ab$.

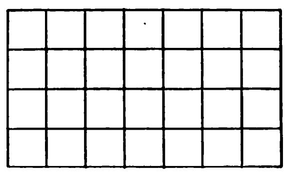
Proof. Let U be the unit of surface. § 313

Then $$\frac{R}{U} = \frac{ab}{1 \times 1} = ab.$$ § 318

But $\frac{R}{U}$ = the *number* of units of surface in R, i.e. the area of R. § 314

$\therefore$ the area of $R = ab$, by Ax. 8. Q.E.D.

321. Practical Measures. When the base and altitude both contain the linear unit an integral number of times, this proposition is rendered evident by dividing the rectangle into squares, each equal to the unit of surface.



Thus, if the base contains seven linear units and the altitude four, the rectangle may be divided into twenty-eight squares, each equal to the unit of surface. Practically this is the way in which we conceive the measure of all rectangles. Even if the sides are incommensurable, we cannot determine this by any measuring instrument. If they seem to be incommensurable with a unit of a thousandth of an inch, they might not seem to be incommensurable with a unit of a millionth of an inch.

EXERCISE 50

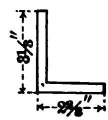
1. A square and a rectangle have equal perimeters, 144 yd., and the length of the rectangle is five times the breadth. Compare the areas of the square and rectangle.

2. On a certain map the linear scale is 1 in. to 10 mi. How many acres are represented by a square $\frac{3}{4}$ in. on a side?

3. Find the ratio of a lot 90 ft. long by 60 ft. wide to a field 40 rd. long by 20 rd. wide.

4. Find the area of a gravel walk 3 ft. 6 in. wide, which surrounds a rectangular plot of grass 40 ft. long and 25 ft. wide. Make a drawing to scale before beginning to compute.

5. Find the number of square inches in this cross section of an L beam, the thickness being $\frac{1}{3}$ in.



6. What is the perimeter of a square field that contains exactly an acre?

7. A machine for planing iron plates planes a space $\frac{1}{2}$ in. wide and 18 ft. long in 1 min. How long will it take to plane a plate 22 ft. 6 in. long and 4 ft. 6 in. wide, allowing 51 min. for adjusting the machine?

8. How many tiles, each 8 in. square, will it take to cover a floor 24 ft. 8 in. long by 16 ft. wide?

9. A rectangle having an area of 48 sq. in. is three times as long as wide. What are the dimensions?

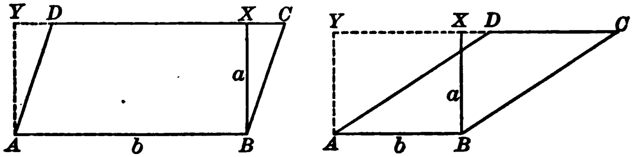
10. The length of a rectangle is four times the width. If the perimeter is 60 ft., what is the area?

11. From two adjacent sides of a rectangular field 60 rd. long and 40 rd. wide a road is cut 4 rd. wide. How many acres are cut off for the road?

12. From one end of a rectangular sheet of iron 10 in. long a square piece is cut off leaving 25 sq. in. in the rest of the sheet. How wide is the sheet?

Proposition IV. Theorem

322. *The area of a parallelogram is equal to the product of its base by its altitude.*



Given the parallelogram *ABCD*, with base *b* and altitude *a*.

To prove that the area of the $\square ABCD = ab$.

Proof. From B draw $BX \perp$ to CD or to CD produced, and from A draw $AY \perp$ to CD produced.

Then $ABXY$ is a rectangle, with base b and altitude a.

Since $AY = BX$, and $AD = BC$, § 125

$\therefore$ the rt. $\triangle$s ADY and BCX are congruent. § 89

From $ABCY$ take the $\triangle BCX$; the $\square ABXY$ is left.

From $ABCY$ take the $\triangle ADY$; the $\square ABCD$ is left,

$\therefore \square ABXY = \square ABCD$. Ax. 2

But the area of the $\square ABXY = ab$. § 320

$\therefore$ the area of the $\square ABCD = ab$, by Ax. 8. Q.E.D.

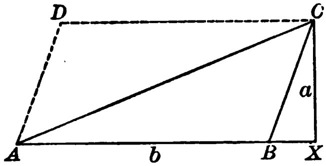
323. Corollary 1. *Parallelograms having equal bases and equal altitudes are equivalent.*

324. Corollary 2. *Parallelograms having equal bases are to each other as their altitudes; parallelograms having equal altitudes are to each other as their bases; any two parallelograms are to each other as the products of their bases by their altitudes.*

This was regarded as very interesting by the ancients, since an ignorant person might think it impossible that the areas of two parallelograms could remain the same although their perimeters differed without limit.

Proposition V. Theorem

325. *The area of a triangle is equal to half the product of its base by its altitude.*



Given the triangle *ABC*, with altitude *a* and base *b*.

To prove that the area of the $\triangle ABC = \frac{1}{2}ab$.

Proof. With AB and BC as adjacent sides construct the parallelogram $ABCD$. § 238

Then $\triangle ABC = \frac{1}{2} \square ABCD$. § 126

But the area of the $\square ABCD = ab$. § 322

$\therefore$ the area of the $\triangle ABC = \frac{1}{2}ab$, by Ax. 4. Q.E.D.

326. Corollary 1. *Triangles having equal bases and equal altitudes are equivalent.*

327. Corollary 2. *Triangles having equal bases are to each other as their altitudes; triangles having equal altitudes are to each other as their bases; any two triangles are to each other as the products of their bases by their altitudes.*

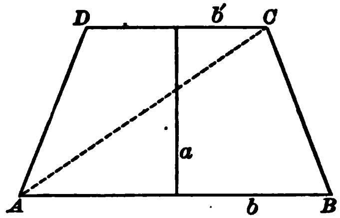
Has this been proved for rectangles? What is the relation of a triangle to a rectangle of equal base and equal altitude? What must then be the relations of triangles to one another?

328. Corollary 3. *The product of the sides of a right triangle is equal to the product of the hypotenuse by the altitude from the vertex of the right angle.*

How is the area of a right triangle found in terms of the sides of the right angle? in terms of the hypotenuse and altitude? How do these results compare?

PROPOSITION VI. THEOREM

329. *The area of a trapezoid is equal to half the product of the sum of its bases by its altitude.*



Given the trapezoid *ABCD*, with bases *b* and *b′* and altitude *a*.

To prove that the area of $ABCD = \frac{1}{2}a(b+b')$.

Proof. Draw the diagonal AC.

Then the area of the $\triangle ABC = \frac{1}{2}ab$,

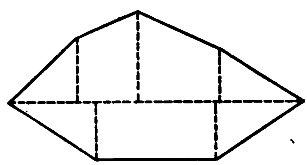
and the area of the $\triangle ACD = \frac{1}{2}ab'$. § 325

$\therefore$ the area of $ABCD = \frac{1}{2}a(b+b')$, by Ax. 1. Q.E.D.

330. COROLLARY. *The area of a trapezoid is equal to the product of the line joining the mid-points of its nonparallel sides by its altitude.*

How is the line joining the mid-points of the nonparallel sides related to the sum of the bases (§ 137)?

331. Area of an Irregular Polygon. The area of an irregular polygon may be found by dividing the polygon into triangles, and then finding the area of each of these triangles separately.



A common method used in land surveying is as follows: Draw the longest diagonal, and let fall perpendiculars upon this diagonal from the other vertices of the polygon. The sum of the right triangles, rectangles, and trapezoids is equivalent to the polygon.

EXERCISE 51

Find the areas of the parallelograms whose bases and altitudes are respectively as follows:

1. 2.25 in., $1\frac{1}{3}$ in.
2. 3.44 in., $1\frac{1}{2}$ in.
3. 2.7 ft., 1.2 ft.
4. 5.6 ft., 2.3 ft.
5. 2 ft. 3 in., 7 in.
6. 3 ft. 6 in., 2 ft.

Find the areas of the triangles whose bases and altitudes are respectively as follows:

7. 1.4 in., $1\frac{1}{2}$ in.
8. 2.5 in., 0.8 in.
9. $6\frac{1}{2}$ ft., 3 ft.
10. 5.4 ft., 1.2 ft.
11. 1 ft. 6 in., 8 in.
12. 3 ft. 8 in., 3 ft.

Find the areas of the trapezoids whose bases are the first two of the following numbers, and whose altitudes are the third numbers:

13. 2 ft., 1 ft., 6 in.
14. $2\frac{1}{2}$ ft., $1\frac{1}{4}$ ft., 9 in.
15. 3 ft. 7 in., 2 ft., 14 in.
16. 5 ft. 6 in., 3 ft., 2 ft.

Find the altitudes of the parallelograms whose areas and bases are respectively as follows:

17. 10 sq. in., 5 in.
18. 6 sq. in., 6 in.
19. 28 sq. ft., 7 ft.
20. 27 sq. ft., 6 ft.
21. 30 sq. ft., 12 ft.
22. 80 sq. in., 16 in.

Find the altitudes of the triangles whose areas and bases are respectively as follows:

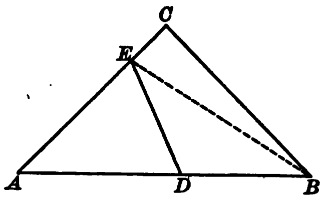
23. 49 sq. in., 14 in.
24. 48 sq. in., 12 in.
25. 50 sq. ft., 10 ft.
26. 160 sq. ft., 20 ft.
27. 110 sq. yd., 10 yd.
28. 176 sq. yd., 32 yd.

Find the altitudes of the trapezoids whose areas and bases are respectively as follows:

29. 33 sq. in., 5 in., 6 in.
30. 15 sq. in., 4 in., 6 in.
31. 13 sq. ft., 9 ft., 5 ft.
32. 70 sq. yd., 9 yd., 11 yd.

Proposition VII. Theorem

332. *The areas of two triangles that have an angle of the one equal to an angle of the other are to each other as the products of the sides including the equal angles.*



Given the triangles *ABC* and *ADE*, with the common angle *A*.

To prove that $$\frac{\triangle ABC}{\triangle ADE}=\frac{AB\times AC}{AD\times AE}.$$

Proof. Draw BE.

Then $$\frac{\triangle ABC}{\triangle ABE}=\frac{AC}{AE},$$

and $$\frac{\triangle ABE}{\triangle ADE}=\frac{AB}{AD}. \qquad \S\ 327$$

(*Triangles having equal altitudes are to each other as their bases.*)

Since we are considering numerical measures of area and length, we may treat all of the terms of these proportions as numbers.

Taking the product of the first members and the product of the second members of these equations, we have

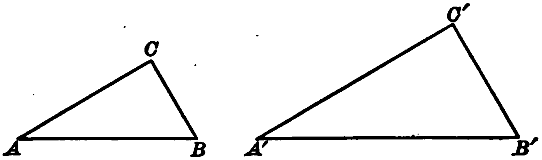
$$\frac{\triangle ABE\times\triangle ABC}{\triangle ADE\times\triangle ABE}=\frac{AB\times AC}{AD\times AE}. \qquad \text{Ax. 3}$$

That is, by canceling $\triangle ABE$, we have the proportion

$$\frac{\triangle ABC}{\triangle ADE}=\frac{AB\times AC}{AD\times AE}. \qquad \text{Q.E.D.}$$

Proposition VIII. Theorem

333. *The areas of two similar triangles are to each other as the squares on any two corresponding sides.*



Given the similar triangles ABC and $A'B'C'$.

To prove that $$\frac{\triangle ABC}{\triangle A'B'C'} = \frac{\overline{AB}^2}{\overline{A'B'}^2}.$$

Proof. Since the triangles are similar, Given

$$\therefore \angle A = \angle A'. \qquad \S\ 282$$

Then $$\frac{\triangle ABC}{\triangle A'B'C'} = \frac{AB \times AC}{A'B' \times A'C'}. \qquad \S\ 332$$

(*The areas of two triangles that have an angle of the one equal to an angle of the other are to each other as the products of the sides including the equal angles.*)

That is, $$\frac{\triangle ABC}{\triangle A'B'C'} = \frac{AB}{A'B'} \times \frac{AC}{A'C'}.$$

But $$\frac{AB}{A'B'} = \frac{AC}{A'C'}. \qquad \S\ 282$$

(*Similar polygons have their corresponding sides proportional.*)

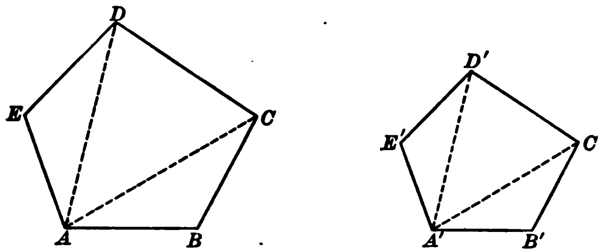
Substituting $\frac{AB}{A'B'}$ for its equal $\frac{AC}{A'C'}$, we have

$$\frac{\triangle ABC}{\triangle A'B'C'} = \frac{AB}{A'B'} \times \frac{AB}{A'B'}, \qquad \text{Ax. 9}$$

or $$\frac{\triangle ABC}{\triangle A'B'C'} = \frac{\overline{AB}^2}{\overline{A'B'}^2}. \qquad \text{Q.E.D.}$$

PROPOSITION IX. THEOREM

334. *The areas of two similar polygons are to each other as the squares on any two corresponding sides.*



Given the similar polygons *ABCDE* and *A'B'C'D'E'*, of area *s* and *s'* respectively.

To prove that $s : s' = \overline{AB}^2 : \overline{A'B'}^2.$

Proof. By drawing all the diagonals from any corresponding vertices A and A', the two similar polygons are divided into similar triangles. § 292

$$\therefore \frac{\triangle ADE}{\triangle A'D'E'} = \frac{\overline{AD}^2}{\overline{A'D'}^2} = \frac{\triangle ACD}{\triangle A'C'D'} = \frac{\overline{AC}^2}{\overline{A'C'}^2} = \frac{\triangle ABC}{\triangle A'B'C'} = \frac{\overline{AB}^2}{\overline{A'B'}^2}. \quad \S\ 333$$

That is, $$\frac{\triangle ADE}{\triangle A'D'E'} = \frac{\triangle ACD}{\triangle A'C'D'} = \frac{\triangle ABC}{\triangle A'B'C'}. \quad \text{Ax. } 8$$

$$\therefore \frac{\triangle ADE + \triangle ACD + \triangle ABC}{\triangle A'D'E' + \triangle A'C'D' + \triangle A'B'C'} = \frac{\triangle ABC}{\triangle A'B'C'} = \frac{\overline{AB}^2}{\overline{A'B'}^2}. \quad \S\ 269$$

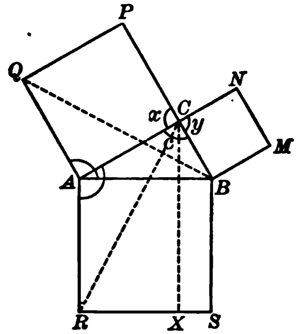
$\therefore s : s' = \overline{AB}^2 : \overline{A'B'}^2$, by Ax. 11. Q.E.D.

335. COROLLARY 1. *The areas of two similar polygons are to each other as the squares on any two corresponding lines.*

336. COROLLARY 2. *Corresponding sides of two similar polygons have the same ratio as the square roots of the areas.*

Proposition X. Theorem

337. *The square on the hypotenuse of a right triangle is equivalent to the sum of the squares on the other two sides.*



Given the right triangle *ABC*, with *AS* the square on the hypotenuse, and *BN*, *CQ* the squares on the other two sides.

To prove that $AS = BN + CQ$.

Proof. Draw CX through $C \parallel$ to BS. § 233

Draw CR and BQ.

Since $\angle$s c and x are rt. $\angle$s, the $\angle PCB$ is a straight angle, § 34

and the line PCB is a straight line. § 43

Similarly, the line ACN is a straight line.

In the $\triangle$s ARC and ABQ,

$AR = AB$,

$AC = AQ$, § 65

and $\angle RAC = \angle BAQ$. Ax. 1

(Each is the sum of a rt. $\angle$ and the $\angle BAC$.)

$\therefore \triangle ARC$ is congruent to $\triangle ABQ$. § 68

Furthermore the $\square AX$ is double the $\triangle ARC$. § 325

(They have the same base AR, and the same altitude RX.)

Again the square CQ is double the $\triangle ABQ$. § 325

(*They have the same base AQ, and the same altitude AC.*)

$\therefore$ the $\square AX$ is equivalent to the square CQ. Ax. 3

In like manner, by drawing CS and AM, it may be proved that the rectangle BX is equivalent to the square BN.

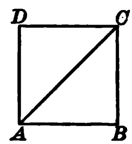
Since square $AS = \square BX + \square AX$, Ax. 11

$\therefore AS = BN + CQ$, by Ax. 9. Q.E.D.

The first proof of this theorem is usually attributed to Pythagoras (about 525 B.C.), although the truth of the proposition was known earlier. It is one of the most important propositions of geometry. Various proofs may be given, but the one here used is the most common. This proof is attributed to Euclid (about 300 B.C.), a famous Greek geometer.

338. Corollary 1. *The square on either side of a right triangle is equivalent to the difference of the square on the hypotenuse and the square on the other side.*

339. Corollary 2. *The diagonal and a side of a square are incommensurable.*



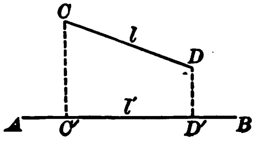
For $\overline{AC}^2 = \overline{AB}^2 + \overline{BC}^2 = 2\,\overline{AB}^2$.

$\therefore AC = AB\sqrt{2}$.

Since $\sqrt{2}$ may be carried to as many decimal places as we please, but cannot be exactly expressed as a rational fraction, it has no common measure with 1. That is, $AC : AB = \sqrt{2}$, an incommensurable number.

340. Projection. If from the extremities of a line-segment perpendiculars are let fall upon another line, the segment thus cut off is called the *projection* of the first line upon the second.

Thus $C'D'$ is the projection of CD upon AB, or l' is the projection of l upon AB.



In general it is convenient to designate by the small letter a the side of a triangle opposite $\angle A$, and so for the other sides; to designate the projection of a by a'; and to designate the height (altitude) by h.

EXERCISE 52

Given the sides of a right triangle as follows, find the hypotenuse to two decimal places:

1. 30 ft., 40 ft.	3. 20 ft., 30 ft.	5. 2 ft. 6 in., 3 ft.
2. 45 ft., 60 ft.	4. 1.5 in., 2.5 in.	6. 3 ft. 8 in., 2 ft.

Given the hypotenuse and one side of a right triangle as follows, find the other side to two decimal places:

7. 50 ft., 40 ft.	9. 10 ft., 6 ft.	11. 3 ft. 4 in., 2 ft.
8. 35 ft., 21 ft.	10. 1.2 in., 0.8 in.	12. 6 ft. 2 in., 5 ft.

13. A ladder 38 ft. 6 in. long is placed against a wall, with its foot 23.1 ft. from the base of the wall. How high does it reach on the wall?

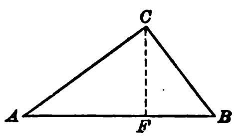
14. Find the altitude of an equilateral triangle with side s.

15. Find the side of an equilateral triangle with altitude h.

16. The area of an equilateral triangle with side s is $\frac{1}{4}s^2\sqrt{3}$.

17. Find the length of the longest chord and of the shortest chord that can be drawn through a point 1 ft. from the center of a circle whose radius is 20 in.

18. The radius of a circle is 5 in. Through a point 3 in. from the center a diameter is drawn, and also a chord perpendicular to the diameter. Find the length of this chord, and the distance (to two decimal places) from one end of the chord to the ends of the diameter.



19. In this figure the angle C is a right angle. From the relations $\overline{AC}^2 = AB \times AF$ (§ 294) and $\overline{CB}^2 = AB \times BF$, show that $\overline{AC}^2 + \overline{CB}^2 = \overline{AB}^2$.

20. If the diagonals of a quadrilateral intersect at right angles, the sum of the squares on one pair of opposite sides is equivalent to the sum of the squares on the other pair.

PROPOSITION XI. THEOREM

341. *In any triangle the square on the side opposite an acute angle is equivalent to the sum of the squares on the other two sides diminished by twice the product of one of those sides by the projection of the other upon that side.*

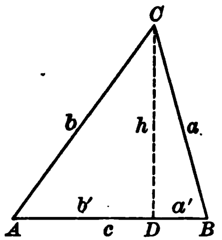


FIG. 1

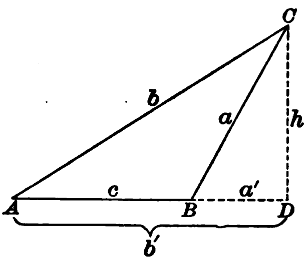


FIG. 2

Given the triangle ABC, A being an acute angle, and a' and b' being the projections of a and b respectively upon c.

To prove that $a^2 = b^2 + c^2 - 2\,b'c.$

Proof. If D, the foot of the $\perp$ from C, falls upon c (Fig. 1),

$$a' = c - b'.$$

If D falls upon c produced (Fig. 2),

$$a' = b' - c.$$

In either case, by squaring, we have

$$a'^2 = b'^2 + c^2 - 2\,b'c. \qquad \text{Ax. 5}$$

Adding h^2 to each side of this equation, we have

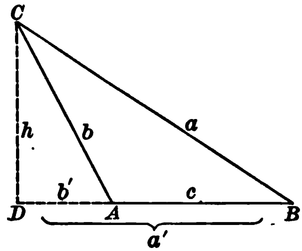
$$h^2 + a'^2 = h^2 + b'^2 + c^2 - 2\,b'c. \qquad \text{Ax. 1}$$

But $h^2 + a'^2 = a^2$, and $h^2 + b'^2 = b^2$. § 337

Putting a^2 and b^2 for their equals in the above equation, we have $a^2 = b^2 + c^2 - 2\,b'c$, by Ax. 9. Q.E.D.

Proposition XII. Theorem

342. *In any obtuse triangle the square on the side opposite the obtuse angle is equivalent to the sum of the squares on the other two sides increased by twice the product of one of those sides by the projection of the other upon that side.*



Given the obtuse triangle *ABC*, *A* being the obtuse angle, and *a'* and *b'* the projections of *a* and *b* respectively upon *c*.

To prove that $a^2 = b^2 + c^2 + 2\,b'c.$

Proof. $a' = b' + c.$ Ax. 11

Squaring, $a'^2 = b'^2 + c^2 + 2\,b'c.$ Ax. 5

Adding h^2 to each side of this equation, we have

$h^2 + a'^2 = h^2 + b'^2 + c^2 + 2\,b'c.$ Ax. 1

But $h^2 + a'^2 = a^2$, and $h^2 + b'^2 = b^2.$ § 337

Putting a^2 and b^2 for their equals in the above equation, we have

$a^2 = b^2 + c^2 + 2\,b'c$, by Ax. 9. Q.E.D.

Discussion. By the Principle of Continuity the last three theorems may be included in one theorem by letting the $\angle A$ change from an acute angle to a right angle and then to an obtuse angle. Let the student explain.

The last three theorems enable us to compute the altitudes of a triangle if the three sides are known; for in Prop. XII we can find b', and from b and b' we can find h.

EXERCISE 53

Find the lengths, to two decimal places, of the diagonals of the squares whose sides are:

1. 7 in. 2. 10 in. 3. 9.2 in. 4. 1 ft. 6 in. 5. 2 ft. 3 in.

Find the lengths, to two decimal places, of the sides of the squares whose diagonals are:

6. 4 in. 7. 8 in. 8. 5 ft. 9. $\sqrt{5}$ in. 10. 2 ft. 6 in.

11. The minute hand and hour hand of a clock are 6 in. and $4\frac{1}{2}$ in. long respectively. How far apart are the ends of the hands at 9 o'clock?

12. A rectangle whose base is 9 and diagonal 15 has the same area as a square whose side is x. Find the value of x.

13. A ring is screwed into a ceiling in a room 10 ft. high. Two rings are screwed into the floor at points 5 ft. and 12 ft. from a point directly beneath the one in the ceiling. Wires are stretched from the ceiling ring to each floor ring. How long are the wires? (Answer to two decimal places.)

14. The sum of the squares on the segments of two perpendicular chords is equivalent to the square on the diameter of the circle.

If AB, CD are the chords, draw the diameter BE, and draw AC, ED, BD. Prove that $AC = ED$.

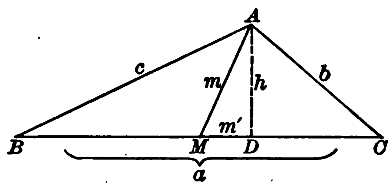
15. The difference of the squares on two sides of a triangle is equivalent to the difference of the squares on the segments of the third side, made by the perpendicular on the third side from the opposite vertex.

16. In an isosceles triangle the square on one of the equal sides is equivalent to the square on any line drawn from the vertex to the base, increased by the product of the segments of the base.

PROPOSITION XIII. THEOREM

343. *The sum of the squares on two sides of a triangle is equivalent to twice the square on half the third side, increased by twice the square on the median upon that side.*

The difference of the squares on two sides of a triangle is equivalent to twice the product of the third side by the projection of the median upon that side.



Given the triangle ABC, the median m, and m' the projection of m upon the side a. Also let c be greater than b.

To prove that 1. $c^2 + b^2 = 2\,\overline{BM}^2 + 2\,m^2$;
2. $c^2 - b^2 = 2\,am'$.

Proof. The $\angle AMB$ is obtuse, and the $\angle CMA$ is acute. § 116

Since $c > b$, M lies between B and D. § 84

Then $c^2 = \overline{BM}^2 + m^2 + 2\,BM \cdot m'$, § 342

and $b^2 = \overline{MC}^2 + m^2 - 2\,MC \cdot m'$. § 341

Adding these equals, and observing that $BM = MC$, we have

$c^2 + b^2 = 2\,\overline{BM}^2 + 2\,m^2$. Ax. 1

Subtracting the second from the first, we have

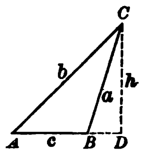
$c^2 - b^2 = 2\,am'$, by Ax. 2. Q.E.D.

Discussion. Consider the proposition when $c = b$.

This theorem may be omitted without interfering with the regular sequence. It enables us to compute the medians when the three sides are known.

EXERCISE 54

1. To compute the area of a triangle in terms of its sides.



At least one of the angles A or B is acute. Suppose A is acute.

In the $\triangle ADC$, $h^2 = b^2 - \overline{AD}^2$. Why?

In the $\triangle ABC$, $a^2 = b^2 + c^2 - 2c \times AD$. Why?

Therefore $AD = \frac{b^2 + c^2 - a^2}{2c}$.

Hence
$$h^2 = b^2 - \frac{(b^2 + c^2 - a^2)^2}{4c^2} = \frac{4b^2c^2 - (b^2 + c^2 - a^2)^2}{4c^2}$$
$$= \frac{(2bc + b^2 + c^2 - a^2)(2bc - b^2 - c^2 + a^2)}{4c^2}$$
$$= \frac{\{(b + c)^2 - a^2\}\{a^2 - (b - c)^2\}}{4c^2}$$
$$= \frac{(a + b + c)(b + c - a)(a + b - c)(a - b + c)}{4c^2}.$$

Let $a + b + c = 2s$, where s stands for semiperimeter.

Then $b + c - a = a + b + c - 2a = 2s - 2a = 2(s - a)$.

Similarly $a + b - c = 2(s - c)$,

and $a - b + c = 2(s - b)$.

Hence
$$h^2 = \frac{2s \times 2(s - a) \times 2(s - b) \times 2(s - c)}{4c^2}.$$

By simplifying, and extracting the square root,
$$h = \frac{2}{c}\sqrt{s(s - a)(s - b)(s - c)}.$$

Hence the area $= \frac{1}{2}ch = \sqrt{s(s - a)(s - b)(s - c)}$.

For example, if the sides are 3, 4, and 5,
$$\text{area} = \sqrt{6(6 - 3)(6 - 4)(6 - 5)} = \sqrt{6 \cdot 3 \cdot 2} = 6.$$

If Ex. 1 has been studied, find the areas, to two decimal places, of the triangles whose sides are:

2. 4, 5, 6. **4.** 6, 8, 10. **6.** 7, 8, 11. **8.** 1.2, 3, 2.1.

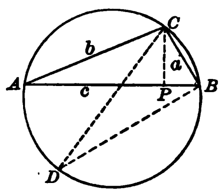
3. 5, 6, 7. **5.** 6, 8, 9. **7.** 9, 10, 11. **9.** 11, 12, 13.

10. To compute the radius of the circle circumscribed about a triangle in terms of the sides of the triangle. (Solve only if § 305 and Ex. 1 have been taken.)

Let CD be a diameter.

By § 305, what do we know about the products $CA \times BC$ and $CD \times CP$?

What does this tell us of ab and $2r \cdot CP$, r being the radius?



From Ex. 1, what does CP equal in terms of the sides?

Is it therefore possible to show that

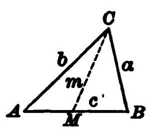
$$r = \frac{abc}{4\sqrt{s(s-a)(s-b)(s-c)}}?$$

If Exs. 1 and 10 have been studied, compute the radii, to two decimal places, of the circles circumscribed about the triangles whose sides are:

11. 3, 4, 5. **12.** 27, 36, 45. **13.** 7, 9, 11. **14.** 10, 11, 12.

15. To compute the medians of a triangle in terms of its sides.

Omit if § 343 has not been taken. What do we know about $a^2 + b^2$ as compared with $2m^2 + 2\left(\frac{c}{2}\right)^2$?



From this relation show that

$$m = \tfrac{1}{2}\sqrt{2(a^2 + b^2) - c^2}.$$

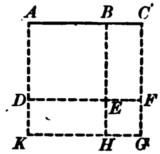
If Ex. 15 has been studied, compute the three medians, to two decimal places, of the triangles whose sides are:

16. 3, 4, 5. **17.** 6, 8, 10. **18.** 6, 7, 8. **19.** 7, 9, 11.

20. If the sides of a triangle are 7, 9, and 11, is the angle opposite the side 11 right, acute, or obtuse?

21. The square constructed upon the sum of two lines is equivalent to the sum of the squares constructed upon these two lines, increased by twice the rectangle of these lines.

Given the two lines *AB* and *BC*, and *AC* their sum. Construct the squares *AKGC* and *ADEB* upon *AC* and *AB* respectively. Produce *BE* and *DE* to meet *KG* and *CG* in *H* and *F* respectively. Then we have the square *EHGF*, with sides each equal to *BC*. Hence the square *AKGC* is the sum of the squares *ADEB* and *EHGF*, and the rectangles *DKHE* and *BEFC*.

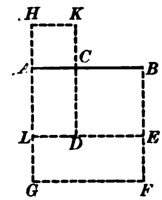


This proves geometrically the algebraic formula

$$(a+b)^2 = a^2 + 2\,ab + b^2.$$

22. The square constructed upon the difference of two lines is equivalent to the sum of the squares constructed upon these two lines, diminished by twice their rectangle.

Given the two lines *AB* and *AC*, and *BC* their difference. Construct the square *AGFB* upon *AB*, the square *ACKH* upon *AC*, and the square *CDEB* upon *BC*. Produce *ED* to meet *AG* in *L*. The dimensions of the rectangles *LGFE* and *HLDK* are *AB* and *AC*, and the square *CDEB* is the difference between the whole figure and the sum of these rectangles.

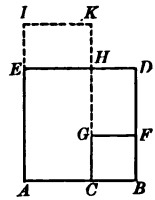


This proves geometrically the algebraic formula

$$(a-b)^2 = a^2 - 2\,ab + b^2.$$

23. The difference between the squares constructed upon two lines is equivalent to the rectangle of the sum and difference of these lines.

Given the squares *ABDE* and *CBFG*, constructed upon *AB* and *BC*. The difference between these squares is the polygon *ACGFDE*, which is composed of the rectangles *ACHE* and *GFDH*. Produce *AE* and *CH* to *I* and *K* respectively, making *EI* and *HK* each equal to *BC*, and draw *IK*. The difference between the squares *ABDE* and *CBFG* is then equivalent to the rectangle *ACKI*, with dimensions *AB* + *BC*, and *AB* − *BC*.

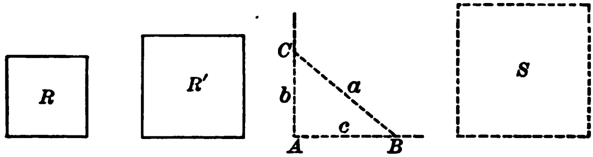


This proves geometrically the algebraic formula

$$a^2 - b^2 = (a+b)(a-b).$$

Proposition XIV. Problem

344. *To construct a square equivalent to the sum of two given squares.*



Given the two squares, R and R'.

Required to construct a square equivalent to $R + R'$.

Construction. Construct the rt. $\angle A$. § 228

On the sides of $\angle A$, take AB, or c, equal to a side of R', and AC, or b, equal to a side of R, and draw BC, or a.

Construct the square S, having a side equal to BC.

Then S is the square required. Q.E.F.

Proof. $a^2 = b^2 + c^2$. § 337

(*The square on the hypotenuse of a rt.* $\triangle$ *is equivalent to the sum of the squares on the other two sides.*)

$\therefore S = R + R'$, by Ax. 9. Q.E.D.

345. Corollary 1. *To construct a square equivalent to the difference of two given squares.*

We may easily reverse the above construction by first drawing c, then erecting a $\perp$ at A, and then with a radius a fixing the point C.

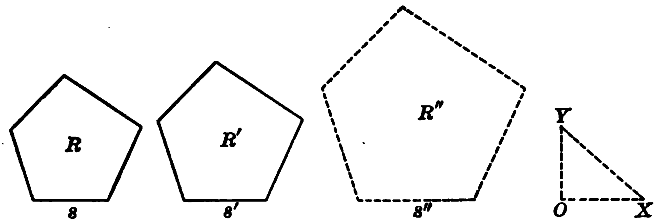
346. Corollary 2. *To construct a square equivalent to the sum of three given squares.*

If a side of the third square is d, we may erect a perpendicular from C to the line BC, take CD equal to d, and join D and B.

Discussion. It is evident that we can continue this process indefinitely, and thus construct a square equivalent to the sum of any number of given squares.

PROPOSITION XV. PROBLEM

347. *To construct a polygon similar to two given similar polygons and equivalent to their sum.*



Given the two similar polygons R and R'.

Required to construct a polygon similar to R and R', and equivalent to $R+R'$.

Construction. Construct the rt. $\angle O$. § 228

Let s and s' be corresponding sides of R and R'.

On the sides of $\angle O$, take OX equal to s', and OY equal to s.

Draw XY, and take s'' equal to XY.

Upon s'', corresponding to s, construct R'' similar to R. § 312

Then R'' is the polygon required. Q.E.F.

Proof. $\overline{OY}^2+\overline{OX}^2=\overline{XY}^2.$ § 337

Putting for OY, OX, and XY their equals s, s', and s'', we have

$$s^2+s'^2=s''^2. \qquad \text{Ax. 9}$$

But
$$\frac{R}{R''}=\frac{s^2}{s''^2},$$

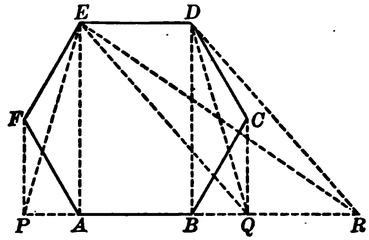
and
$$\frac{R'}{R''}=\frac{s'^2}{s''^2}. \qquad \S\ 334$$

By addition,
$$\frac{R+R'}{R''}=\frac{s^2+s'^2}{s''^2}=1. \qquad \text{Ax. 1}$$

$\therefore R''=R+R'$, by Ax. 3. Q.E.D

Proposition XVI. Problem

348. *To construct a triangle equivalent to a given polygon.*



Given the polygon *ABCDEF*.

Required to construct a triangle equivalent to ABCDEF.

Construction. Let B, C, and D be any three consecutive vertices of the polygon. Draw the diagonal DB.

From C draw a line $\parallel$ to DB. § 233

Produce AB to meet this line at Q, and draw DQ.

Again, draw EQ, and from D draw a line $\parallel$ to EQ, meeting AB produced at R, and draw ER.

In like manner continue to reduce the number of sides of the polygon until we obtain the $\triangle EPR$.

Then $\triangle EPR$ is the triangle required. Q.E.F.

Proof. The polygon $AQDEF$ has one side less than the polygon $ABCDEF$.

Furthermore, in the two polygons, the part $ABDEF$ is common,

and the $\triangle BQD = \triangle BCD$. § 326

(For the base DB is common, and their vertices C and Q are in the line CQ ∥ to the base.)

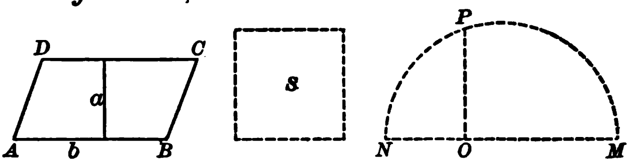
$\therefore AQDEF = ABCDEF$. Ax. 1

In like manner it may be proved that

$AREF = AQDEF$, and $EPR = AREF$. Q.E.D.

Proposition XVII. Problem

349. *To construct a square equivalent to a given parallelogram.*



Given the parallelogram ***ABCD.***

Required to construct a square equivalent to the $\square ABCD$.

Construction. Upon any convenient line take NO equal to a, and OM equal to b, the altitude and base respectively of $\square$ $ABCD$.

Upon NM as a diameter describe a semicircle.

At O erect $OP \perp$ to NM, meeting the circle at P. § 228

Construct the square S, having a side equal to OP.

Then S is the square required. Q.E.F.

Proof. $NO : OP = OP : OM.$ § 297

$\therefore \overline{OP}^2 = NO \times OM.$ § 261

That is, $\overline{OP}^2 = ab.$ Ax. 9

But $S = \overline{OP}^2,$

and $\square ABCD = ab.$ § 322

$\therefore S = \square ABCD$, by Ax. 9. Q.E.D.

350. Corollary 1. *To construct a sqiare equivalent to a given triangle.*

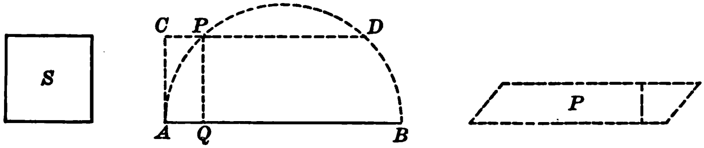
Take for a side of the square the mean proportional between the base and half the altitude of the triangle.

351. Corollary 2. *To construct a square equivalent to a given polygon.*

First reduce the polygon to an equivalent triangle, and then construct a square equivalent to the triangle.

Proposition XVIII. Problem

352. *To construct a parallelogram equivalent to a given square, and having the sum of its base and altitude equal to a given line.*



Given the square *S*, and the line *AB*.

Required to construct a ▱ equivalent to S, with the sum of its base and altitude equal to AB.

Construction. Upon AB as a diameter describe a semicircle.

At A erect $AC \perp$ to AB and equal to a side of the given square S. § 228

Draw $CD \parallel$ to AB, cutting the circle at P. § 233

Draw $PQ \perp$ to AB. § 227

Then any ▱, as P, having AQ for its altitude and QB for its base is equivalent to S. Q.E.F.

Proof. $AQ : PQ = PQ : QB$. § 297

$\therefore \overline{PQ}^2 = AQ \times QB$. § 261

Furthermore PQ is $\parallel$ to CA. § 95

$\therefore PQ = CA$. § 127

$\therefore \overline{PQ}^2 = \overline{CA}^2$. Ax. 5

$\therefore AQ \times QB = \overline{CA}^2$. Ax. 8

But $P = AQ \times QB$, § 322

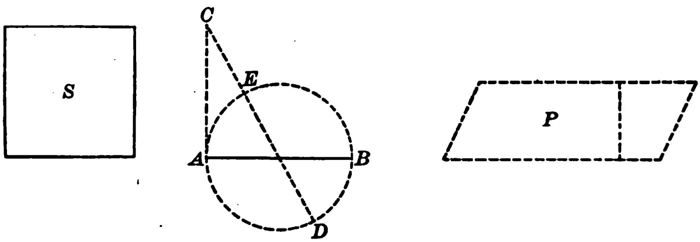
and $S = \overline{CA}^2$. § 320

$\therefore P = S$, by Ax. 8. Q.E.D.

Thus is solved geometrically the algebraic problem, given $x + y = a$, $xy = b$, to find x and y.

PROPOSITION XIX. PROBLEM

353. *To construct a parallelogram equivalent to a given square, and having the difference of its base and altitude equal to a given line.*



Given the square *S*, and the line *AB*.

Required to construct a ▱ equivalent to S, with the difference of its base and altitude equal to AB.

Construction. Upon AB as a diameter describe a circle.

From A draw AC, tangent to the circle, § 246
and equal to a side of the given square S.

Through the center of the circle draw CD intersecting the circle at E and D.

Then any ▱, as P, having CD for its base and CE for its altitude, is equivalent to S. Q. E. F.

Proof. $CD : CA = CA : CE$. § 302

$\therefore \overline{CA}^2 = CD \times CE$, § 261

and the difference between CD and CE is the diameter of the circle, that is, AB.

But $P = CD \times CE$, § 322

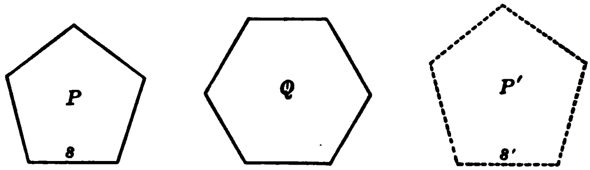
and $S = \overline{CA}^2$. § 320

$\therefore P = S$, by Ax. 8. Q. E. D.

Thus is solved geometrically the algebraic problem, given $x - y = a$, $xy = b$, to find x and y.

Proposition XX. Problem

354. *To construct a polygon similar to a given polygon and equivalent to another given polygon.*



Given the polygons *P* and *Q*.

Required to construct a polygon similar to P and equivalent to Q.

Construction. Construct squares equivalent to P and Q, § 351
and let m and n respectively denote their sides.

Let s be any side of P.

Find s', the fourth proportional to m, n, and s. § 307

Upon s', corresponding to s,
construct a polygon P' similar to the polygon P. § 312

Then P' is the polygon required. Q.E.F.

Proof. Since $m : n = s : s'$, Const.

$\therefore m^2 : n^2 = s^2 : s'^2$. § 270

But $P = m^2$, and $Q = n^2$. Const.

$\therefore P : Q = s^2 : s'^2$. Ax. 9

But $P : P' = s^2 : s'^2$. § 334

(*The areas of two similar polygons are to each other as the squares on any two corresponding sides.*)

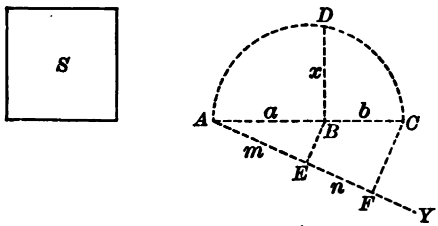
$\therefore P : Q = P : P'$. Ax. 8

$\therefore P' = Q$. § 263

$\therefore$ **P', being similar to P, is the polygon required.** Q.E.D.

PROPOSITION XXI. PROBLEM

355. *To construct a square which shall have a given ratio to a given square.*



Given the square S, and the ratio $\frac{n}{m}$.

Required to construct a square which shall be to S as n is to m.

Construction. Take AB equal to a side of S, and draw AY, making any convenient angle with AB.

On AY take AE equal to m units and EF equal to n units.

Draw EB.

From F draw a line ∥ to EB, meeting AB produced at C. § 233

On AC as a diameter describe a semicircle.

At B erect $BD \perp$ to AC, meeting the semicircle at D. § 228

Then BD is a side of the square required. Q.E.F.

Proof. Denote AB by a, BC by b, and BD by x.

Then $a : x = x : b$. § 297

$\therefore a : b = a^2 : x^2$. § 271

But $a : b = m : n$. § 273

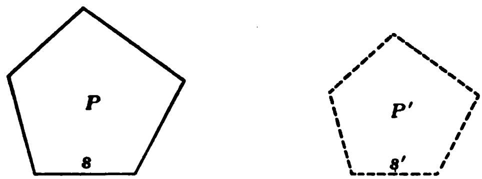
$\therefore a^2 : x^2 = m : n$. Ax. 8

By inversion, $x^2 : a^2 = n : m$. § 266

Hence the square on BD will have the same ratio to S as n has to m. Q.E.D.

Proposition XXII. Problem

356. *To construct a polygon similar to a given polygon and having a given ratio to it.*



Given the polygon P and the ratio $\frac{n}{m}$.

Required to construct a polygon similar to P, which shall be to P as n is to m.

Construction. Let s be any side of P.

Draw a line s', such that the square on s' shall be to the square on s as n is to m. § 355

Upon s' as a side corresponding to s construct the polygon P' similar to P. § 312

(*Upon a given line corresponding to a given side of a given polygon, to construct a polygon similar to the given polygon.*)

Then P' is the polygon required. Q.E.F.

Proof. $P' : P = s'^2 : s^2$. § 334

(*The areas of two similar polygons are to each other as the squares on any two corresponding sides.*)

But $s'^2 : s^2 = n : m$. Const.

Therefore $P' : P = n : m$, by Ax. 8. Q.E.D.

This problem enables us to construct a square that is twice a given square or half a given square, to construct an equilateral triangle that shall be any number of times a given equilateral triangle, and in general to enlarge or to reduce any figure in a given ratio. An architect's drawing, for example, might need to be enlarged so as to be double the area of the original, and the scale could be found by this method.

EXERCISE 55

Problems of Computation

1. The sides of a triangle are 0.7 in., 0.6 in., and 0.7 in. respectively. Is the largest angle acute, right, or obtuse?

2. The sides of a triangle are 5.1 in., 6.8 in., and 8.5 in. respectively. Is the largest angle acute, right, or obtuse?

3. Find the area of an isosceles triangle whose perimeter is 14 in. and base 4 in. (One decimal place.)

4. Find the area of an equilateral triangle whose perimeter is 18 in. (One decimal place.)

5. Find the area of a right triangle, the hypotenuse being 1.7 in. and one of the other sides being 0.8 in.

6. Find the ratio of the altitudes of two triangles of equal area, the base of one being 1.5 in. and that of the other 4.5 in.

7. The bases of a trapezoid are 34 in. and 30 in., and the altitude is 2 in. Find the side of a square having the same area.

8. What is the area of the isosceles right triangle in which the hypotenuse is $\sqrt{2}$?

9. What is the area of the isosceles right triangle in which the hypotenuse is $7\sqrt{2}$?

10. If the side of an equilateral triangle is $2\sqrt{3}$, what is the altitude of the triangle? the area of the triangle?

11. If the side of an equilateral triangle is 1 ft., what is the area of the triangle?

12. If the area of an equilateral triangle is 43.3 sq. in., what is the base of the triangle? (Take $\sqrt{3} = 1.732$.)

13. The sides of a triangle are 2.8 in., 3.5 in., and 2.1 in. respectively. Draw the figure carefully and see what kind of a triangle it is. Verify this conclusion by applying a geometric test, and find the area of the triangle.

EXERCISE 56

Theorems

1. The area of a rhombus is equal to half the product of its diagonals.

2. Two triangles are equivalent if the base of the first is equal to half the altitude of the second, and the altitude of the first is equal to twice the base of the second.

3. The area of a circumscribed polygon is equal to half the product of its perimeter by the radius of the inscribed circle.

4. Two parallelograms are equivalent if their altitudes are reciprocally proportional to their bases.

5. If equilateral triangles are constructed on the sides of a right triangle, the triangle on the hypotenuse is equivalent to the sum of the triangles on the other two sides.

6. If similar polygons are constructed on the sides of a right triangle, as corresponding sides, the polygon on the hypotenuse is equivalent to the sum of the polygons on the other two sides.

Ex. 6 is one of the general forms of the Pythagorean Theorem.

7. If lines are drawn from any point within a parallelogram to the four vertices, the sum of either pair of triangles with parallel bases is equivalent to the sum of the other pair.

8. Every line drawn through the intersection of the diagonals of a parallelogram bisects the parallelogram.

9. The line that bisects the bases of a trapezoid divides the trapezoid into two equivalent parts.

10. If a quadrilateral with two sides parallel is bisected by either diagonal, the quadrilateral is a parallelogram.

11. The triangle formed by two lines drawn from the midpoint of either of the nonparallel sides of a trapezoid to the opposite vertices is equivalent to half the trapezoid.

EXERCISE 57

Problems of Construction

1. Given a square, to construct a square of half its area.

2. To construct a right triangle equivalent to a given oblique triangle.

3. To construct a triangle equivalent to the sum of two given triangles.

4. To construct a triangle equivalent to a given triangle, and having one side equal to a given line.

5. To construct a rectangle equivalent to a given parallelogram, and having its altitude equal to a given line.

6. To construct a right triangle equivalent to a given triangle, and having one of the sides of the right angle equal to a given line.

7. To construct a right triangle equivalent to a given triangle, and having its hypotenuse equal to a given line.

8. To divide a given triangle into two equivalent parts by a line through a given point P in the base.

9. To draw from a given point P in the base AB of a triangle ABC a line to AC produced, so that it may be bisected by BC.

10. To find a point within a given triangle such that the lines from this point to the vertices shall divide the triangle into three equivalent triangles.

11. To divide a given triangle into two equivalent parts by a line parallel to one of the sides.

12. Through a given point to draw a line so that the segments intercepted between the point and perpendiculars drawn to the line from two other given points may have a given ratio.

13. To find a point such that the perpendiculars from it to the sides of a given triangle shall be in the ratio p, q, r.

EXERCISE 58

Review Questions

1. What is meant by the area of a surface? Illustrate.

2. What is the difference between equivalent figures and congruent figures?

3. State two propositions relating to the ratio of one rectangle to another.

4. Given the base and altitude of a rectangle, how is the area found? Given the area and base, how is the altitude found?

5. How do you justify the expression, "the product of two lines"? "the quotient of an area by a line"?

6. Can a triangle with a perimeter of 10 in. have the same area as one with a perimeter of 1 in.? Is the same answer true for two squares?

7. Can a parallelogram with a perimeter of 10 in. have the same area as a rectangle with a perimeter of 1 in.? Is the same answer true for two rectangles?

8. Explain how the area of an irregular field with straight sides may be found by the use of the theorems of Book IV.

9. A triangle has two sides 5 and 6, including an angle of 70°, and another triangle has two sides 2 and $7\frac{1}{2}$, including an angle of 70°. What is the ratio of the areas of the triangles?

10. Two similar triangles have two corresponding sides 5 in. and 15 in. respectively. The larger triangle has how many times the area of the smaller?

11. Given the hypotenuse of an isosceles right triangle, how do you proceed to find the area?

12. Given three sides of a triangle, what test can you apply to determine whether or not it is a right triangle?

13. Suppose you wish to construct a square equivalent to a given polygon, how do you proceed?