PLANE GEOMETRY

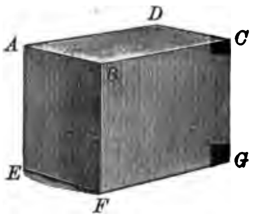
INTRODUCTION

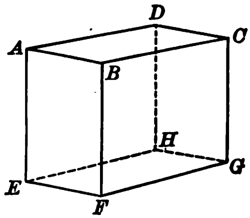
1. The Nature of Arithmetic. In arithmetic we study computation, the working with numbers. We may have a formula expressed in algebraic symbols, such as $a = bh$, where a may stand for the area of a rectangle, and b and h respectively for the number of units of length in the base and height; but the actual computation involved in applying such formula to a particular case is part of arithmetic.

2. The Nature of Algebra. In algebra we generalize the arithmetic, and instead of saying that the area of a rectangle with base 4 in. and height 2 in. is 4×2 sq. in., we express a general law by saying that $a = bh$. In arithmetic we may have an equality, like $2 \times 16 + 17 = 49$, but in algebra we make much use of equations, like $2x + 17 = 49$. Algebra, therefore, is a generalized arithmetic.

3. The Nature of Geometry. We are now about to begin another branch of mathematics, one not chiefly relating to numbers although it uses numbers, and not primarily devoted to equations although using them, but one that is concerned principally with the study of forms, such as triangles, parallelograms, and circles. Many facts that are stated in arithmetic and algebra are proved in geometry. For example, in geometry it is proved that the square on the hypotenuse of a right triangle equals the sum of the squares on the other two sides, and that the circumference of a circle equals 3.1416 times the diameter.

4. Solid. The block here represented is called a *solid;* it is a limited portion of space filled with matter. In geometry, however, we have nothing to do with the matter of which a





body is composed; we study simply its *shape* and *size*, as in the second figure.

That is, a physical solid can be touched and handled; a geometric solid is the space that a physical solid is conceived to occupy. For example, a stick is a physical solid; but if we put it into wet plaster, and then remove it, the hole that is left may be thought of as a geometric solid although it is filled with air.

5. Geometric Solid. A limited portion of space is called a *geometric solid.*

6. Dimensions. The block represented in § 4 extends in three principal directions:

(1) From left to right, that is, from A to D;
(2) From back to front, that is, from A to B;
(3) From top to bottom, that is, from A to E.

These extensions are called the *dimensions* of the block, and are named in the order given, *length, breadth* (or width), and *thickness* (height, altitude, or depth). Similarly, we may say that every solid has three dimensions.

Very often a solid is of such shape that we cannot point out the length, or distinguish it from the breadth or thickness, as an irregular block of coal. In the case of a round ball, where the length, breadth, and thickness are all the same in extent, it is impossible to distinguish one dimension from the others.

7. Surface. The block shown in § 4 has six flat faces, each of which is called a *surface*. If the faces are made smooth by polishing, so that when a straight edge is applied to any one of them the straight edge in every part will touch the surface, each face is called a *plane surface*, or a *plane*.

These surfaces are simply the boundaries of the solid. They have no thickness, even as a colored light shining upon a piece of paper does not make the paper thicker. A board may be planed thinner and thinner, and then sandpapered still thinner, thus coming nearer and nearer to representing what we think of as a geometric plane, but it is always a solid bounded by surfaces.

That which has length and breadth without thickness is called a *surface*.

8. Line. In the solid shown in § 4 we see that two adjacent surfaces intersect in a line. A line is therefore simply the boundary of a surface, and has neither breadth nor thickness.

That which has length without breadth or thickness is called a *line*.

A telegraph wire, for example, is not a line. It is a solid. Even a pencil mark has width and a very little thickness, so that it is also a solid. But if we think of a wire as drawn out so that it becomes finer and finer, it comes nearer and nearer to representing what we think of and speak of as a geometric line.

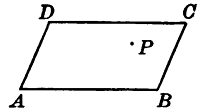
9. Magnitudes. Solids, surfaces, and lines are called *magnitudes*.

10. Point. In the solid shown in § 4 we see that when two lines meet they meet in a point. A point is therefore simply the boundary of a line, and has no length, no breadth, and no thickness.

That which has only position, without length, breadth, or thickness, is called a *point*.

We may think of the extremity of a line as a point. We may also think of the intersection of two lines as a point, and of the intersection of two surfaces as a line.

11. Representing Points and Geometric Magnitudes. Although we only imagine such geometric magnitudes as lines or planes, we may represent them by pictures.



Thus we represent a point by a fine dot, and name it by a letter, as *P* in this figure.

We represent a line by a fine mark, and name it by letters placed at the ends, as *AB*.

We represent a surface by its boundary lines, and name it by letters placed at the corners or in some other convenient way, as *ABCD*.

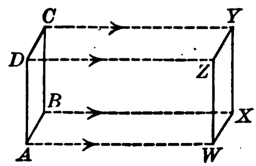
We represent a solid by the boundary faces or by the lines bounding the faces, as in § 4.

12. Generation of Geometric Magnitudes. We may think of

(1) A line as generated by a moving point;
(2) A surface as generated by a moving line;
(3) A solid as generated by a moving surface.

For example, as shown in the figure let the surface *ABCD* move to the position *WXYZ*. Then

(1) *A* generates the line *AW*;
(2) *AB* generates the surface *AWXB*;
(3) *ABCD* generates the solid *AY*.



Of course a point will not generate a line by simply turning over, for this is not motion for a point; nor will a line generate a surface by simply sliding along itself; nor will a surface generate a solid by simply sliding upon itself.

13. Geometric Figure. A point, a line, a surface, a solid, or any combination of these, is called a *geometric figure*.

A geometric figure is generally called simply a *figure*.

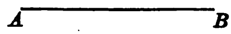
14. Geometry. The science of geometric figures is called *geometry*.

Plane geometry treats of figures that lie wholly in the same plane, that is, of plane figures.

Solid geometry treats of figures that do not lie wholly in the same plane.

15. Straight Line. A line such that any part placed with its ends on any other part must lie wholly in the line is called a *straight line.*

For example, AB is a straight line, for if we take, say, a half inch of it, and place it in any way on any other part of AB, but so that its ends lie in AB, then the whole of the half inch of line will lie in AB. This is well shown by using tracing paper. The word *line* used alone is understood to mean a straight line.



Part of a straight line is called a *segment* of the line. The term *segment* is applied also to certain other magnitudes.

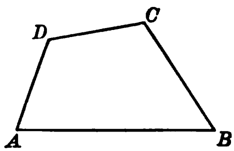
16. Equality of Lines. Two straight-line segments that can be placed one upon the other so that their extremities coincide are said to be *equal.*

In general, two geometric magnitudes are equal if they can be made to coincide throughout their whole extent. We shall see later that some figures that coincide are said to be *congruent.*

17. Broken Line. A line made up of two or more different straight lines is called a *broken line.*

For example, CD is a broken line.

18. Rectilinear Figure. A plane figure formed by a broken line is called a *rectilinear figure.*

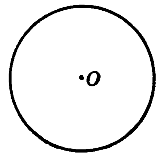


For example, $ABCD$ is a rectilinear figure.

19. Curve Line. A line no part of which is straight is called a *curve line,* or simply a *curve.*

For example, EF is a curve line.

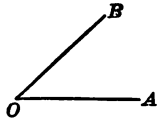
20. Curvilinear Figure. A plane figure formed by a curve line is called a *curvilinear figure.*



For example, O is a curvilinear figure with which we are already familiar.

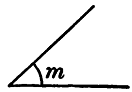
Some curvilinear figures are surfaces bounded by curves and others are the curves themselves.

21. Angle. The opening between two straight lines drawn from the same point is called an *angle*.



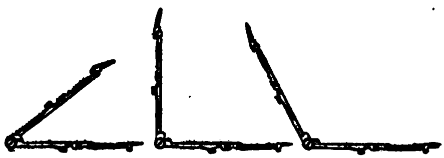
Strictly speaking, this is a *plane* angle. We shall find later that there are angles made by curve lines and angles made by planes.

The two lines are called the *sides* of the angle, and the point of meeting is called the *vertex*.



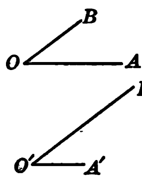
An angle may be read by naming the letters designating the sides, the vertex letter being between the others, as the angle *AOB*. An angle may also be designated by the vertex letter, as the angle *O*, or by a small letter within, as the angle *m*. A curve is often drawn to show the particular angle meant, as in angle *m*.

22. Size of Angle. The size of an angle depends upon the amount of turning necessary to bring one side into the position of the other.



One angle is greater than another angle when the amount of turning is greater. Thus in these compasses the first angle is smaller than the second, which is also smaller than the third. The length of the sides has nothing to do with the size of the angle.

23. Equality of Angles. Two angles that can be placed one upon the other so that their vertices coincide and the sides of one lie along the sides of the other are said to be *equal*.



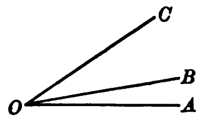
For example, the angles *AOB* and *A′O′B′* (read "*A* prime, *O* prime, *B* prime") are equal. It is well to illustrate this by tracing one on thin paper and placing it upon the other.

24. Bisector. A point, a line, or a plane that divides a geometric magnitude into two equal parts is called a *bisector* of the magnitude.

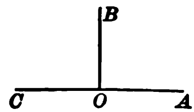
For example, *M*, the mid-point of the line *AB*, is a bisector of the line.

25. Adjacent Angles. Two angles that have the same vertex and a common side between them are called *adjacent angles.*

For example, the angles AOB and BOC are adjacent angles, and in § 26 the angles AOB and BOC are adjacent angles.



26. Right Angle. When one straight line meets another straight line and makes the adjacent angles equal, each angle is called a *right angle.*

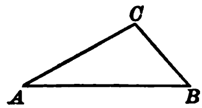


For example, angles AOB and BOC in this figure. If CO is cut off, angle AOB is still a right angle.

27. Perpendicular. A straight line making a right angle with another straight line is said to be *perpendicular* to it.

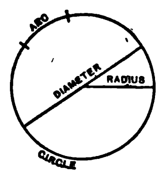
Thus OB is perpendicular to CA, and CA to OB. OB is also called *a perpendicular* to CA, and O is called the *foot* of the perpendicular OB.

28. Triangle. A portion of a plane bounded by three straight lines is called a *triangle.*



The lines AB, BC, and CA are called the *sides* of the triangle ABC, and the sides taken together form the *perimeter.* The points A, B, and C are the *vertices* of the triangle, and the angles A, B, and C are the *angles* of the triangle. The side AB upon which the triangle is supposed to rest is the *base* of the triangle. Similarly for other plane figures.

29. Circle. A closed curve lying in a plane, and such that all of its points are equally distant from a fixed point in the plane, is called a *circle.*

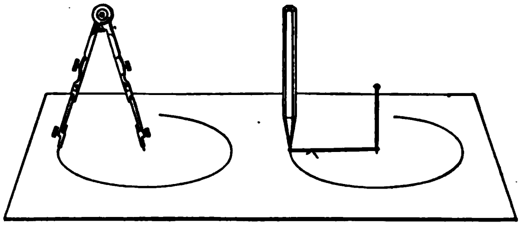


The length of the circle is called the *circumference.* The point from which all points on the circle are equally distant is the *center.* Any portion of a circle is an *arc.* A straight line from the center to the circle is a *radius.* A straight line through the center, terminated at each end by the circle, is a *diameter.*

Formerly in elementary geometry *circle* was taken to mean the space inclosed, and the bounding line was called the circumference. Modern usage has conformed to the definition used in higher mathematics.

30. Instruments of Geometry. In geometry only two instruments are necessary besides pencil and paper. These are a straight edge, or ruler, and a pair of compasses.

It is evident that *all radii of the same circle are equal.*



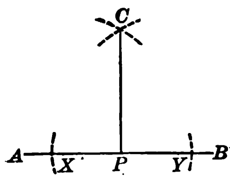
In the absence of compasses, and particularly for blackboard work, a loop made of string may be used. For the accurate transfer of lengths, however, compasses are desirable.

31. Exercises in using Instruments. The following simple exercises are designed to accustom the pupil to the use of instruments. No proofs are attempted, these coming later in the course.

This section may be omitted if desired, without affecting the course.

EXERCISE 1

1. From a given point on a given straight line required to draw a perpendicular to the line.



Let AB be the given line and P be the given point.

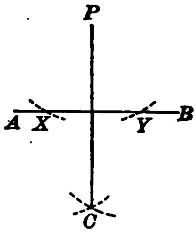
It is required to draw from P a line perpendicular to AB.

With P as a center and any convenient radius draw arcs cutting AB at X and Y.

With X as a center and XY as a radius draw a circle, and with Y as a center and the same radius draw another circle, and call one intersection of the circles C.

With a straight edge draw a line from P to C, and this will be the perpendicular required.

2. From a given point outside a given straight line required to let fall a perpendicular to the line.



Let AB be the given straight line and P be the given point.

It is required to draw from P a line perpendicular to AB.

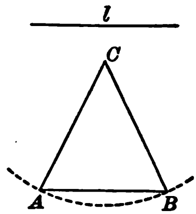
With P as a center and any convenient radius draw an arc cutting AB at X and Y.

With X as a center and any convenient radius draw a circle, and with Y as a center and the same radius draw another circle, and call one intersection of the circles C.

With a straight edge draw a straight line from P to C, and this will be the perpendicular required.

It is interesting to test the results in Exs. 1 and 2, by cutting the paper and fitting the angles together.

3. Required to draw a triangle having two sides each equal to a given line.



Let l be the given line.

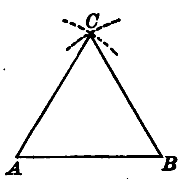
It is required to draw a triangle having two sides each equal to l.

With any center, as C, and a radius equal to l draw an arc.

Join any two points on the arc, as A and B, with each other and with C by straight lines.

Then ABC is the triangle required.

4. Required to draw a triangle having its three sides each equal to a given line.



Let AB be the given line.

It is required to draw a triangle having its three sides each equal to AB.

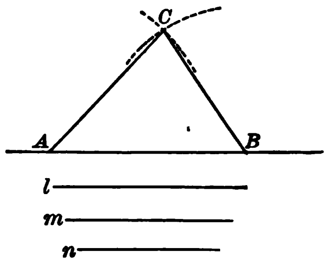
With A as a center and AB as a radius draw a circle, and with B as a center and the same radius draw another circle.

Join either intersection of the circles with A and B by straight lines.

Then ABC is the triangle required.

In such cases draw the arcs only long enough to show the point of intersection.

5. Required to draw a triangle having its sides equal respectively to three given lines.



Let the three lines be *l*, *m*, and *n*.

What is now required?

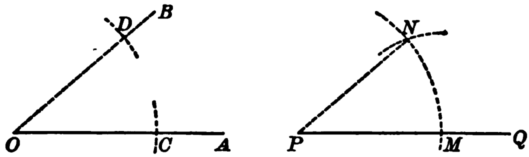
Upon any line mark off with the compasses a line-segment *AB* equal to *l*.

With *A* as a center and *m* as a radius draw a circle; with *B* as a center and *n* as a radius draw a circle.

Draw *AC* and *BC*.

Then *ABC* is the required triangle.

6. From a given point on a given line required to draw a line making an angle equal to a given angle.



Let *P* be the given point on the given line *PQ*, and let angle *AOB* be the given angle.

What is now required?

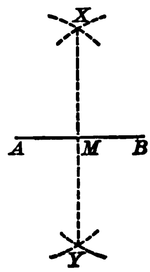
With *O* as a center and any radius draw an arc cutting *AO* at *C* and *BO* at *D*.

With *P* as a center and *OC* as a radius draw an arc cutting *PQ* at *M*.

With *M* as a center and the straight line joining *C* and *D* as a radius draw an arc cutting the arc just drawn at *N*, and draw *PN*.

Then angle *MPN* is the required angle.

7. Required to bisect a given straight line.



Let *AB* be the given line.

It is required to bisect *AB*.

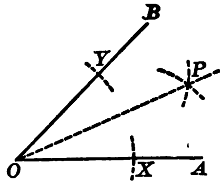
With *A* as a center and *AB* as a radius draw a circle, and with *B* as a center and the same radius draw a circle.

Call the two intersections of the circles *X* and *Y*.

Draw the straight line *XY*.

Then *XY* bisects the line *AB* at the point of intersection *M*.

8. Required to bisect a given angle.



Let AOB be the given angle.

It is required to bisect the angle AOB.

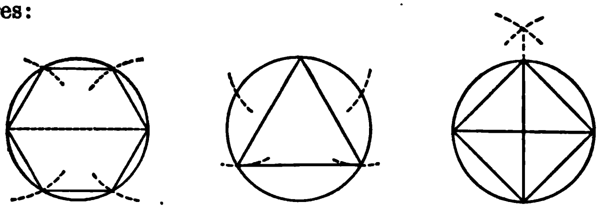
With O as a center and any convenient radius draw an arc cutting OA at X and OB at Y.

With X as a center and a line joining X and Y as a radius draw a circle, and with Y as a center and the same radius draw a circle, and call one point of intersection of the circles P.

Draw the straight line OP.

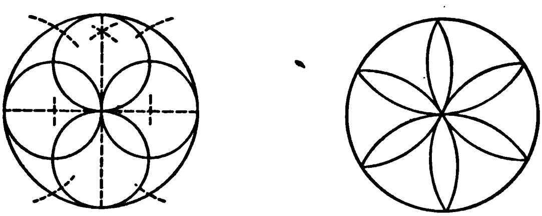
Then OP is the required bisector.

9. By the use of compasses and ruler draw the following figures:



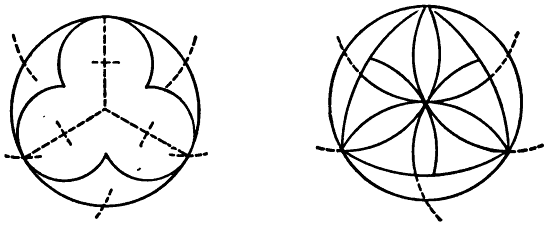
The dotted lines show how to fix the points needed in drawing the figure, and they may be erased after the figure is completed. In general, in geometry, auxiliary lines (those needed only as aids) are indicated by dotted lines.

10. By the use of compasses and ruler draw the following figures:

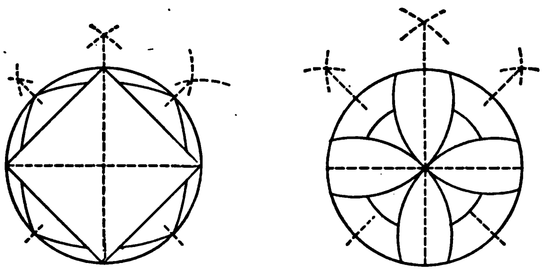


It is apparent from the figures in Exs. 9 and 10 that the radius of the circle may be used in describing arcs that shall divide the circle into six equal parts.

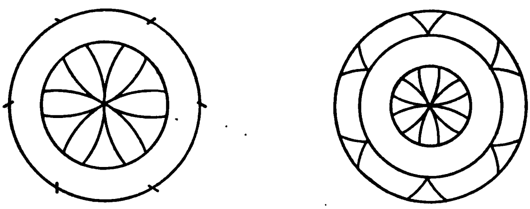
11. By the use of compasses and ruler draw the following figures:



12. By the use of compasses and ruler draw the following figures:



13. By the use of compasses and ruler draw the following figures:



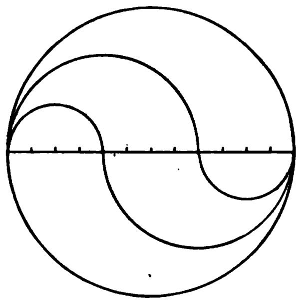
In such figures artistic patterns may be made by coloring various portions of the drawings. In this way designs are made for stained-glass windows, for oilcloth, for colored tiles, and for other decorations.

14. Draw a triangle of which each side is $1\frac{1}{4}$ in.

15. Draw two lines bisecting each other at right angles.

16. Bisect each of the four right angles formed by two lines bisecting each other at right angles.

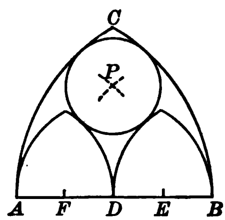
17. Draw a line $1\frac{1}{2}$ in. long and divide it into eighths of an inch, using the ruler. Then with the compasses draw this figure.



It is easily shown, when we come to the measurement of the circle, that these two curve lines divide the space inclosed by the circle into parts that are exactly equal to one another.

By continuing each semicircle to make a complete circle another interesting figure is formed. Other similar designs are easily invented, and students should be encouraged to make such original designs.

18. In planning a Gothic window this drawing is needed. The arc BC is drawn with A as a center and AB as a radius. The small arches are described with A, D, and B as centers and AD as a radius. The center P is found by taking A and B as centers and AE as a radius. How may the points D, E, and F be found? Draw the figure.



19. Draw a triangle of which each side is 1 in. Bisect each side, and with the points of bisection as centers and with radii $\frac{1}{2}$ in. long draw three circles.

20. A baseball diamond is a square 90 ft. on a side. Draw the plan, using a scale of $\frac{1}{16}$ in. tc a foot. Locate the pitcher 60 ft. from the home plate.

21. A man travels from A directly east 1 mi. to B. He then turns and travels directly north $1\frac{3}{4}$ mi. to C. Draw the plan and find by measurement the distance AC to the nearest quarter of a mile. Use a scale of $\frac{1}{2}$ in. to a mile.

22. A double tennis court is 78 ft. long and 36 ft. wide. The net is placed 39 ft. from each end and the service lines 18 ft. from each end. Draw the plan, using a scale of $\frac{1}{16}$ in. to a foot, making the right angles as shown in Ex. 1. The accuracy of the construction may be tested by measuring the diagonals, which should be equal.

23. At the entrance to New York harbor is a gun having a range of 12 mi. Draw a line inclosing the range of fire, using a scale of $\frac{1}{16}$ in. to a mile.

24. Two forts are placed on opposite sides of a harbor entrance, 13 mi. apart. Each has a gun having a range of 10 mi. Draw a plan showing the area exposed to the fire of both guns, using a scale of $\frac{1}{16}$ in. to a mile.

25. Two forts, *A* and *B*, are placed on opposite sides of a harbor entrance, 16 mi. apart. On an island in the harbor, 12 mi. from *A* and 11 mi. from *B*, is a fort *C*. The fort *A* has a gun with a range of 12 mi., fort *B* one with a range of 11 mi., and fort *C* one with a range of 10 mi. Draw a plan of the entrance to the harbor, showing the area exposed to the fire of each gun.

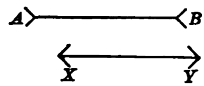
26. A horse, tied by a rope 25 ft. long at the corner of a lot 50 ft. square, grazes over as much of the lot as possible. The next day he is tied at the next corner, the third day at the third corner, and the fourth day at the fourth corner. Draw a plan showing the area over which he has grazed during the four days, using a scale of $\frac{1}{8}$ in. to 5 ft.

27. A gardener laid out a flower bed on the following plan: He made a triangle *ABC*, 16 ft. on a side, and then bisected two of the angles. From the point of intersection of the bisectors, *P*, he drew perpendiculars to the three sides of the triangle, *PX*, *PY*, and *PZ*. Then he drew a circle with *P* as a center and *PX* as a radius, and found that it just fitted in the triangle. Draw the plan, using a scale of $\frac{1}{4}$ in. to a foot.

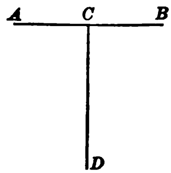
32. Necessity for Proof. Although part of geometry consists in drawing figures, this is not the most important part. It is essential to prove that the figures are what we claim them to be. The danger of trusting to appearances is seen in Exercise 2.

EXERCISE 2

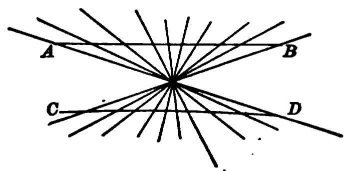
1. Estimate which is the longer line, *AB* or *XY*, and how much longer. Then test your estimate by measuring with the compasses or with a piece of paper carefully marked.



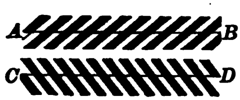
2. Estimate which is the longer line, *AB* or *CD*, and how much longer. Then test your estimate by measuring as in Ex. 1.



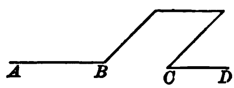
3. Look at this figure and state whether *AB* and *CD* are both straight lines. If one is not straight, which one is it? Test your answer by using a ruler or the folded edge of a piece of paper.



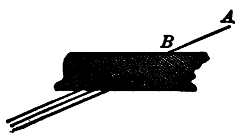
4. Look at this figure and state whether *AB* and *CD* are the same distance apart at *A* and *C* as at *B* and *D*. Then test your answer as in Ex. 1.



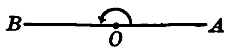
5. Look at this figure and state whether *AB* will, if prolonged, lie on *CD*. Also state whether *WX* will, if prolonged, lie on *YZ*. Then test your answer by laying a ruler along the lines.



6. Look at this figure and state which of the three lower lines is *AB* prolonged. Then test your answer by laying a ruler along *AB*.



33. Straight Angle. When the sides of an angle extend in opposite directions, so as to be in the same straight line, the angle is called a *straight angle*.

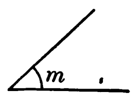


For example, the angle AOB, as shown in this figure, is a straight angle. The angle BOA, below the line, is also a straight angle.

34. Right Angle and Straight Angle. It follows from the definition of right angle (§ 26) that *a right angle is half of a straight angle.*

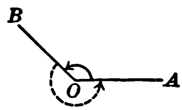
In like manner, it follows that *a straight angle equals twice a right angle.*

35. Acute Angle. An angle less than a right angle is called an *acute angle.*



For example, the angle m, as shown in this figure, is an acute angle.

36. Obtuse Angle. An angle greater than a right angle and less than a straight angle is called an *obtuse angle.*



For example, the angle AOB, as shown in this figure, is an obtuse angle.

37. Reflex Angle. An angle greater than a straight angle and less than two straight angles is called a *reflex angle.*

For example, the angle BOA, marked with a dotted curve line in the figure in § 36, is a reflex angle.

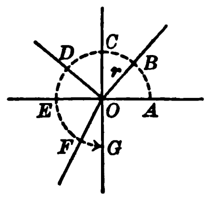
When we speak of an angle formed by two given lines drawn from a point we mean the smaller angle unless the contrary is stated.

38. Oblique Angles. Acute angles and obtuse angles are called *oblique angles.*

The sides of oblique angles are said to be *oblique* to each other, and are called *oblique lines.*

Evidently if we bisect a straight angle, we form two right angles; if we bisect a right angle or an obtuse angle, we form two acute angles; if we bisect a reflex angle, we form two obtuse angles.

39. Generation of Angles. Suppose the line r to revolve from the position OA about the point O as a vertex to the position OB. Then r describes or generates the *acute angle* AOB, and, as we have seen (§ 22) *the size of the angle depends upon the amount of rotation*, the angle being greater as the amount of turning is greater.



If r rotates still further, to the position OC, it has then generated the *right angle* AOC and is perpendicular to OA.

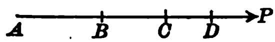
If r rotates still further, to the position OD, it has then generated the *obtuse angle* AOD.

If r rotates to the position OE, it has then generated the *straight angle* AOE.

If r rotates to the position OF, it has then generated the *reflex angle* AOF.

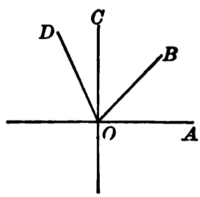
If r rotates still further, past OG to the position OA again, it has made a complete revolution and has generated two straight angles or four right angles.

40. Sums and Differences of Magnitudes. If the straight line AP has been generated by a point P moving from A to P, the segments AB, BC, CD, and so on, having been generated in succession, then we call AC the *sum* of AB and BC. That is,



$$AC = AB + BC, \text{ whence } AC - BC = AB.$$

If the angle AOD has been generated by the line OA revolving about O as a vertex from the position OA, the angles AOB, BOC, and COD having been generated in succession, then we call angle AOC the *sum* of angles AOB and BOC. That is, considering angles,



$$AOC = AOB + BOC, \text{ whence } AOC - BOC = AOB.$$

In the same way that we may have the sum or the difference of lines or of angles we may have the sum or the difference of surfaces or of solids.

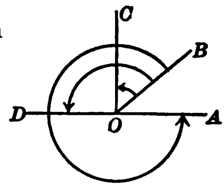
41. Perigon. The whole angular space in a plane about a point is called a *perigon.*

It therefore follows that a perigon equals the sum of two straight angles or the sum of four right angles.

42. Complements, Supplements, and Conjugates. If the sum of two angles is a right angle, each angle is called the *complement* of the other.

If the sum of two angles is a straight angle, each angle is called the *supplement* of the other.

If the sum of two angles is a perigon, each angle is called the *conjugate* of the other.



Thus, with respect to angle AOB,
the *complement* is angle BOC,
the *supplement* is angle BOD,
the *conjugate* is angle BOA (reflex).

43. Properties of Supplementary Angles. It is sufficiently evident to be taken without proof that

1. *The two adjacent angles which one straight line makes with another are together equal to a straight angle.*

2. *If the sum of two adjacent angles is a straight angle, their exterior sides are in the same straight line.*

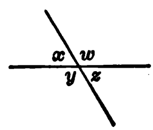
44. Angle Measure. Angles are measured by taking as a unit $\frac{1}{360}$ of a perigon. This unit is called a *degree.*

The degree is divided into 60 equal parts, called *minutes*, and the minute into 60 equal parts, called *seconds.*

We write $5° 13' 12''$ for 5 degrees 13 minutes 12 seconds.

It is evident that a right angle equals 90°, a straight angle equals 180°, and a perigon equals 360°.

45. Vertical Angles. When two angles have the same vertex, and the sides of the one are prolongations of the sides of the other, those angles are called *vertical angles.*



In the figure the angles x and z are vertical angles, as are also the angles w and y.

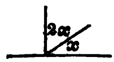
EXERCISE 3

1. Find the complement of 72°; of 65° 30′; of 22° 20′ 15″.

2. What is the supplement of 45°? of 120°? of 145° 5′? of 22° 20′ 15″?

3. What is the conjugate of 240°? of 280°? of 312° 10′ 40″?

4. The complement of a certain angle x is $2x$. How many degrees are there in x?



5. The complement of a certain angle x is $3x$. How many degrees are there in x?

6. What is the angle of which the complement is four times the angle itself?

7. The supplement of a certain angle x is $5x$. How many degrees are there in x?

8. The supplement of a certain angle x is $14x$. How many degrees are there in x?

9. What is the angle of which the supplement equals half of the angle itself?

10. How many degrees in an angle that equals its own complement? in one that equals its own supplement?

11. The conjugate of a certain angle x is $\frac{3}{5}x$. How many degrees are there in x?

12. The conjugate of a certain angle x is $\frac{1}{2}x$. How many degrees are there in x?

13. How many degrees in an angle that equals a third of its own conjugate? in one that equals its own conjugate?

14. Find two angles, x and y, such that their sum is 90° and their difference is 10°.

15. Find two complementary angles such that their difference is 30°.

16. Find two supplementary angles such that one is 20° greater than the other.

17. The angles x and y are conjugate angles, and their difference is a straight angle. How many degrees are there in each?

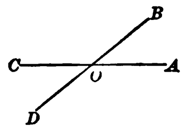
18. The angles x and y are conjugate angles, and their difference is zero. How many degrees are there in each?

19. Of two complementary angles one is four fifths of the other. How many degrees are there in each?

20. Of two supplementary angles one is five times the other. How many degrees are there in each?

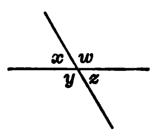
21. How many degrees are there in the smaller angle formed by the hands of a clock at 5 o'clock?

22. How many degrees are there in the smaller angle formed by the hands of a clock at 10 o'clock?



23. In this figure, if angle AOB is 38°, how many degrees in angle BOC? How many in angle COD? How many in angle DOA?

24. In the same figure, if angle AOB is equal to a third of angle BOC, how many degrees in each of the four angles?



25. In the angles of this figure, if $w = 2x$, how many degrees in each? How many degrees in y? How many degrees in z?

26. Find the angle whose complement decreased by 30° equals the angle itself.

27. Find the angle whose complement divided by 2 equals the angle itself.

28. Draw a figure to show that if two adjacent angles have their exterior sides in the same straight line, their sum is a straight angle.

29. Draw a figure to show that the sum of all the angles on the same side of a straight line, at a given point, is equal to two right angles.

30. Draw a figure to show that the complements of equal angles are equal.

46. Axiom. A general statement admitted without proof to be true is called an *axiom.*

For example, it is stated in algebra that "if equals are added to equals the sums are equal." This is so simple that it is generally accepted without proof. It is therefore an axiom.

47. Postulate. In geometry a geometric statement admitted without proof to be true is called a *postulate.*

For example, it is so evident that all straight angles are equal, that this statement is a postulate. It is also evident that a straight line may be drawn and that a circle may be described, and these statements are therefore postulates of geometry.

Axioms are therefore general mathematical assumptions, while geometric postulates are the assumptions peculiar to geometry. Postulates and axioms are the assumptions upon which the whole science of mathematics rests.

48. Theorem. A statement to be proved is called a *theorem.*

For example, it is stated in arithmetic that the square on the hypotenuse of a right triangle equals the sum of the squares on the other two sides. This statement is a theorem to be proved in geometry.

49. Problem. A construction to be made so that it shall satisfy certain given conditions is called a *problem.*

For example, required to construct a triangle all of whose sides shall be equal. This construction was made in § 31, Ex. 4, and later it will be proved that the construction was correct.

50. Proposition. A statement of a theorem to be proved or a problem to be solved is called a *proposition.*

In geometry, therefore, a proposition is either a theorem or a problem. We shall find that most of the propositions at first are theorems. After we have proved a number of theorems so that we can prove that the solutions of problems are correct, we shall solve some problems.

51. Corollary. A truth that follows from another with little or no proof is called a *corollary.*

For example, since we admit that all straight angles are equal, it follows as a corollary that all right angles are equal, since a right angle is half of a straight angle.

52. Axioms. The following are the most important axioms used in geometry:

1. *If equals are added to equals the sums are equal.*
2. *If equals are subtracted from equals the remainders are equal.*
3. *If equals are multiplied by equals the products are equal.*
4. *If equals are divided by equals the quotients are equal.*

In division the divisor is never zero.

5. *Like powers or like positive roots of equals are equal.*

We learn from algebra that the square root of 4 is $+2$ or -2, but of course these are not equal. In geometry we shall use only the positive roots.

6. *If unequals are operated on by positive equals in the same way, the results are unequal in the same order.*

Taking $a > b$ and taking x and y as equal positive quantities, this axiom states that

$$a + x > b + y, \quad a - x > b - y, \quad ax > by, \quad \frac{a}{x} > \frac{b}{y}, \text{ etc.}$$

7. *If unequals are added to unequals in the same order, the sums are unequal in the same order; if unequals are subtracted from equals the remainders are unequal in the reverse order.*

If $a > b$, $c > d$, and $x = y$, then $a + c > b + d$, and $x - a < y - b$.

8. *Quantities that are equal to the same quantity or to equal quantities are equal to each other.*
9. *A quantity may be substituted for its equal in an equation or in an inequality.*

Thus if $x = b$ and if $a + x = c$, then $a + b = c$; and if $a + x > c$, then $a + b > c$. Axiom 8 is used so often that it is stated separately, although it is really included in Axiom 9.

10. *If the first of three quantities is greater than the second, and the second is greater than the third, then the first is greater than the third.*

Thus if $a > b$, and if $b > c$, then $a > c$.

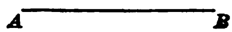
11. *The whole is greater than any of its parts, and is equal to the sum of all of its parts.*

53. Postulates. The following are among the most important postulates used in geometry. Others will be introduced as needed.

1. *One straight line and only one can be drawn through two given points.*

2. *A straight line may be produced to any required length.*

To produce AB means to extend it through B; to produce BA means to extend it through A.



3. *A straight line is the shortest path between two points.*

4. *A circle may be described with any given point as a center and any given line as a radius.*

5. *Any figure may be moved from one place to another without altering its size or shape.*

6. *All straight angles are equal.*

54. Corollary 1. *Two points determine a straight line.*

This is only a brief way of stating Postulate 1.

55. Corollary 2. *Two straight lines can intersect in only one point.*

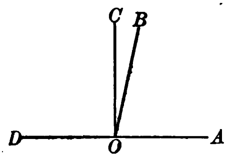
For if they had two points in common they would coincide (Post. 1).

56. Corollary 3. *All right angles are equal.*

For all straight angles are equal (Post. 6), and a straight angle (§ 34) is twice a right angle. Hence Axiom 4 applies.

57. Corollary 4. *From a given point in a given line only one perpendicular can be drawn to the line.*

For if there could be two perpendiculars to DA at O, as OB and OC, we should have angles AOB and AOC both right angles, which is impossible (§ 56).



58. Corollary 5. *Equal angles have equal complements, equal supplements, and equal conjugates.*

59. Corollary 6. *The greater of two angles has the less complement, the less supplement, and the less conjugate.*

EXERCISE 4

1. If $10^\circ + \angle x = 27^\circ\ 30'$, find the value of $\angle x$.
2. If $\angle x + 37^\circ = \frac{1}{2} \angle x + 40^\circ$, find the value of $\angle x$.
3. If $\frac{2}{3} \angle x + \angle b = 5 \angle b$, find the value of $\angle x$.
4. If $\angle x + \angle a = 4 \angle a - \angle x$, find the value of $\angle x$.

Find the value of $\angle x$ in each of the following equations:

5. $\angle x + 13^\circ = 39^\circ$.
6. $\angle x - 17^\circ = 46^\circ$.
7. $2 \angle x = \angle x + 23^\circ$.
8. $5 \angle x = 2 \angle x + 21^\circ$.
9. $4 \angle x = \frac{1}{2} \angle x + 70^\circ$.
10. $\angle x = 0.7 \angle x + 33^\circ$.
11. $\angle x = 0.1 \angle x + 18^\circ$.
12. $\frac{3}{4} \angle x = \frac{1}{2} \angle x + 2\frac{1}{2}^\circ$.
13. $\frac{4}{5} \angle x = 0.1 \angle x + 14^\circ$.
14. $\frac{2}{3} \angle x = \frac{1}{2} \angle x + 2^\circ$.
15. $12 \angle x + 17^\circ = 9 \angle x + 32^\circ$.
16. $5 \angle x - 22^\circ\ 30' = 2 \angle x + 11^\circ$.
17. $51^\circ\ 20' - \frac{2}{3} \angle x = 5^\circ\ 1' + 3 \angle x$.
18. $73^\circ\ 21'\ 4'' - \angle x = 3^\circ\ 3'\ 12'' + 4 \angle x$.

19. If $x + 20^\circ = y$ and $y - 5^\circ = 2x$, what is the value of x and of y?

Find the value of x and of y in each of the following sets of equations:

20. $x + y = 45^\circ$,
    $x - y = 35^\circ$.
21. $x - 8y = 0^\circ$,
    $x + 8y = 80^\circ$.
22. $2x + y = 64^\circ$,
    $3x - y = 88^\circ$.
23. $x + 2y = 21^\circ$,
    $x + 3y = 26^\circ\ 15'$.
24. $x + y = 9^\circ\ 20'\ 15''$,
    $2x - y = 12^\circ\ 25'\ 15''$.
25. $x - y = 5'\ 5''$,
    $3x + 4y = 14^\circ\ 50'\ 50''$.

26. If $x < 10^\circ$ and $y = 7^\circ\ 30'$, what can be said as to the value of $x + y$?

27. In Ex. 26, what can be said as to the value of $x - y$?