MATH LEVEL 2
LESSON PLAN 1

NUMBER TO INTEGER

Section 1: Natural Numbers

1. **Natural numbers** are counting numbers. Counting starts from **one**. We use these numbers to count things, such as, cups. The count increases by **one** each time. This counting can go on forever.

2. The thing that is counted one at a time is called the **unit**. If one is counting houses, then each house is a unit. If one is counting inches in a length, then each inch is a unit.

3. The smallest natural number is **one (1)**. The largest natural number is as large as can be thought of.

Section 2: Whole Numbers

4. When there is nothing to count, we represent it by **zero**. Zero means “nothing”. When counting is referenced from zero, the numbers are called **whole numbers**.

5. Zero (0) is the smallest whole number. It is not a natural number. But all other whole numbers are also natural numbers.

6. Two numbers may be added in any order. For example,

   5 + 8 = 8 + 5

   ($$$$$) + ($$$$$$$$) = ($$$$$$$$) + ($$$$$)

7. Two numbers may be multiplied in any order. For example,

   5 x 8 = 8 x 5

   This can be shown by arranging items in rows and columns.

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8. Three numbers may be associated in equivalent order as follows.

   3 + (5 + 8) = (3 + 5) + 8

   3 x (5 x 8) = (3 x 5) x 8

   Thus, three or more numbers may be added or multiplied in different orders without changing the outcome.

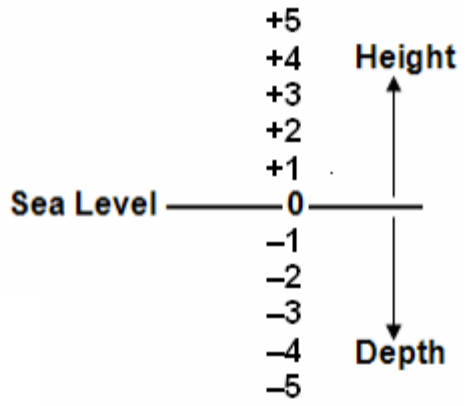
9. Numbers may be multiplies by separating them into TEN and ONE.

23 x 7 = (20 + 3) x 7 = 20 x 7 + 3 x 7

Section 3: Integers

10. Numbers may be counted from zero in two oppose directions, such as, increasing (positive), and decreasing (negative). Such numbers are called **integers**.

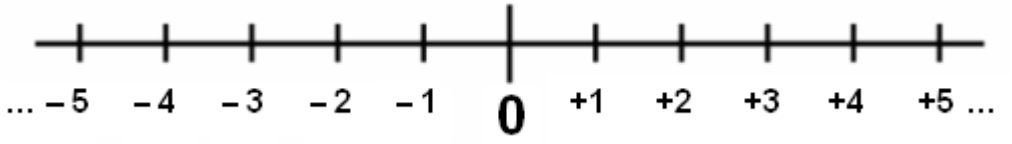
    For example, we mark the sea level as 0, and the measure the height of mountains by positive (+) numbers, and the depth of seas by negative (–) numbers



11. Positive numbers are greater than zero. Negative numbers are less than zero.

+5	**=**	**0 + 5**	(such as, 5 feet above water)
–5	**=**	**0 – 5**	(such as, 5 feet below water)

12. We may show integers on a Number Line. The integers increase to the right (+), and decrease to the left (–).



    Such integers extend indefinitely in either direction from zero. Zero "0" is called the reference point.

13. Integers on the right are larger than the integers on the left on the number line.

☺ EXERCISE

1. Describe the reference point of zero as used for

(a) The depth of seas
(b) The height of a tree
(c) Your height
(d) Your age
(e) The temperature
(f) The year

Answer: (a) Sea level (b) The ground (c) The floor you are standing on (d) Your birth (e) Melting point of ice (f) Since the death of Christ.

2. Describe the following integers as positive and negative numbers

(a) A tree 50 feet high
(b) A ditch 4 feet deep
(c) A temperature 20 °C warmer than the freezing water
(d) A shortage of $25 in bank account

Answer: (a) +50 (b) -4 (c) +20 (d) -25

3. **Place the correct symbol, > (greater than) or < (less than), between the two numbers.**

(a) 3 … 5 **(c) 7 … 4** **(e) –2 … –5** **(g) –6 … –1**
(b) –2 …13 **(d) 3 … –7** **(f) 5 … –6** **(h) –11 … –8**

Answer: (a) < (b) < (c) > (d) > (e) > (f) > (g) < (h) <

4. **Write the following integers from smallest to the largest**

9, –3, +5, –6, –1, +8, –4, +7, –2

Answer: **– 6, – 4, – 3, – 2, – 1, + 5, + 7, + 8, + 9**

Section 4: Combining of Integers as Addition & Subtraction

14. Combination of integers is equivalent to addition and subtraction of corresponding whole numbers. Note that the signs of addition and subtraction are associated with the number that follows it.

+8 and +3 = 8 + 3 = 11 or +11
+8 and –3 = 8 – 3 = 5 or +5

15. Combination of same integer of opposite signs is like subtracting a number from itself; therefore the outcome is zero.

+8 and –8 = 8 – 8 = 0

16. Combining integers of **like signs** requires finding their sum. The same sign is attached to the sum

+3 and +5 = + (sum of 3 and 5) = +8
–3 and –5 = – (sum of 3 and 5) = –8

17. Combining integers of **unlike signs** requires finding their difference. The sign of the larger number is attached to the sum

+5 and –3 = + (difference of 5 and 3) = +2
+3 and –5 = – (difference of 5 and 3) = –2

18. A string of addition and subtractions may be converted into integers first. Then integers may be gathered into positive and negative groups to be reduced as follows.

3 – 5 + 7 – 2 + 17 – 23 = +3 –5 +7 –2 +17 –23
= + (sum of 3, 7, 17) – (sum of 5, 2, 23)
= +27 –30
= –3

19. In a string of addition and subtractions the same number with opposite signs may simply be cancelled.

8 – 5 + 3 – 4 – 3 + 7 + 4 – 8 = 8 – 5 + 3 – 4 – 3 + 7 + 4 – 8
= 0 – 5 + 7
= 2

☺ EXERCISE

1. Combine the following integers.

(a) –8 +7 = ___ **(c) +9 –4 = ___** **(e) –7 –2 = ___** **(g) +8 +6 = ___**
(b) –2 –8 = ___ **(d) +5 +4 = ___** **(f) –7 +15 = ___** **(h) +11 –27 = ___**

Solution: (a) –1 (b) –10 (c) +5 (d) +9 (e) –9 (f) +8 (g) +14 (h) –16

2. Combine the following integers.

(a) +4 –16 –9 +25 –3 –11 +7
(b) +8 –4 –14 +18 –5 +7 +2

Solution: (a) –3 (b) +12

Section 5: The Rule of Consecutive signs

20. The positive of an integer is the same integer.

+ (+5) = +5
+ (–3) = –3

21. The negative of an integer is the opposite integer.

– (+5) = –5
– (–3) = +3

22. We may say that,

(a) When the consecutive signs are "like", the resulting sign is "+"
(b) When the consecutive signs are "unlike", the resulting sign is "–"

+ (+) = + and + (–) = –
– (–) = + and – (+) = –

23. Adding integers is the same as combining integers.

(+3) + (+5) = +3 +5 = +8
(–3) + (–5) = –3 –5 = –8
(+3) + (–5) = +3 –5 = –2
(–3) + (+5) = –3 +5 = +2

24. To subtract an integer we simply reverse its sign and then combine the integers.

(+3) – (+5) = +3 –5 = –2
(–3) – (–5) = –3 +5 = +2
(+3) – (–5) = +3 +5 = +8
(–3) – (+5) = –3 –5 = –8

25. We may simplify an expression made up of integers as follows.

+3 + (–5) – (+2) – (–9) + (–3) = +3 –5 –2 +9 –3
= +12 –10
= +2

☺ EXERCISE

1. Place the resulting sign for the number

(a) – (+7) = …7 **(c) + (–4) = …4** **(e) – (–2) = …2** **(g) + (+6) = …6**

(b) – (–8) = …8 **(d) + (+1) = …1** **(f) – (+15) = …15** **(h) + (–27) = …27**

Answer: (a) – 7 (b) +8 (c) –4 (d) +1 (e) +2 (f) –15 (g) +6 (h) – 27

2. Add and subtract the following integers.

(a) –8 + (–7) = __ **(c) +9 – (–4) = __** **(e) –7 – (–2) = __** **(g) +8 + (–6) = __**

(b) –2 – (–8) = __ **(d) –5 + (+4) = __** **(f) +7 – (+15) = __** **(h) +11 + (–27) = __**

Answer: (a) –15 (b) +6 (c) +13 (d) –1 (e) –5 (f) –8 (g) +2 (h) –16

3. Solve the following mixed operations.

(a) –8 + (–7) + (+5) – (–4) – (+5) = __ **(c) –7 – (–2) + (–3) + (–2) – (–3) = __**

(b) –2 – (–8) – (+7) + (–6) + (+7) = __ **(d) +7 – (+15) + (+11) – (–27) + (–17) = __**

Answer: (a) –11 (b) 0 (c) –7 (d) +13

Section 6: Multiplying & Dividing Integers

26. In multiplication and division of integers, we separate the signs from the integers and then combine them.

(+12)	**x (+4)**	**=**	**(+)(+)**	**(12 x 4)**	**=**	**+48**
(+12)	**x (–4)**	**=**	**(+)(–)**	**(12 x 4)**	**=**	**–48**
(–12)	**x (–4)**	**=**	**(–)(–)**	**(12 x 4)**	**=**	**+48**
(–12)	**x (+4)**	**=**	**(–)(+)**	**(12 x 4)**	**=**	**–48**
(+12)	**÷ (+4)**	**=**	**(+)(+)**	**(12 ÷ 4)**	**=**	**+3**
(+12)	**÷ (–4)**	**=**	**(+)(–)**	**(12 ÷ 4)**	**=**	**–3**
(–12)	**÷ (–4)**	**=**	**(–)(–)**	**(12 ÷ 4)**	**=**	**+3**
(–12)	**÷ (+4)**	**=**	**(–)(+)**	**(12 ÷ 4)**	**=**	**–3**

27. When simplifying a term containing integers, we combine the consecutive signs from left to right two at a time.

(+9) x (–2) ÷ (–6) = (+)(–)(–) (9 x 2 ÷ 6) = (–)(–) (18 ÷ 6) = +3

☺ EXERCISE

4. Multiply the following

(a) (–7)(–5) **(c) (–7)(+5)** **(e) (–9)(+6)** **(g) (+4)(+5)(–2)**

(b) (+7)(–5) **(d) (+9)(–6)** **(f) (+9)(+6)** **(h) (–2)(–7)(+5)**

Answer: (a) +35 (b) –35 (c) –35 (d) –54 (e) –54 (f) +54 (g) –40 (h) +70

5. Divide the following

(a) (–10) ÷ (–5) **(c) (–10) ÷ (+5)** **(e) (–9) ÷ (+3)** **(g) (+45) ÷ (–9)**

(b) (+10) ÷ (–5) **(d) (+9) ÷ (–3)** **(f) (+9) ÷ (+3)** **(h) (–91) ÷ (+13)**

Answer: (a) +2 (b) –2 (c) –2 (d) –3 (e) –3 (f) +3 (g) –5 (h) –7

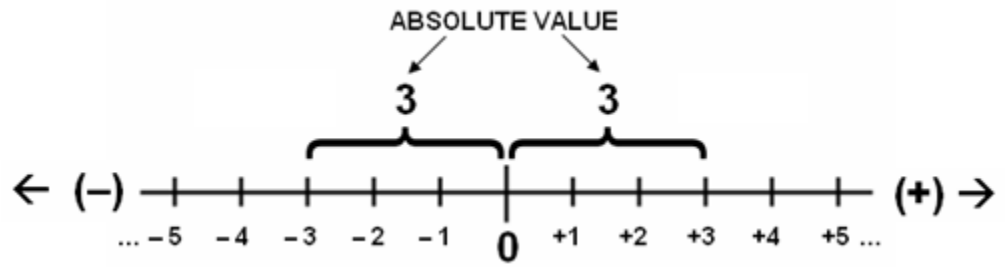
6. Reduce the following

(a) (+3) x (–4) ÷ (+6) **(b) (–15) ÷ (–5) x (+4)** **(c) (–21) ÷ (+14) x (+2)**

Answer: (a) –2 (b) +12 (c) –3

Section 7: Integers & Absolute Value

28. Integers are symmetrical about 0. This means that +3 and –3 are at equal distance from 0. We call this distance the absolute value of the integer. We represent the absolute value by a counting number (no signs)



29. We place two bars around the integer to show that we are talking about the absolute value of the integer.

|–3| = Absolute value of –3 = 3
|+3| = Absolute value of +3 = 3

30. A counting number is treated as positive. Therefore, the absolute value is understood as a positive number.

☺ EXERCISE

Identify absolute values from integers

+15, –15, |+15|, 3, –7, |–8|, +9, 12, |–12|, –19,

Answer: Absolute values are |+15|, 3, |–8|, 12, and |–12|. The integers are +15, –15, –7, +9, and –19

☺ Lesson *Plan 8: Check your Understanding*

1. **What is the purpose of 0 on a number line containing integers?**
2. **How are integers defined by 0?**
3. **Arrange the following integers from the smallest to the largest.**
   +2, –3, +4, –5, +6, –7

Check your answers against the answers given below.

Lesson Plan 8: Answer

1) The “0” always provides the reference point for the integers.
2) Positive integers are greater than 0. Negative integers are less than 0.
3) –7, –5, –3, +2, +4, +6