

MATH LEVEL 1
LESSON PLAN 6

UNITS

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Section 1: Number, Unit & Quantity

1. A QUANTITY is made up of a number and a unit.

Example: 7 is an abstract number; but “7 books” is a quantity of books.

Example: 5 is an abstract number; but “5 inches” is a quantity of length.

2. A UNIT qualifies a number. It is what you count one at a time.

**Example: When you say “7 cats” you have counted “cats” one at a time up to 7.
Each “cat” is a unit.**

Example: When you say “5 feet” you have counted the measure of “foot” one at a time up to 5. Each “foot” is a unit.

☺ **EXERCISE**

Identify number and units in your environment repeatedly.

“There are _____ (number) of _____ (unit).”

Do this until you are happy with the concepts of “number” and “unit”.

Section 2: “Like” and “Unlike” Quantities

3. When two quantities have the same units, they are “like” quantities.

Example: The quantities “3 cats” and “4 cats” are like quantities.

4. When two quantities have different units they are “unlike” quantities.

Example: The quantities “3 cats” and “4 dogs” are unlike quantities.

☺ **EXERCISE**

Identify like and unlike units in your environment repeatedly.

Do this until you are happy with the concepts of “like units” and “unlike unit”.

Section 3: Law of Addition and Subtraction

5. We may add and subtract numbers with “like” units only.

$$\begin{array}{l} 3 \text{ apples} + 2 \text{ apples} = 5 \text{ apples} \\ 10 \text{ oranges} - 2 \text{ oranges} = 8 \text{ oranges} \end{array}$$

6. We cannot add or subtract numbers with “unlike” units.

$$\begin{array}{l} 3 \text{ apples} + 2 \text{ oranges} = ? \quad \text{(Cannot be added)} \\ 10 \text{ apples} - 2 \text{ oranges} = ? \quad \text{(Cannot be subtracted)} \end{array}$$

7. To add and subtract numbers we must convert them to “like” unit first.

$$\begin{array}{l} 3 \text{ apples} + 2 \text{ oranges} = 3 \text{ fruits} + 2 \text{ fruits} = 5 \text{ fruits} \\ 3 \text{ dimes} - 2 \text{ nickels} = 3 \text{ dimes} - 1 \text{ dime} = 2 \text{ dimes} \end{array}$$

8. When we do not mention the units in addition and subtraction, we imply that they are “like” units to be understood from the context.

😊 EXERCISE

1. Add the following to get a sum

(a) 3 horses and 5 camels

(b) 3 horses and 5 horses

(c) 5 horses, 4 camels, 2 horses and 3 camels

Answer: (a) 3 horses and 5 camels (b) 8 horses (c) 7 horses and 7 camels

2. Add the following using a common unit

(a) 4 apples and 3 bananas

(b) 5 men and 5 women

(c) 3 \$10-bills and 5 \$1-bills

Answer: (a) 7 fruits (b) 10 people (c) 8 bills or \$35

Section 4: Conversion of Units

9. We may express the same quantity in different units as follows.

$$1 \text{ dime} = 10 \text{ pennies}$$

$$1 \text{ foot} = 12 \text{ inches}$$

10. A “conversion factor” is a quantity divided by itself in different units. Therefore, a conversion factor is always equal to 1.

$$\frac{10 \text{ pennies}}{1 \text{ dime}} = 1;$$

$$\frac{12 \text{ inches}}{1 \text{ foot}} = 1;$$

$$\frac{1 \text{ dime}}{10 \text{ pennies}} = 1;$$

$$\frac{1 \text{ foot}}{12 \text{ inches}} = 1;$$

11. To reduce a quantity to a different unit we multiply it by the conversion factor. Since the conversion factor is always 1, it does not change the quantity, only the units. In the conversion factor the new unit appears at the top and the old unit at bottom, so that we can cancel out the old unit.

$$5 \text{ feet} = 5 \text{ feet} \times \frac{12 \text{ inches}}{1 \text{ foot}} = 5 \times 12 \text{ inches} = 60 \text{ inches}$$

$$84 \text{ inches} = 84 \text{ inches} \times \frac{1 \text{ foot}}{12 \text{ inches}} = 84 \div 12 \text{ feet} = 7 \text{ feet}$$

$$8 \text{ dimes} = \cancel{8 \text{ dimes}} \times \frac{10 \text{ pennies}}{1 \cancel{\text{ dime}}} = 8 \times 10 \text{ pennies} = 80 \text{ pennies}$$

$$120 \text{ pennies} = 120 \cancel{\text{ pennies}} \times \frac{1 \cancel{\text{ dime}}}{10 \text{ pennies}} = 120 \div 10 \text{ dimes} = 12 \text{ dimes}$$

12. When we reduce a quantity from a larger to a smaller unit, the number always increases. Therefore the conversion always results in a multiplication.

$$\begin{array}{l} 5 \text{ feet} = (5 \times 12) \text{ inches} = 60 \text{ inches} \\ 8 \text{ dimes} = (8 \times 10) \text{ pennies} = 80 \text{ pennies} \end{array}$$

13. When we reduce a quantity from a smaller to a larger unit, the number always decreases. Therefore the conversion always results in a division.

$$\begin{array}{l} 84 \text{ inches} = (84 \div 12) \text{ feet} = 7 \text{ feet} \\ 120 \text{ pennies} = (120 \div 10) \text{ dimes} = 12 \text{ dimes} \end{array}$$

14. To add and subtract “unlike” quantities, we always convert them to “like” quantities first.

$$\begin{array}{l} 8 \text{ feet} - 24 \text{ inches} = 8 \text{ feet} - 2 \text{ feet} = 6 \text{ feet} \\ 3 \text{ dimes} + 23 \text{ pennies} = 30 \text{ pennies} + 23 \text{ pennies} = 53 \text{ pennies} \end{array}$$

😊 EXERCISE

1. Provide 3 examples of conversion factors.

2. Reduce the following:

- (a) 7 feet to inches (c) 10 dimes to pennies (e) 3000 pennies to dimes
 (b) 36 inches to feet (d) 50 pennies to dimes (f) 70,716 inches to feet

Answer: (a) 84 inches (b) 3 feet (c) 100 pennies (d) 5 dimes (e) 300 dimes (f) 5,893 feet

3. Add or subtract the following:

- (a) 6 feet + 12 inches
 (b) 4 dimes – 20 pennies
 (c) 108 inches – 5 feet

Answer: (a) 7 feet or 84 inches (b) 2 dimes or 20 pennies (c) 4 feet or 48 inches

Section 5: Convert Combined Units

15. To convert a speed from miles per hour to feet per second, we use conversion factors of length and time separately. We apply conversion factors in such a way that the old and intermediate units cancel out and only the desired units remain.

EXAMPLE: Convert 60 miles per hour into feet per second.

$$\begin{aligned}
60 \text{ mph} &= \frac{60 \text{ miles}}{1 \text{ hour}} \times \frac{1760 \text{ yards}}{1 \text{ mile}} \times \frac{3 \text{ feet}}{1 \text{ yard}} \times \frac{1 \text{ hour}}{60 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ sec}} \\
&= \frac{60 \times 1760 \times 3 \text{ feet}}{60 \times 60 \text{ sec}} \\
&= 88 \text{ ft/sec}
\end{aligned}$$

😊 EXERCISE

Convert the following:

(a) 72 kilometers/hour to meters/second

(b) 150 miles/hour to feet/second

(c) 180 kilometers/hour to meters / second

Answer: (a) 20 meters/sec (b) 220 feet/sec (c) 50 meters/sec

Section 6: Units in Cost

16. The cost of an item is the money one spends for a single item. For example, the cost of a book is \$10 per book.

17. When we know the cost of one item we can find the cost of multiple items by multiplying the cost with the number of items.

$$\begin{aligned}
\text{Cost of 1 book} &= \frac{\$10}{1 \text{ book}} \\
\text{Therefore, cost of 7 books} &= \frac{\$10}{1 \text{ book}} \times 7 \text{ books} = \$70
\end{aligned}$$

18. When we know the cost of multiple items we can find the cost of one item by dividing the cost with the number of items.

$$\begin{aligned}
\text{Cost of 8 pencils} &= 40 \text{ cents} \\
\text{Therefore, cost of 1 pencil} &= 40 \text{ cents} \div 8 \text{ pencils} = \frac{5 \text{ cents}}{1 \text{ pencil}}
\end{aligned}$$

19. Knowing the cost of a number of items, we can calculate the cost of a different number of items.

EXAMPLE: If 5 oranges cost 45 cents, how much will 8 oranges cost?

$$\begin{aligned}
\text{Cost of 5 oranges} &= 45 \text{ cents} \\
\text{Therefore, cost of 1 orange} &= 45 \text{ cents} \div 5 \text{ oranges} = \frac{9 \text{ cents}}{1 \text{ orange}} \\
\text{Therefore, cost of 8 oranges} &= \frac{9 \text{ cents}}{1 \text{ orange}} \times 8 \text{ oranges} = 72 \text{ cents}
\end{aligned}$$

😊 EXERCISE

(a) If 10 chairs cost \$150, how much will 13 chairs cost?

(b) If 12 books cost \$132, how much will 15 books cost?

(c) If 7 apples cost 91 cents, how much will 18 apples cost?

Answer: (a) \$195 (b) \$165 (c) \$2 and 34 cents

Section 7: Simple and Compound Numbers

20. A **simple number** is a quantity expressed with a single unit, such as,

5 inches; or 25 minutes

21. A **compound number** is a quantity expressed by two or more units, such as,

6 feet 4 inches; or 3 hours 10 minutes

😊 EXERCISE

1. Identify simple and compound numbers:

(a) 6 feet (b) 7 inches (c) 6 feet 7 inches (d) 5 days (e) 3 months 15 days

Answer: (a) simple (b) simple (c) compound (d) simple (e) compound

Section 8: Reducing Compound Lengths

22. The units of length are as follows.

12 inches (in.)	make	1 foot (ft.)
3 feet	make	1 yard (yd.)
1760 yards	make	1 mile (m.)

23. When reducing smaller units to larger units, the conversion requires division. The quotient is expressed in the larger unit, and the remainder remains in the smaller unit.

150 inches = 150 ÷ 12 feet = 12 R6 = 12 feet 6 inches

24. When reducing larger units to smaller units, the conversion requires multiplication. We get a simple number as follows.

6 feet 4 inches = (6 x 12 inches) + 4 inches = 76 inches

25. Here are some conversions from larger to smaller units.

50 miles	=	50 miles x 1760 yards/mile	=	88,000 yards
	=	88,000 yards x 3 feet/yard	=	264,000 feet
	=	264,000 feet x 12 inches/feet	=	3,168,000 inches
16 yd 2 ft 9 in	=	16 yd x 3 ft/yd + 2 ft + 9 inches		
	=	50 ft + 9 in		
	=	50 ft x 12 in/ft + 9 in		
	=	609 in		

26. Here are some conversions from smaller to larger units.

400 inches	=	12	 	400 in	
		3	 	33 ft	4 in
				11 yd	0 ft

$$\begin{aligned}
 &= 11 \text{ yards } 4 \text{ inches} \\
 1,000,000 \text{ in} &= \begin{array}{r} 12 \overline{)1,000,000 \text{ in}} \\ 3 \overline{)83,333 \text{ ft}} \quad 4 \text{ in} \\ 1760 \overline{)27,777 \text{ yd}} \quad 2 \text{ ft} \\ \quad \quad \quad 15 \text{ m} \quad 1377 \text{ yd} \end{array} \\
 &= 15 \text{ miles } 1377 \text{ yards } 2 \text{ feet } 4 \text{ inches}
 \end{aligned}$$

☺ EXERCISE

1. Reduce to inches

(a) 20 yd 1 ft 11 in (b) 1 mile 1043 yd 5 in (c) 83 yd 2 ft 7 in

Answer: (a) 743 in (b) 100,913 in (c) 3,019 in

2. Reduce to higher units

(a) 500 inches (b) 6,258 inches (c) 35,723 inches

Answer: (a) 13 yd, 2 ft, 8 in (b) 173 yd, 2 ft, 6 in (c) 992 yd, 11 in

Section 9: Reducing Compound Times

27. The units of time are as follows.

60 seconds (sec)	make	1 minute (min.)
60 minutes	make	1 hour (hr.)
24 hours	make	1 day (d.)
7 days	make	1 week (wk.)
30 days	make	1 month (mo.)
365 days	make	1 year (yr.)

28. Here are some conversions from larger to smaller units.

3 years	=	3 x 365 days	=	1,095 days
	=	1,095 x 24 hours	=	26,280 hours
	=	26,280 x 60 minutes	=	1,576,800 minutes
	=	1,576,800 x 60 seconds	=	94,608,000 seconds
8 mo, 11 d, 7 hr	=	(8 x 30 + 11) d, 7 hr	=	251 d, 7 hr
	=	(251 x 24 + 7) hr	=	6031 hr
	=	(6031 x 60) min	=	361,860 min
	=	(361,860 x 60) sec	=	21,711,600 sec

29. Here are some conversions from smaller to larger units.

$$\begin{aligned}
 10,000,000 \text{ sec} &= \begin{array}{r} 60 \overline{)10,000,000 \text{ sec}} \\ 60 \overline{)166,666 \text{ min}} \quad 40 \text{ sec} \\ 24 \overline{)2,777 \text{ hr}} \quad 46 \text{ min} \\ 30 \overline{)115 \text{ d}} \quad 17 \text{ hr} \\ \quad \quad \quad 3 \text{ mo} \quad 25 \text{ d} \end{array} \\
 &= 3 \text{ months } 25 \text{ days } 17 \text{ hours } 46 \text{ minutes } 40 \text{ seconds} \\
 12,345,678 \text{ sec} &= 60 \overline{)12,345,678 \text{ sec}}
 \end{aligned}$$

60	205,761 min	18 sec
24	3,429 hr	21 min
30	142 d	21 hr
	4 mo	22 d

= 4 months 22 days 21 hours 21 minutes 18 seconds

☺ **EXERCISE**

1. Reduce to lower units

- (a) 20 hr 16 min 37 sec to seconds
- (b) 3 mo 23 d 13 hr to hours
- (c) 29 d 37 min to minutes

Answer: (a) 72,997 sec (b) 2725 hr (c) 41,797 min

2. Reduce to higher units

- (a) 3,888,333 sec (b) 6,258 min (c) 35,723 hr

Answer: (a) 1 mo, 15 d, 5 min, 33 sec (b) 4 d, 8 hr, 18 min (c) 4 yr, 18 d, 11 hr

Section 10: Adding & Subtracting Compound Numbers

30. To add compound numbers, line up the numbers by units and add them. Then reduce the numbers starting from the smallest unit.

EXAMPLE: Add 17 hr 25 min 19 sec and 15 hr 43 min 56 sec.

	17 hr	25 min	19 sec	
+	15 hr	43 min	56 sec	
	32 hr	68 min	75 sec	
	32 hr	68 min	1 min 15 sec	
	32 hr	69 min	15 sec	
	32 hr	1 hr 9 min	15 sec	
	33 hr	9 min	15 sec	
1 day	9 hr	9 min	15 sec	<i>Answer</i>

31. To subtract a smaller from a larger compound number, line up the numbers by units, and then use borrowing to subtract them.

EXAMPLE: From 8 yd 1 ft 3 in subtract 5 yd 2 ft 11 in

	8 yd	1 ft	3 in
-	5 yd	2 ft	11 in

Or,

	8 yd	0 ft	15 in
-	5 yd	2 ft	11 in

Or,

	7 yd	3 ft	15 in	
-	5 yd	2 ft	11 in	
	2 yd	1 ft	4 in	<i>Answer</i>

Section 11: Multiplying & Dividing Compound Numbers

32. Multiplication is “repeated addition”. One multiplies each unit of the compound number first and then reduces the numbers among those units, starting from the smallest unit.

EXAMPLE: Multiply 17 hr, 25 min, 19 sec by 5.

	17 hr	25 min	19 sec	
			x 5	
	85 hr	125 min	95 sec	
	85 hr	125 min	1 min 35 sec	
	85 hr	126 min	35 sec	
	85 hr	2 hr 6 min	35 sec	
	87 hr	6 min	35 sec	
3 days	15 hr	6 min	35 sec	

33. In division, one starts with the largest unit. The remainder is then reduced to the next unit, which is then divided.

EXAMPLE: Divide 3 days, 4 hr, 32 min, 15 sec by 5.

	15 hr	18 min	27 sec	
5 3 days				
	4 hr	32 min	15 sec	
	76 hr	32 min	15 sec	
	<u>75 hr</u>			
	1 hr	32 min	15 sec	
		92 min	15 sec	
		<u>90 min</u>		
		2 min	15 sec	
			135 sec	
			<u>135 sec</u>	
			0 remainder	

☺ **Lesson Plan 9: Check your Understanding**

1. What is a unit?
2. Add 1 yd 2 ft 6 in, 7 yd 1 ft 3 in, and 4 yd 2 ft 9 in.
3. Divide 17 hr 52 min 35 sec by 5.
4. If 7 chairs cost \$56, how much will 15 chairs cost?
5. Convert 120 mph to ft/sec.

Check your answers against the answers given below.

Lesson Plan 9: Answer

- 1) A UNIT is what you count one at a time.
- 2) 14 yd 6 in
- 3) 3 hr 34 min 31 sec
- 4) \$120
- 5) 176 ft/sec.