DECIMAL FRACTIONS

Section 1:   The Decimal Number

1. The word DECIMAL comes from a Latin word, which means "ten." The Decimal system uses ten digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 to write numbers.

    A number is written with one or more digits. From right to left the orders of digits increase in a tenfold ratio. Thus, in the number 5,329 the rightmost place denotes ONES. The subsequent place values to the left increase tenfold each time as TENS, HUNDREDS, THOUSANDS, and so on. This may continue to the left forever.

    From left to right the orders of digits decrease in a tenfold ratio. However, we stop at "ONES" place because the whole numbers end there.

2. If the decrease in a tenfold ratio continues to the right, we get the place values as unit fractions of TENTHS, HUNDREDTHS, THOUSANDTHS, and so on. This may continue to the right forever.

3. A DECIMAL POINT is used to separate the whole number portion from the fraction portion. It is placed before the TENTHS place value to distinguish the fraction.

    The expanded form of this number is as follows.

    $$5329.468 \;=\; 5000 + 300 + 20 + 9 + \frac{4}{10} + \frac{6}{100} + \frac{8}{1000}$$

    This number is read with all its place values as follows.

$$5\ 3\ 2\ 9\ .\ 4\ 6\ 8$$

- 8 Thousandths
- 6 Hundredths
- 4 Tenths
- 9 Ones
- 2 Tens
- 3 Hundreds
- 5 Thousands

Section 2:   The Decimal Fraction

4. A common fraction is one who denominator may be any number; as 2, 3, 4, etc. A decimal fraction is one whose denominator is 10, and multiples of 10; as 10, 100, 1000, etc. The unit decimal fractions correspond to the place values as follows.

$\frac{1}{10}$ is written   **.1**

$\frac{1}{100}$ is written   **.01**

$\frac{1}{1000}$ is written   **.001**

$\frac{1}{10000}$ is written   **.0001**

$\frac{1}{100000}$ is written   **.00001**, and so on.

The denominator of a decimal fraction is 1 with as many zeroes as there are decimal places in the number. The number **.036254** has 6 decimal places (6 places after the decimal point), therefore

$$.036254 \quad = \quad \frac{36254}{1000000}$$

5. The decimal fractions may be combined as follows.

    1 tenth and 1 hundredth are 11 hundredth

$$\frac{1}{10} + \frac{1}{100} = \frac{11}{100}; \qquad .1 + .01 = .11$$

    2 tenths, 6 hundredths and 5 thousandths are 265 thousandths

$$\frac{2}{10} + \frac{6}{100} + \frac{5}{1000} = \frac{265}{1000}; \qquad .2 + .06 + .005 = .265$$

6. The decimal fractions may be written as follows.

    265 millionths is **0.000265**   (5 is in the position of millionth)

265 hundredths is **2.65**   (5 is in the position of hundredth)

2 units and 17 thousandths is **2.017**   (This is a "mixed" decimal number)

7. When the decimal fractions are too large, they are read literally as follows.

   28.10065701 is **Twenty-eight point one, zero, zero, six, five, seven, zero, one.**

☺ EXERCISE

Write the following decimal numbers.

1. **Twenty-six *hundredths***

2. **Five *thousandths***

3. **Three hundred and four *thousandths***

4. **Seven *millionths***

5. **Read the following decimal numbers.**

   .028, 50.005, 3.0205, 70.1200764

6. **Change the following common fractions to decimals.**

   $$\frac{3}{10}, \quad \frac{97}{100}, \quad \frac{487}{1000}, \quad \frac{101}{10000}, \quad \frac{53}{100000}$$

7. **Change the following decimals to common fractions.**

   .9, .73, .691, .00079, .000068

Answer: (1) .26   (2) .005   (3) .304   (4) .000007   (5) Twenty-eight thousandths, Fifty and five thousandths, Three and two hundred and five ten-thousandth, Seventy point one two zero zero seven six four   (6) .3, .97, .487, .0101, .00053   (7) 9/10. 73/100, 691/1000, 79/100000, 68/1000000

Section 3:   Reduction of Decimals

8. Attaching zeroes before or after a number does not change its value.

   EXAMPLE: **007** is same as **7** in value**,** which is same as **7.00**

   EXAMPLE: **.70** is same as **.7;**

   for $\frac{70}{100} = \frac{7}{10}$

9. To reduce a decimal to a common fraction, write the decimal as a common fraction, and then reduce the fraction to its lowest terms.

   EXAMPLE: Reduce **.75** to a common fraction.

   $$.75 \quad = \quad \frac{75}{100} \quad = \quad \frac{3}{4}$$

10. To reduce a common fraction to a decimal, attach decimal zeroes to the numerator and divide by the denominator. Line up the decimal point.

EXAMPLE: Reduce **3/4** to a decimal.

$$\frac{3}{4} \quad = \quad 3.00 \div 4 \quad = \quad 4\overline{)3.00}^{\;.75} \quad = \quad 0.75$$

EXAMPLE: Reduce **4/3** to a decimal.

$$\frac{4}{3} \quad = \quad 4.000 \div 3 \quad = \quad 3\overline{)4.000\ldots}^{\;1.333\ldots} \quad = \quad 1.333\ldots$$

In this example the decimal continues interminably.

☺ EXERCISE

1. **Reduce the following decimals to common fractions.**

   (a) .6   (b) .25   (c) .375   (d) .5625   (e) 8.415

2. **Reduce the following decimals to common fractions.**

   (a) $\frac{4}{5}$ (b) $\frac{5}{8}$ (c) $\frac{7}{25}$ (d) $\frac{15}{16}$ (e) $\frac{9}{400}$

Answer: 1. (a) 3/5   (b) 1/4   (c) 3/8   (d) 9/16   (e) 8 43/200
        2. (a) .8   (b) .625   (c) .28   (d) .9375   (e) .0225

Section 4:   Addition of Decimals

11. To add decimals, write the numbers so that the decimal points and place values of the same order may stand in the same column. Add as in simple numbers. Place the decimal point in the sum under the column of decimal points.

EXAMPLE: Add .08, .3, .0009, .803, .05.

```
    0 . 0 8              0 . 0 8 0 0
    0 . 3                0 . 3 0 0 0
    0 . 0 0 0 9          0 . 0 0 0 9
    0 . 8 0 3            0 . 8 0 3 0
  + 0 . 0 5            + 0 . 0 5 0 0
  _____          _____
                        1 . 2 3 3 9
```

☺ EXERCISE

1. **Add 21.611; 6888.32; 3.4167.**

2. **Add 37.1065; 432.07; 4.20733; 11.706.**

3. **Add 375.83; 49.627; 5842.1963; 813.9762.**

4. **Add 6.61; 636.1; 6516.14; 67.1234; 5.1233.**

5. Add 35 ten-thousandths; .00035; 35 millionths; and 35 ten-millionths.

Section 5:   Subtraction of Decimals

12. To subtract decimals, write the numbers so that the decimal points and place values of the same order may stand in the same column. Subtract as in simple numbers. Place the decimal point in the remainder under the column of decimal points.

    EXAMPLE: From **11.0010** subtract **4.0028**.

```
    1 1 . 0 0 1 0
  −    4 . 0 0 2 8
    ─────────────
       6 . 9 9 8 2
    ─────────────
```

    EXAMPLE: From **4.1835** subtract **2.03716.** Place zero for the blank.

```
      4 . 1 8 3 5
  −   2 . 0 3 7 1 6
    ─────────────
      2 . 1 4 6 3 4
```

☺ **EXERCISE**

1. **From 97.5168 subtract 38.25942.**

2. **From 27.014 subtract 7.0021.**

3. **From 5.03 subtract 2.115.**

4. **From 24.0042 subtract 13.7013.**

5. **From 1 thousand subtract 1 ten-thousandth.**

Section 6:   Multiplication of Decimals

13. Multiply together the numbers without the decimal points as in simple numbers. Point off as many decimal places in the product as there are decimal places in both factors.

    EXAMPLE: Multiply **2.149** by **6.34**.

    Multiply 2149 by 634 as in simple numbers. There are 3 and 2 decimal places in the factors respectively. Their sum is 5. Hence there must be 5 decimal places in the product.

    2149 x 634   =   1362466 (apply 5 decimal places)   **=   13.62466**

EXAMPLE: Multiply **.0276** by **.035**.

Multiply 276 by 35 as in simple numbers. There are 4 and 3 decimal places in the factors respectively. There sum is 7. Hence there must be 7 decimal places in the product.

If there are more decimal places than the digits then prefix the digits by zeroes. You may omit zeros at the right of the decimal part of the product.

276 x 35   =   9660 (apply 7 decimal places)   **=   .0009660   =   .000966**

EXAMPLE: Multiply **2.075** by **100.**

To multiply a decimal by 10, 100, 1000, etc., move the decimal point as many places to the right as there are zeroes in the multiplier. In this multiplication example we move the decimal point 2 places to the right.

2.075 x 100   =   207.5

EXAMPLE: Multiply **254.5** by **100.**

To move the decimal point 2 places to the right, attach a zero at the end.

254.5 x 100   =   25450

☺ EXERCISE

1. **Multiply 33.21 by 4.41.**
2. **Multiply 32.16 by 22.5.**
3. **Multiply .125 by 9.**
4. **Multiply .135 by .005.**
5. **Multiply 43 thousandths by 21 ten-thousandths.**

Answer: (1) 146.4561   (2) 723.6   (3) 1.125   (4) .000675   (5) .0000903

Section 7:   Division of Decimals

14. Move the decimal points in the dividend and divisor as many places to the right as there are decimal places in the divisor. Then divide keeping the decimal point in the quotient in the same column as in the dividend.

EXAMPLE: Divide **2.125** by **.5**.

There is 1 decimal place in the divisor. Therefore, move decimal points in both dividend and divisor one place to the right, and divide.

2.125 ÷ .5   =   21.25 ÷ 5   =   4.25

EXAMPLE: Divide **.048** by **.006**.

There are 3 decimal places in the divisor. Therefore, move decimal points in both dividend and divisor 3 places to the right, and divide.

.048 ÷ .006   =   48 ÷ 6   =   8

EXAMPLE: Divide **.3** by **.004**

There are 3 decimal places in the divisor. Therefore, move decimal points in both dividend and divisor 3 places to the right, and divide.

.3 ÷ .006   =   .300 ÷ .006   =   300 ÷ 6   =   50

EXAMPLE: Divide **83.1** by **4**

Here we do not have to shift the decimal points, but we attach 2 zeroes to the dividend to make the division exact.

83.1 ÷ 4   =   83.100 ÷ 4   =   20.775

EXAMPLE: Divide **2.11** by **3**

Again, we do not have to shift the decimal points, but we attach 1 or more zeroes to the dividend to carry out the division as far as is wanted..

2.11 ÷ 3   =   2.11000 ÷ 3   =   .70333…

EXAMPLE: Divide **475.6** by **100**

To divide a decimal by 10, 100, 1000, etc., move the decimal point as many places to the left as there are zeroes in the divisor. In this division example we move the decimal point 2 places to the left.

475.6 ÷ 100   =   4.756

☺ **EXERCISE**

1. **Divide 21 by .5.**
2. **Divide 2 by .008.**
3. **Divide 1.125 by .03.**
4. **Divide .21318 by .19.**
5. **Divide .08 by 100.**

Answer: (1) 42   (2) 250   (3) 37.5   (4) 1.122   (5) .0008

☺ **L2 Lesson Plan 5: Check your Understanding**

1. **How a place value is related to the place value (a) on the immediate right (b) on the immediate left?**

2. **How far do the place values extend on either side of the number?**

3. **Where exactly is the decimal point placed in a number?**

4. **Write the expanded form of the number 66,063.0508.**

Check your answers against the answers given below.

Answer:
1) A place value is (a) Ten times the place value to its right, (b) Tenth of the place value to its left.
2) The place values extend to either side without limit.
3) The decimal point is always placed to the right of the ONES.
4) $60,000 + 6000 + 60 + 3 + 5/100 + 8/10,000$