# MATH LEVEL 1 <br> LESSON PLAN 5 <br> DIVISION 

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## Section 1: Basic Division

1. DIVISION is the number of times a number can be taken out of another as if through "repeated subtraction".

When there are 30 pennies on the table, you can take a group of 6 pennies away only 5 times, with no pennies remaining. We write this as follows.

$$
30 \div 6=5 ; \quad \text { because } \quad \begin{array}{r}
30-6-6-6-6-6 \\
\text { Subtract five times }
\end{array}=0
$$

This is called EXACT DIVISION since there is no remainder.
But when there are 33 pennies on the table and you take groups of 6 pennies away, then 3 pennies are left in the end.

$$
33 \div 6=5 \text { R3; because } \quad \begin{gathered}
33-6-6-6-6-6 \\
\text { Subtract five times }
\end{gathered}=\underset{\text { Remainder }}{3}
$$

This is called INEXACT DIVISION since there is a remainder.
2. The original number of pennies on the table are called dividend. The pennies taken away at a time are called the divisor. The maximum number of times the divisor can be taken out of the dividend is called the quotient. And the pennies left at the end, because they are less than the divisor, are called the remainder.

$$
\underset{\text { Dividend }}{33} \div \underset{\text { Divisor }}{6}=\underset{\text { Quotient }}{5} \quad \begin{aligned}
& \text { R3 } \\
& \text { Remainder }
\end{aligned}
$$

3. A number can be taken out of itself exactly 1 time.

$$
6 \div 6=1 ; \quad 30 \div 30=1
$$

1 can be taken out of a number as many times as the number.

$$
6 \div 1=6 ; \quad 30 \div 1=30
$$

When there is nothing (0), you cannot take anything out of it at all..

$$
0 \div 6=0 ; \quad 0 \div 30=0
$$

When you are taking nothing (0) out of a number, it can be "taken out" any number of times.

$$
6 \div 0=\text { undefined; } \quad 30 \div 0=\text { undefined }
$$

## EXERCISE

A. Find the quotient with remainder, if any. Tell if the division is exact or inexact.
(a) $9 \div 3$
(c) $16 \div 4$
(e) $25 \div 5$
(g) $12 \div 4$
(b) $9 \div 2$
(d) $16 \div 5$
(f) $23 \div 6$
(h) $12 \div 10$

Answer: (a) 3 exact (b) 4 R1 inexact (c) 4 exact (d) 3 R1 inexact (e) 5 exact (f) 3 R5 inexact (g) 3 exact (h) 1 R2 inexact
B. Find the quotients for the following:
(a) $9 \div 9$
(c) $9 \div 0$
(e) $0 \div 23$
(g) $24 \div 1$
(b) $8 \div 1$
(d) $8 \div 8$
(f) $28 \div 0$
(h) $0 \div 4$

Answer: (a) 1 (b) 8 (c) undefined $\begin{array}{llllll} & \text { (d) } 1 & \text { (e) } 0 & \text { (f) undefined } & \text { (g) } 24 & \text { (h) } 0\end{array}$
C. Find the quotient using multiplication tables
(a) $27 \div 3$
(c) $48 \div 6$
(e) $72 \div 9$
(g) $36 \div 4$
(b) $56 \div 8$
(d) $45 \div 5$
(f) $16 \div 2$
(h) $35 \div 7$

Answer: (a) 9 (b) 7 (c) 8 (d) 9 (e) 8 (f) 8 (g) 9 (h) 5

## Section 2: Dividing by Single-Digit Divisors (Short Division)

4. We divide larger numbers from left to right by breaking them into their place values.

$$
\begin{aligned}
486 \div 2 & =(400+80+6) \div 2 \\
& =200+40+3 \\
& =243
\end{aligned}
$$

We may also divide larger numbers by writing them as follows.


Hundreds: From 4 we can take 2 out 2 times (no remainder)
Tens: $\quad$ From 8 we can take 2 out 4 times (no remainder)
Hundreds: From 6 we can take 2 out 3 times (no remainder)
Therefore, $\quad 486 \div \mathbf{2}=\mathbf{2 4 3}$
To check, multiply 243 by 2 . You should get back 486 because multiplication is reverse of division.

The 'long form' of division is shown on the right. We pull down the next digit and place it to the right of the remainder. We then divide the resulting number.

$$
\begin{array}{ll}
\text { Therefore, } & 572 \div 2=286 \\
& \text { (Check } 286 \times 2=572)
\end{array}
$$



The 'short form' of division is shown on the right. We place the remainder from a column to the left of the next digit. We then divide the resulting number.

$$
\text { 2) } \begin{array}{c|c|c|}
\hline 2 & 8 & 6 \\
\hline 5 & 17 & 12
\end{array}
$$

5. When the first digit of the dividend cannot be divided, then start with the first two digits. Make sure you write the digit of the quotient in the correct column.

$$
\begin{array}{ll}
\text { Therefore, } & 564 \div 6=94 \\
& \text { (Check } 94 \times 6=564 \text { ) }
\end{array}
$$


6. When a number in a column cannot be divided, put a 0 in the quotient for that column.

Therefore, $\quad 728 \div 7=104$
(Check $104 \times 7=728$ )

7. We may also write division with quotient below the dividend as shown on the right. This has some advantages when factoring numbers (see Lesson Divisor 2 | 5 | 17 | 12 |
| ---: | ---: | ---: | ---: |
| 2 | 8 | 6 | Dividend plan on FACTORS).

## EXERCISE

In the following exercises either use a Multiplication Table, or make the table for the divisor on the spot.
A. Divide the following and check your answers by multiplying back.
(a) $396 \div 3$
(c) $444 \div 3$
(e) $435 \div 5$
(g) $450 \div 6$
(b) $327 \div 3$
(d) $612 \div 6$
(f) $315 \div 7$
(h) $224 \div 4$

Answer: (a) 132
(b) 109
(c) 148
(d) 102
$\begin{array}{ll}\text { (e) } 87 & \text { (f) } 45\end{array}$
(g) 75
(h) 56
B. Divide the following and check your answers by multiplying back.
(a) $844 \div 4$
(c) $732 \div 6$
(e) $57 \div 3$
(g) $3,174 \div 3$
(b) $992 \div 8$
(d) $894 \div 6$
(f) $847 \div 7$
(h) $9,945 \div 5$

Answer:
$\begin{array}{llll}\text { (a) } 211 & \text { (b) } 124 & \text { (c) } 122 & \text { (d) } 149\end{array}$
(e) 19 (f) 121 (g) 1,058
(h) 1,989
8. When there is a final remainder at the end of division, you write it next to the quotient as shown. To check your answer multiply back as before and then add the
 remainder.

Therefore, $\mathbf{7 6 4 \div 3} \mathbf{=} \mathbf{2 5 4} \mathbf{R 2}$
[Check $(254 \times 3)+2=764]$. Note that we add remainder to get back the dividend.

## EXERCISE

A. Check your answer by multiplying back and the adding the remainder.
(a) $15 \div 4$
(c) $139 \div 6$
(e) $578 \div 3$
(g) $3,177 \div 2$
(b) $92 \div 8$
(d) $793 \div 7$
(f) $897 \div 5$
(h) $9,745 \div 9$

## Section 3: Dividing by Double-Digit Divisors (Long Division)

9. When divisors are large, the long form division is preferred because the remainders are large.

## Example: Divide $10853 \div 16$

The steps are:
(a) The Divisor is 16 . We first make the table of 10 multiples for 16 as follows:

| $16 \times 1$ | $(10+6) \times 1$ | = | $10+6$ | = | 16 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $16 \times 2$ | $(10+6) \times 2$ | = | $20+12$ | = | 32 |
| $16 \times 3$ | $(10+6) \times 3$ | = | $30+18$ |  | 48 |
| $16 \times 4$ | $(10+6) \times 4$ | = | $40+24$ |  | 64 |
| $16 \times 5$ | $(10+6) \times 5$ | = | $50+30$ | = | 80 |
| $16 \times 6$ | $(10+6) \times 6$ | = | $60+36$ |  | 96 |
| $16 \times 7$ | $(10+6) \times 7$ | = | $70+42$ |  | 112 |
| $16 \times 8$ | $(10+6) \times 8$ | = | $80+48$ | = | 128 |
| $16 \times 9$ | $(10+6) \times 9$ | = | $90+54$ | = | 144 |
| $16 \times 10$ | $(10+6) \times 10$ | = | $100+60$ | = | 160 |

(b) 16 cannot be taken out of the first two digits. Try taking it out the first 3 digits.
(c) 16 can be taken out of 108 six times $(16 \times 6=96)$. Place 6 in the column above 8. Subtract 96 from 108. The remainder is 12 . Bring down the next digit 5 . We get 125 .
(d) 16 can be taken out of 125 seven times ( $16 \times 7=112$ ). Place 7 in the column above 5. Subtract 112 from 125. The remainder is 13. Bring down the next digit 3 . We get 133.
(e) 16 can be taken out of 133 eight times $(16 \times 8=128)$. Place 8 in the column above 3. Subtract 128 from 133.

## Long Form

 The remainder is 5 . There are no more digits.
(f) The quotient is 678 , and there is a remainder of 5 .

Therefore, $10853 \div 16=678$ R5
[Check $(678 \times 16)+5=10853]$

## EXERCISE

Divide by first writing down the multiples of the divisor
(a) $108 \div 12$
(d) $3225 \div 12$
(g) $976 \div 11$
(b) $911 \div 11$
(e) $1111 \div 11$
(h) $322 \div 13$
(c) $432 \div 14$
(f) $4555 \div 15$
(i) $3567 \div 18$
10. Instead of making the table of multiples, we may first guess the needed multiple.

Divide, $563 \div 62$
Round to tens as follows:
563 / 62 is about 560 / 60; the "tens" are 56 / 6.
We can take 6 out of 56 nine times.
Therefore, 62 may be taken out of 563 about 9 times.
Compute $62 \times 9=558$. This gives us, $563 \div 62=9$ R5

## Divide, $396 \div 44$

Round to tens as follows:
396 / 44 is about 400 / 40; the "tens" are 40 / 4.
We can take 4 out of 40 ten times.
Therefore, 44 may be taken out of 396 about 10 times.
Compute $44 \times 10=440$ (more than 396)
Compute $44 \times 9=396$. This gives us, $396 \div 44=9$

## Divide, $611 \div 87$

Round to tens as follows:
611 / 87 is about 610 / 90 ; the "tens" are 61 / 9.
We can take 9 out of 61 six times.
Therefore, 87 may be taken out of 611 about 6 times.
Compute 87 x $6=522$ (much smaller than 611)
Compute $87 \times 7=609$. This gives us, $611 \div 87=7$ R2

## Divide $630526 \div 37$

The steps are
(a) For $63 \div 37$ round them to 60 and 40 . We can take out the tens (4 out of 6) one time. Check $37 \times 1=37$; and $37 \times 2=74$. We can take 37 out of 63 one time. Write 1 above 63 (line it up to the right). Subtract $63-37=26$.
(b) Pull the next digit (0) down and place it next to 26, making it 260.
(c) For $260 \div 37$ round them to 260 and 40 . We can take out the tens (4 out of 26) 6 times. Check $37 \times 6=222$; and $37 \times 7=$
 259. We can take 37 out of 260 , seven times. Write 7 above 0 . Subtract $260-259=1$.
(d) Pull the next digit (5) down and place it next to 1 , making it 15.
(e) For $15 \div 37$ we obviously get 0 . Write 0 above 5 . Subtract $15-0=15$.
(f) Pull the next digit (2) down and place it next to 15, making it 152.
(g) For $152 \div 37$ round to 150 and 40 . We can take out the tens ( 4 out of 15 ) 3 times. Check $37 \times 3=111$; and $37 \times 4=148$. We can take 37 out of 152 , four times. Write 4 above 2. Subtract $152-148=4$.
(h) Pull the next digit (6) down and place it next to 4, making it 46 .
(i) For $46 \div 37$, we can 37 out of 46 one time. Write 1 above 6 . Subtract $46-37=9$. There are no more digits, so the final remainder is 9 .

Therefore, $630526 \div 37=17041$ R9
[Check (17041 x 37) $+9=630526$ ]

## EXERCISE

Divide by approximating the quotient.
(a) $144 \div 16$
(d) $217 \div 45$
(g) $7488 \div 61$
(b) $123 \div 23$
(e) $318 \div 53$
(h) $82593 \div 71$
(c) $259 \div 37$
(f) $419 \div 59$
(i) $994720 \div 89$

Answer: (a) 9 (b) 5 R8 $\quad$ (c) 7 (d) 4 R37 $\begin{array}{llllll}\text { (e) } 6 & \text { (f) } 7 \text { R6 } & \text { (g) } 122 \text { R46 } & \text { (h) } 1163 \text { R20 } & \text { (i) } 11176 \text { R56 }\end{array}$

## Section 4: Word Problems

11. Word problems require a translation of words into mathematical language.
(a) Example 1: If 6 apples cost 54 cents, how much would 9 apples cost?

LOGIC: First find out how much one apple costs. Then you can find out how much 9 apples would cost.

Cost of 6 apples $=54$ cents
Cost of 1 apple $=54 \div 6=9$ cents
Cost of 9 apple $=9 \times 9=81$ cents Answer
(b) Example 2: Joe, Bob and Lisa shared a prize of $\$ 108$ equally among themselves. Lisa then shared her money equally with her two sisters. How much money did Lisa get?
LOGIC: First find out how much Lisa got before sharing the prize with her sisters. Then you can find out how much Lisa got after sharing with her sisters.

Amount Lisa got initially $=\$ 108 \div 3=\$ 36$
Amount Lisa got finally $=\$ 36 \div 3=\$ 12$ Answer

## EXERCISE

Do the following word problems. Check your answer against the answers given.
(a) You bought 5 bananas for 30 cents. How much will 1 banana cost? 8 bananas cost?
(b) You bought 14 pencils for 70 cents. How much will 1 pencil cost? 21 pencils cost?
(c) You won a lottery of $\$ 600$, which you shared with 4 of your friends. You then shared your winnings equally with your brother and sister. How much money did you end up with?

## © Lesson Plan 5: Check your Understanding

1. How is division related to multiplication?
2. Divide (a) $47,971 \div 7$ (b) $299,997 \div 3$ (c) $751,941 \div 9$
3. Divide (a) $47,971 \div 27$ (b) $299,997 \div 63$ (c) $751,941 \div 89$

Check your answers against the answers given below.

## Answers:

1) Division is "reverse multiplication."
2) (a) 6,853 (b) 99,999 (c) 83,549
3) (a) $1,776 \mathrm{R} 19$ (b) $4,761 \mathrm{R} 54$ (c) $8,448 \mathrm{R} 69$.
