SOLUTIONS: Stage I Question Set 4

Solution to Question #1:

Multiply each side by x. $\frac{4}{x}(x) = \frac{x}{4}(x)$, so $4 = \frac{x^2}{4}$. $16 = x^2$, so $x = +4$ or $x = -4$.

Since x < 0, x = -4 , The correct answer is (d) .

Solution to Question #2:

$2^3 + 2^4 + 1 = 8 + 16 + 1 = 25$

The correct answer is (b) .

Solution to Question #3:

$x - 1 = 0, \sqrt{x} = 1, \sqrt{x} + 1 = 2, \frac{1}{x} = 1$

The correct answer is (a) .

Solution to Question #4:

40 x 2 = twice the rain which fell in Drydale = 80 cm 80 cm of rain fell last year in Raintown.
The correct answer is (b) .

Solution to Question #5:

$\frac{\sqrt{x}}{x} + \sqrt{x} = \frac{\sqrt{16}}{16} + \sqrt{16} = \frac{4}{16} + 4 = 4\frac{1}{4}$

The correct answer is (a) .

Solution to Question #6:

a) The product of 6 and 2 is 6 x 2 = 12. The sum of 4 and 8 is 4 + 8 = 12.
The two amounts are equal, so this statement is false.

b) The sum of -5 and 3 is -5 + 3 = -2. The difference of 8 and 10 is 8 - 10 = -2.
The two amounts are equal, so this statement is false.

c) The quotient of 5 and 3 is 5 ÷ 3 = 1 2/3. 1 2/3 < 2, so this statement is false.

d) The product of 3 and 5 and 4 is 3 x 5 x 4 = 60. The quotient of 120 and 3 is 120 ÷ 3 = 40.
60 > 40, so this statement is true.

e) Since d) is true, e) is false.

The correct answer is (d) .

Solution to Question #7:

a) If the last two digits of a number are divisible by 4, that number is divisible by 4. Since 24 is divisible by 4, 724 is divisible by 4. Alternatively, you could do the problem 724 ÷ 4 = 181, proving again that 724 is divisible by 4. This statement is true.

b) If the digits of a number add up to a number divisible by 3, the number is divisible by 3. With 1011, (1 + 0 + 1 + 1) = 3, and since 3 is divisible by 3, 1011 is divisible by 3. Alternatively, you can do the division problem 1011 ÷ 3 = 337, which also demonstrates that 1011 is divisible by 3.

c) To find out if 1836 is divisible by 12, find out if it is divisible by both 3 and 4. The last two digits of 1836, that is 36, are divisible by 4, so the number 1836 is divisible by 4. To find out if 1836 is divisible by 3, add the digits together (1 + 8 + 3 + 6) = 18, which is divisible by 3. Since 1836 is divisible by both 3 and 4, it is divisible by 12. Alternatively, you can do the problem 1836 ÷ 12 = 153. This statement is true.

d) a), b) & c) are all true, so d is the correct answer.

e) This statement is false. The correct answer is (d) .

Solution to Question #8:
The prime numbers between 12 and 24 are 13, 17, 19, and 23. There are 4 prime numbers between 12 and 24. The correct answer is (c) .

Solution to Question #9:
p = original price of the computer. The marked-down price is the original price less the 10% discount.
$1,800 = p - 0.1p = 0.9p; 1800 ÷ 0.9 = p = 2000
The original price was $2,000. $2,000 was not listed in the choices (a)-(d)
The correct answer is (e) .

Solution to Question #10:
For the first portion of his journey, if Walter flew 1200 air-miles at a rate of 500 miles/hour, the time he took is 1200 ÷ 500 = 2.4 hours. For the second portion of his journey, if Walter flew 3000 air-miles at a rate of 600 miles/hour, the time he took is 3000 ÷ 600 = 5 hours. The total time taken is 5 + 2.4 = 7.4 hours. The correct answer is (b) .

Solution to Question #11:
B = area of the two squares which are cut away = $2a^2$
$64 = 48 + B$; $B = 16 = 2a^2$; $a^2 = 8$; Since a is positive, $a = 2\sqrt{2}$ cm
The correct answer is (a)

Solution to Question #12:
O is the centre of both circles. OA = 10 cm. OB = 4 cm.
D = The area of the doughnut-shaped region
  = the area of the large circle minus the area of the small circle.
  = $\pi(10 \times 10) - \pi(4 \times 4) = 84\pi$ cm²
Therefore, (c) is the correct answer.

Solution to Question #13:
If a + b = 14, and b = 3, then a = 11.
If a = 11, and x + a = 24, then x = 13.
The correct answer is (b) .

Solution to Question #14:
Since the three angles of a triangle add up to 180°, 180 = 120 + 40 + x, x = 20°
The value of c can be deduced since 120 + c = 180 (since the measure of a straight line is 180). c = 60°
You deduce a by looking at the triangle with the angles a, c, and 60.
180 = a + c + 60 = a + 60 + 60, a = 60°
You know that x + a + b = 180. x + a + b = 20 + 60 + b = 180. b = 100°

Alternatively, the straight angle, (b + a + x) = 180. Therefore, b = 180 - (a+x).
From the larger triangle, (a + x) + 60 + 40 = 180. Therefore, 180 - (a + x) = 60 + 40 = 100.
Hence, b = 180 - (a+x) = 100. The correct answer is (d) .

Solution to Question #15:
The area of triangle ACD = [area of triangle ABC] - [area of triangle ABD]
The area of triangle ABD is ½(base x height) = ½(1)(4) = 2
The area of triangle ABC is ½(1 + 2)(4) = 6.
The area of triangle ACD = 6 - 2 = 4 units²
The correct answer is (c) .

Solution to Question #16:
a) If Diane has exactly 6 quarters, that is $1.50. The remaining 50 cents can be made up with 5 dimes, so this statement is false.

b) If Diane has exactly 7 quarters, that is $1.75. The remaining 25 cents cannot be made up with dimes only, so there must be at least one nickle. This statement is true. (???)
c) The 7 dimes make 70 cents. The remaining amount is $1.30. 5 quarters = $1.25, so there must be at least one nickle. This statement is true.
d) Since a is false, d is false.
e) Since b & c are true, this is the correct answer. (???}

Solution to Question #17:
a) This statement is true.
b) The volume of a cube which is 3 x 3 x 3 is 27 m^3. The volume of a rectangular solid which is 1 x 2 x 4 is 8 m^3.
c) The surface area of each surface of the cube is 3 x 3. Since there are 6 identical surfaces, the surface area of the entire cube is 6(3 x 3) = 54 m^2 . The surface area of the rectangular solid is 2(1 x 2) + 2(1 x 4) + 2(2 x 4) = 4 + 8 + 16 = 28 m^2. Since the surface area of the cube is greater than the surface area of the rectangular solid, this statement is true.
d) Since b is also correct, this statement is false.
e) This statement is true.
The correct answer is (e) .

Solution to Question #18:
The difference between the volume of the cube and the rectangular solid is 5, so
$s^3 - (s - 1)(s)(s + 1) = s^3 - (s^3 - s) = 5 = s$
Since the rectangular solid has dimensions (s - 1) x s x (s + 1), the dimensions are 4 x 5 x 6.
The correct answer is (c) .

Solution to Question #19:
The radius of the smaller circle is 4, since the line segment from the centre to the edge is 4.
The formula for the circumference of a circle is $C = 2\pi r$. The circumference of the smaller circle is $2\pi(4) = 8\pi$. The circumference of the larger circle is 3 times as much as the circumference of the smaller circle so it is 24π. $24\pi = 2\pi R$. R = radius of the larger circle = 12 units. Since $A = \pi R^2$, the area of the larger circle is $A = \pi(12)^2 = 144\pi$. The correct answer is (b) .

Solution to Question #20:
First, convert all of the dimensions to meters from centimeters. 40 cm = 0.4 m. 3 cm = 0.03 m.
The surface area of the two large surfaces of the door is (2.5 x 0.4) x 2 = 2 m^2.
The surface area of the top and bottom surfaces of the door is 2 x (0.4 x 0.03) = 2 x 0.012 = 0.024 m^2.
The surface area of the side surfaces of the door is 2 x (2.5 x 0.03) = 2 x 0.075 = 0.15 m^2
The total surface area of the door is (2 + 0.024 + 0.15) = 2.174 m^2.
The correct answer is (d) .