PERIODIC OR REPEATING DECIMALS

FACT 1: Repeating decimals are written with a bar over the repeating digit.

Repeating decimals occur whenever the denominator contains a factor other than 2 and 5.

$$\frac{2}{3} = 0.66666\ldots = 0.\overline{6} \quad \text{(Read as "zero point period six")}$$

$$\frac{3}{7} = 0.428571\,428571\,428571\ldots = 0.\overline{428571}$$

FACT 2: We convert a repeating decimal to a common fraction by subtracting out the repeating portion.

Express 0.55555… in decimal notation.

$$0.\overline{5} = 0.5555555\ldots \quad \text{Equation (1)}$$
$$10 \times 0.\overline{5} = 5.5555555\ldots \quad \text{Equation (2)}$$

By subtracting (1) from (2), we cancel out the repeating portion.

$$(10 - 1) \times 0.\overline{5} = 5.55555\ldots - 0.55555\ldots$$
$$\text{or,} \quad 9 \times 0.\overline{5} = 5$$
$$\text{or,} \quad 0.\overline{5} = \frac{5}{9}$$

A short cut is to divide the repeating digits of the decimal fraction by as many 9's.

$$0.\overline{148} = 0.148148148\ldots = \frac{148}{999} = \frac{4}{27}$$

$$0.\overline{142857} = \frac{142857}{999999} = \frac{1}{7}$$

FACT 3: We convert a mixed repeating decimal to a common fraction in a similar manner.

Express 0.954545454… in decimal notation.

$$0.9\overline{54} = 0.95454545454\ldots \quad (1)$$
$$10 \times 0.9\overline{54} = 9.5454545454\ldots \quad (2)$$
$$1000 \times 0.9\overline{54} = 954.5454545454\ldots \quad (3)$$

Subtracting (2) from (3), we cancel out the repeating portion

$$(1000 - 10) \times 0.9\overline{54} = (954 - 9)$$
$$\text{or,} \quad 990 \times 0.9\overline{54} = 945$$
$$\text{or,} \quad 0.9\overline{54} = \frac{945}{990} = \frac{21}{22}$$

A short cut would be to write a fraction where the numerator is "total (non-repeating and repeating) digits, minus the non-repeating digits," and the denominator is "as many 9's as total digits minus as many 9's as the non-repeating digits."

$$0.7\overline{3} = \frac{73-7}{99-9} = \frac{66}{90} = \frac{11}{15}$$

$$0.37\overline{6} = \frac{376-37}{999-99} = \frac{339}{900} = \frac{113}{300}$$

1. Express the following common fractions as decimal fractions. Use the periodic notation to express repeating decimal fractions.

(a) 4 / 9	(d) 4 / 11	(g) 5 / 6	(j) 11 / 15
(b) 4 /11	(e) 5 / 13	(h) 1 / 6	(k) 9 / 13
(c) 4 / 7	(f) 7 / 9	(i) 5 / 11	(l) 2 / 11

Answer: (a) $0.\overline{4}$ (b) $0.\overline{36}$ (c) $0.\overline{571428}$ (d) $0.\overline{81}$ (e) $0.\overline{384615}$ (f) $0.\overline{7}$ (g) $0.8\overline{3}$ (h) $0.1\overline{6}$ (i) $0.\overline{45}$ (j) $0.7\overline{3}$ (k) $0.\overline{692307}$ (l) $0.\overline{18}$

2. Convert the following periodic decimals to common fractions.

(a) $0.\overline{3}$	(d) $0.\overline{42}$	(g) $0.\overline{621}$	(j) $0.\overline{714285}$
(b) $0.\overline{36}$	(e) $0.\overline{648}$	(h) $0.\overline{216}$	(k) 0.1111…
(c) $0.\overline{6}$	(f) $0.\overline{108}$	(i) $0.\overline{567}$	(l) $0.\overline{428571}$

Answer: (a) 1/3 (b) 4/11 (c) 2/3 (d) 14/33 (e) 24/37 (f) 4/37 (g) 23/37 (h) 8/37 (i) 21/37 (j) 5/7 (k) 1/9 (l) 3/7

End of Lesson