

## LEAST COMMON MULTIPLE

To convert 'unlike fractions' into equivalent 'like fractions' one requires the least common multiple of the unlike denominators.

**FACT 1: The product of denominators provides a common multiple.**

A common multiple of 10 and 15 is  $10 \times 15 = 150$ .

150 is a multiple of 10.

150 is also a multiple of 15

**FACT 2: The least common multiple (LCM) is the smallest common multiple of the denominators.**

150 is not the least common multiple of 10 and 15.

Multiples of 10 are: 10, 20, **30** ...

Multiples of 15 are: 15, **30** ...

Thus, the least common multiple of 10 and 15 is **30**.

**FACT 3: We obtain the LCM by eliminating any repeat occurrence of a common factor from the product of denominators.**

One repeat occurrence of the common factor '5' is eliminated to get the LCM.

$$\begin{array}{rcl} \text{Product} & = & 10 \times 15 = (2 \times 5) \times (3 \times 5) = 150 \\ \text{Least Common Multiple} & = & (2 \times 5) \times (3 \times \cancel{5}) = 30 \end{array}$$

**FACT 4: We may eliminate the repeat occurrence of common factor by applying side by side division to the denominators as follows.**

We divide the denominators by common factor. The LCM is the product of the common factor with the remaining factors in the bottom row.

$$\begin{array}{r} 5 \mid \underline{10, \quad 15} \\ \quad \quad \quad 2, \quad 3 \\ \text{LCM} \quad = \quad 5 \times 2 \times 3 \quad = \quad 30 \end{array} \quad \text{(Divide by the common factor 5)}$$

Find the LCM of 42 and 63.

$$\begin{array}{r} 7 \mid \underline{42, \quad 63} \quad (7 \text{ is a common factor}) \\ 3 \mid \underline{6, \quad 9} \quad (3 \text{ is a common factor}) \\ \quad \quad \quad 2, \quad 3 \quad (\text{There are no more common factors}) \\ \text{LCM} \quad = \quad 7 \times 3 \times 2 \times 3 \quad = \quad 126 \end{array}$$

**FACT 5: For more than two denominators, a common prime factor to any two of the denominators may be taken out as follows.**

Find the LCM of 9, 14 and 21.

$$\begin{array}{r}
 3 | \underline{9, \quad 14, \quad 21} \quad (3 \text{ is a prime factor common to 9 and 21, bring 14 down as-is}) \\
 7 | \underline{3, \quad 14, \quad 7} \quad (7 \text{ is a prime factor common to 14 and 7, bring 3 down as-is}) \\
 \quad \quad \quad 3, \quad 2, \quad 1 \quad (\text{No prime factor is common to any two of these numbers})
 \end{array}$$

$$\text{LCM} = 3 \times 7 \times 3 \times 2 \times 1 = 126$$

**FACT 6: We use LCM is the common denominator of the equivalent “like” fractions.**

To compare  $\frac{7}{10}$  to  $\frac{11}{15}$ , we compute the LCM of 10 and 15 as 30 (see FACT 4).

$$\begin{array}{rcl}
 \frac{7}{10} = \frac{7 \times 3}{10 \times 3} = \frac{21}{30} \quad \text{and} \quad \frac{11}{15} = \frac{11 \times 2}{15 \times 2} = \frac{22}{30} \\
 \text{Since } \frac{21}{30} < \frac{22}{30}, \text{ therefore } \frac{7}{10} < \frac{11}{15}
 \end{array}$$

**1. Find the LCM (Least Common Multiple) of the following set of numbers:**

(a) 4 and 9      (b) 6 and 9      (c) 14 and 42      (d) 36 and 60  
 (e) 6, 15 and 18      (f) 6, 13 and 26      (g) 26, 33, 39 and 44

Answer: (a) 36 (b) 18 (c) 42 (d) 180 (e) 90 (f) 78 (g) 1716

**2. Use to LCM to find the equivalent like fractions for the following pairs of fractions.**

$$\begin{array}{llll}
 \text{(a)} \frac{3}{5}, \frac{3}{10} & \text{(d)} \frac{5}{9}, \frac{7}{12} & \text{(g)} \frac{7}{15}, \frac{11}{25} & \text{(j)} \frac{9}{14}, \frac{11}{21} \\
 \text{(b)} \frac{5}{6}, \frac{8}{9} & \text{(e)} \frac{3}{10}, \frac{4}{15} & \text{(h)} \frac{1}{6}, \frac{1}{8} & \text{(k)} \frac{19}{24}, \frac{11}{16} \\
 \text{(c)} \frac{3}{4}, \frac{1}{6} & \text{(f)} \frac{3}{8}, \frac{5}{12} & \text{(i)} \frac{5}{9}, \frac{7}{15} & \text{(l)} \frac{13}{20}, \frac{11}{15}
 \end{array}$$

Answer: (a) 6/10, 3/10 (b) 15/18, 16/18 (c) 9/12, 2/12 (d) 20/36, 21/36  
 (e) 9/30, 8/30 (f) 9/24, 10/24 (g) 35/75, 33/75 (h) 4/24, 3/24  
 (i) 25/45, 21/45 (j) 27/42, 22/42 (k) 38/48, 33/48 (l) 39/60, 44/60

**End of Lesson**