

## LEAST COMMON MULTIPLE

To convert 'unlike fractions' into equivalent 'like fractions' one requires the least common multiple of the unlike denominators.

**FACT 1: The product of denominators provides a common multiple.**

A common multiple of 10 and 15 is  $10 \times 15 = 150$ .

150 is a multiple of 10.

150 is also a multiple of 15

**FACT 2: The least common multiple (LCM) is the smallest common multiple of the denominators.**

150 is not the least common multiple of 10 and 15.

Multiples of 10 are: 10, 20, **30** ...

Multiples of 15 are: 15, **30** ...

Thus, the least common multiple of 10 and 15 is **30**.

**FACT 3: We obtain the LCM by eliminating any repeat occurrence of a common factor from the product of denominators.**

One repeat occurrence of the common factor '5' is eliminated to get the LCM.

$$\text{Product} = 10 \times 15 = (2 \times 5) \times (3 \times 5) = 150$$

$$\text{Least Common Multiple} = (2 \times 5) \times (3 \times \cancel{5}) = 30$$

**FACT 4: We may eliminate the repeat occurrence of common factor by applying side by side division to the denominators as follows.**

We divide the denominators by common factor. The LCM is the product of the common factor with the remaining factors in the bottom row.

$$\begin{array}{r} 5 \mid \underline{10, 15} \\ 2, 3 \\ \text{LCM} = 5 \times 2 \times 3 = 30 \end{array} \quad \text{(Divide by the common factor 5)}$$

Find the LCM of 42 and 63.

$$\begin{array}{r} 7 \mid \underline{42, 63} \\ 3 \mid \underline{6, 9} \\ 2, 3 \end{array} \quad \begin{array}{l} (7 \text{ is a common factor}) \\ (3 \text{ is a common factor}) \\ (\text{There are no more common factors}) \end{array}$$

$$\text{LCM} = 7 \times 3 \times 2 \times 3 = 126$$

**FACT 5: For more than two denominators, a common prime factor to any two of the denominators may be taken out as follows.**

Find the LCM of 9, 14 and 21.

$$\begin{array}{r|rrr}
 3 & 9 & 14 & 21 \\
 7 & 3 & 14 & 7 \\
 & 3 & 2 & 1
 \end{array}
 \begin{array}{l}
 (3 \text{ is a prime factor common to } 9 \text{ and } 21, \text{ bring } 14 \text{ down as-is}) \\
 (7 \text{ is a prime factor common to } 14 \text{ and } 7, \text{ bring } 3 \text{ down as-is}) \\
 (\text{No prime factor is common to any two of these numbers})
 \end{array}$$

$$\text{LCM} = 3 \times 7 \times 3 \times 2 \times 1 = 126$$

**FACT 6:** We use LCM is the common denominator of the equivalent “like” fractions.

To compare  $7/10$  to  $11/15$ , we compute the LCM of 10 and 15 as 30 (see FACT 4).

$$\frac{7}{10} = \frac{7 \times 3}{10 \times 3} = \frac{21}{30} \quad \text{and} \quad \frac{11}{15} = \frac{11 \times 2}{15 \times 2} = \frac{22}{30}$$

$$\text{Since } \frac{21}{30} < \frac{22}{30}, \text{ therefore } \frac{7}{10} < \frac{11}{15}$$

**1. Find the LCM (Least Common Multiple) of the following set of numbers:**

- (a) 4 and 9      (b) 6 and 9      (c) 14 and 42      (d) 36 and 60  
 (e) 6, 15 and 18      (f) 6, 13 and 26      (g) 26, 33, 39 and 44

Answer: (a) 36 (b) 18 (c) 42 (d) 180 (e) 90 (f) 78 (g) 1716

**2. Use to LCM to find the equivalent like fractions for the following pairs of fractions.**

- (a)  $\frac{3}{5}, \frac{3}{10}$       (d)  $\frac{5}{9}, \frac{7}{12}$       (g)  $\frac{7}{15}, \frac{11}{25}$       (j)  $\frac{9}{14}, \frac{11}{21}$   
 (b)  $\frac{5}{6}, \frac{8}{9}$       (e)  $\frac{3}{10}, \frac{4}{15}$       (h)  $\frac{1}{6}, \frac{1}{8}$       (k)  $\frac{19}{24}, \frac{11}{16}$   
 (c)  $\frac{3}{4}, \frac{1}{6}$       (f)  $\frac{3}{8}, \frac{5}{12}$       (i)  $\frac{5}{9}, \frac{7}{15}$       (l)  $\frac{13}{20}, \frac{11}{15}$

Answer: (a) 6/10, 3/10 (b) 15/18, 16/18 (c) 9/12, 2/12 (d) 20/36, 21/36  
 (e) 9/30, 8/30 (f) 9/24, 10/24 (g) 35/75, 33/75 (h) 4/24, 3/24  
 (i) 25/45, 21/45 (j) 27/42, 22/42 (k) 38/48, 33/48 (l) 39/60, 44/60

**End of Lesson**