SIMPLE EQUATIONS,

CONTAINING THREE OR MORE UNKNOWN QUANTITIES.

162. Equations involving three or more unknown quantities may be solved by either of the three methods of elimination already explained.

Suppose we have the following equations, in which it is required to find the values of x, y, and z.

$$x+2y+z=20 \quad (1.)$$
$$2x+y+3z=31 \quad (2.)$$
$$3x+4y+2z=44 \quad (3.)$$

SOLUTION BY SUBSTITUTION.

From equation (1), $x=20-2y-z$.

Substituting this in equation (2), we have

$$2(20-2y-z)+y+3z=31;$$
$$\text{or,}\quad 40-4y-2z+y+3z=31$$
$$3y-z=9 \quad (4.)$$

Substituting the same value of x in equation (3), we have

$$3(20-2y-z)+4y+2z=44;$$
$$\text{or,}\quad 60-6y-3z+4y+2z=44.$$
$$2y+z=16 \quad (5.)$$
$$3y-z=9 \quad (4.)$$

The values of y and z are found, by Rule, Art. 158, to be 5 and 6; substituting these values in equation (1), $x=4$.

SOLUTION BY COMPARISON.

From equation (1), $x=20-2y-z$.

" " (2), $x=\frac{31-y-3z}{2}$.

" " (3), $x=\frac{44-4y-2z}{3}$.

Comparing the first and second values of x, we have

$$20-2y-z=\frac{31-y-3z}{2};$$

$$\text{or,}\quad 40-4y-2z=31-y-3z;$$

$$\text{or,}\quad 3y-z=9\quad(4.)$$

Comparing the first and third values of x, we have

$$20-2y-z=\frac{44-4y-2z}{3};$$

$$\text{or,}\quad 60-6y-3z=44-4y-2z.$$

$$2y+z=16\quad(5.)$$

From equations (4) and (5), the values of y and z, and then x, may be found by the Rule, Art. 159.

SOLUTION BY ADDITION AND SUBTRACTION.

Multiplying equation (1) by 2, we have

$$2x+4y+2z=40$$

Equation (2) is $2x+y+3z=31$

By subtracting, $3y-z=9$ (4.)

Next, multiplying equation (1) by 3, we have

$$3x+6y+3z=60$$

Equation (3) is $3x+4y+2z=44$

By subtracting, $2y+z=16$ (5.)

$$3y-z=9\quad(4.)$$

By adding, $5y=25$

$$y=5$$

Then, $10+z=16$, and $z=6$.
And $x+10+6=20$, and $x=4$.

It will be found, in practice, that the method of elimination by addition and subtraction is generally to be preferred; we shall, therefore, illustrate it by another example.

$$v+2x+3y+4z=30\quad(1.)$$

$$2v+3x+y+z=15\quad(2.)$$

$$3v+x+2y+3z=23\quad(3.)$$

$$4v+2x-y+14z=61\quad(4.)$$

Let us first eliminate v; this may be done thus:

$2v+4x+6y+8z=60$, by multiplying equation (1) by 2.
$2v+3x+y+z=15$ (2.)

$x+5y+7z=45$ (5.), by subtracting.

$3v+6x+9y+12z=90$, by multiplying equation (1) by 3.
$3v+x+2y+3z=23$ (3.)

$5x+7y+9z=67$ (6.), by subtracting.

$4v+8x+12y+16z=120$, by multiplying equation (1) by 4.
$4v+2x-y+14z=61$ (4.)

$6x+13y+2z=59$ (7.), by subtracting.

Collecting into one place the new equations (5), (6), and (7), we find that the number of unknown quantities, as well as the number of equations, is *one* less.

$x+5y+7z=45$ (5.)
$5x+7y+9z=67$ (6.)
$6x+13y+2z=59$ (7.)

The next step is to eliminate x, in a similar manner.

$5x+25y+35z=225$, by multiplying equation (5) by 5.
$5x+7y+9z=67$, equation (6).

$18y+26z=158$ (8.), by subtracting.

$6x+30y+42z=270$, by multiplying equation (5) by 6.
$6x+13y+2z=59$, equation (7).

$17y+40z=211$ (9.), by subtracting.

Bringing together equations (8) and (9), the number of equations, as well as of unknown quantities, is *two* less.

$18y+26z=158$ (8.)
$17y+40z=211$ (9.)

$306y+720z=3798$, by multiplying equation (9) by 18.
$306y+442z=2686$, by multiplying equation (8) by 17.

$278z=1112$, by subtracting.
$z=4$

Substituting the value of z, in equation (9), we get

$17y+160=211$; and $17y=51$; and $y=3$.

Substituting the values of y and z, in equation (5), we get

$$x+15+28=45$$
$$x=2$$

Substituting the values of x, y, and z, in equation (1), we have

$$v+4+9+16=30$$
$$v=1$$

From the preceding example, we derive the following

GENERAL RULE,

FOR ELIMINATION BY ADDITION AND SUBTRACTION.

1. *Eliminate the same unknown quantity from each of the equations; the number of equations and of unknown quantities will be one less.*

2. *Proceed in the same way with another unknown quantity; the number of equations and of unknown quantities will be two less.*

3. *Continue this series of operations until a single equation is obtained, containing but one unknown quantity.*

4. *By going back and substituting, the values of the other unknown quantities may be readily found.*

REMARK.—When one or more of the equations contains but one or two of the unknown quantities, the method of substitution will generally be found the shortest.

In literal equations and some others, the method of comparison may be most convenient. After solving several examples by each method, the pupil will be able to appreciate their relative excellence in different cases.

SOLVE BY EITHER METHOD OF ELIMINATION.

1. $\begin{cases} x+y=50. \\ x+z=28. \\ y+z=42. \end{cases}$ . . . . Ans. $\begin{cases} x=18. \\ y=32. \\ z=10. \end{cases}$

2. $\begin{cases} 3x+5y=76. \\ 4x+6z=108. \\ 5z+7y=106. \end{cases}$ . . . . Ans. $\begin{cases} x=12. \\ y=8. \\ z=10. \end{cases}$

3. $\begin{cases} x+y+z=26. \\ x+y-z=-6. \\ x-y+z=12. \end{cases}$ . . . . Ans. $\begin{cases} x=3. \\ y=7. \\ z=16. \end{cases}$

4. $\begin{cases} x+\frac{y}{2}=100. \\ y+\frac{z}{3}=100. \\ z+\frac{x}{4}=100. \end{cases}$ . . . . Ans. $\begin{cases} x=64. \\ y=72. \\ z=84. \end{cases}$

5. $\begin{cases} 2x-y+z=9. \\ x-2y+3z=14. \\ 3x+4y-2z=7. \end{cases}$ . . . . Ans. $\begin{cases} x=3. \\ y=2. \\ z=5. \end{cases}$

6. $\begin{cases} \frac{x}{3}-\frac{y}{2}+z=3. \\ \frac{x}{6}+\frac{y}{4}-\frac{z}{3}=1. \\ \frac{x}{2}-\frac{y}{4}+z=5. \end{cases}$ . . . . Ans. $\begin{cases} x=6. \\ y=4. \\ z=3. \end{cases}$

PROBLEMS PRODUCING EQUATIONS CONTAINING THREE OR MORE UNKNOWN QUANTITIES.

163. When a problem contains *three* or more unknown quantities, the equations may be formed according to the directions given in Art. 156 and 161.

REMARK.—When one or more of the unknown quantities can be expressed in terms of another, it is best to reduce the number of equations and symbols by doing so.

REVIEW.—162. What is the general rule for elimination by addition and subtraction? When is elimination by substitution to be preferred? When that by comparison?

1. A has 3 ingots, composed of different metals in different proportions; 1 lb. of the first contains 7 oz. of silver, 3 of copper, and 6 of tin; 1 lb. of the second contains 12 oz. of silver, 3 of copper, and 1 of tin; and 1 lb. of the third contains 4 oz. of silver, 7 of copper, and 5 of tin. How much of each must be taken to form an ingot of 1 lb. weight, containing 8 oz. of silver, $3\frac{3}{4}$ of copper, and $4\frac{1}{4}$ of tin?

Let x, y, z, represent the number of oz. taken of the 3 ingots respectively.

Then, since 16 oz. of the first contains 7 oz. of silver, 1 oz. will contain $\frac{7}{16}$ oz. of silver; and x oz. will contain $\frac{7x}{16}$ oz. of silver

In like manner, y oz. of the second will contain $\frac{12y}{16}$ oz. of silver; and z oz. of the third will contain $\frac{4z}{16}$ oz. of silver.

But, by the question, the number of oz. of silver in a pound of the new ingot, is to be 8; hence,

$$\frac{7x}{16}+\frac{12y}{16}+\frac{4z}{16}=8.$$

Or, by clearing it of fractions,

$$7x+12y+4z=128 \quad (1.)$$

Reasoning in a similar manner with reference to the copper and the tin, we have the two following equations:

$$3x+3y+7z=60 \quad (2.)$$
$$6x+y+5z=68 \quad (3.)$$

The terms containing y being the simplest, will be most easily eliminated.

Multiplying (2) by 4, and subtracting (1), we have

$$5x+24z=112 \quad (4.)$$

Multiplying (3) by 3, and subtracting (2), we have

$$15x+8z=144 \quad (5.)$$

REVIEW.—163. Upon what principle are equations formed, when a problem contains three or more unknown quantities? When may we reduce the number of symbols?

Multiplying (5) by 3, and subtracting (4), there results

$$40x=320$$
$$x=8$$

Substituting this value of x in equation (5), we have

$$120+8z=144$$
$$z=3$$

And substituting these values of x and z in equation (3),

$$48+y+15=68$$
$$y=5$$

Hence, the new ingot will contain 8 oz. of the first, 5 of the second, and 3 of the third.

2. The sums of three numbers, taken two and two, are 27, 32, and 35; required the numbers.

Ans. 12, 15, and 20.

3. The sum of three numbers is 59; $\frac{1}{2}$ the difference of the first and second is 5, and $\frac{1}{2}$ the difference of the first and third is 9; required the numbers.

Ans. 29, 19, and 11.

4. A person bought three silver watches; the price of the first, with $\frac{1}{2}$ the price of the other two, was \$25; the price of the second, with $\frac{1}{3}$ the price of the other two, was \$26; and the price of the third, with $\frac{1}{2}$ the price of the other two, was \$29; required the price of each.

Ans. \$8, \$18, and \$16.

5. Find three numbers, such that the first with $\frac{1}{3}$ of the other two, the second with $\frac{1}{4}$ of the other two, and the third with $\frac{1}{5}$ of the other two, shall each equal 25.

Ans. 13, 17, and 19.

6. A boy bought at one time 2 apples and 5 pears, for 12 cts.; at another, 3 pears and 4 peaches, for 18 cts.; at another, 4 pears and 5 oranges, for 28 cts.; and at another, 5 peaches and 6 oranges, for 39 cts.; required the cost of each kind of fruit.

Ans. Apples 1, pears 2, peaches 3, oranges 4 cts. each.

7. A and B together possess only $\frac{2}{3}$ as much money as C; B and C together have 6 times as much as A; and B has $680 less than A and C together; how much has each? Ans. A $200, B $360, and C $840.

8. A, B, and C compare their money; A says to B, "give me $700, and I shall have twice as much as you will have left." B says to C, "give me $1400, and I shall have three times as much as you will have left." C says to A, "give me $420, and I shall have five times as much as you will have left." How much has each?

Ans. A $980, B $1540, and C $2380.

9. A certain number is expressed by three figures, whose sum is 11; the figure in the place of units is double that in the place of hundreds; and if 297 be added to the number, its figures will be inverted; required the number.

Ans. 326.

10. The sum of 3 numbers is 83; if from the first and second you subtract 7, the remainders are as 5 to 3; but if from the second and third you subtract 3, the remainders are to each other as 11 to 9; required the numbers. Ans. 37, 25, and 21.

11. Divide $180 among three persons, A, B, and C, so that twice A's share plus $80, three times B's share plus $40, and four times C's share plus $20, may be all equal to each other. Ans. A $70, B $60, C $50.

12. If A and B can perform a certain work in 12 days, A and C in 15 days, and B and C in 20 days, in what time could each do it alone?

Ans. A 20, B 30, and C 60 days.

13. A number expressed by three figures, when divided by the sum of the figures plus 9, gives a quotient of 19; the middle figure equals half the sum of the first and third; and if 198 be added to the number, we obtain a number with the same figures in an inverted order; what is the number? Ans. 456.

14. A farmer mixes barley at 28 cents, with rye at 36, and wheat at 48 cents per bu., so that the whole is 100 bu., and worth 40 cents per bu. Had he put twice as much rye, and 10 bu. more wheat, the whole would have been worth exactly the same per bu.; how much of each was there? Ans. Barley 28, rye 20, wheat 52 bu.

15. A, B, and C killed 96 birds, which they wish to share equally; to do this, A, who has the most, gives to B and C as many as they already had; next, B gives to A and C as many as they had after the first division; lastly, C gives to A and B as many as they both had after the second division, and each then had the same number; how many had each at first? Ans. A 52, B 28, and C 16.

GENERAL REVIEW.

What two parts in the solution of a problem? What are explicit conditions? Implied conditions? Rule for forming an equation. On what condition may you change the sign of one term in an equation?

Define elimination. How many methods of elimination? Define elimination by substitution—by comparison—by addition and subtraction. Rule for each method. How state a problem containing two unknown quantities? How one containing three or more unknown quantities? When is the first method of elimination preferred? When the second? The third? Rule for elimination in three or more unknown quantities.

Give two rules for rendering a complex fraction simple. State the eight theorems, Arts. 80 to 85. Rule for exponents in multiplication. In division. Difference between subtraction in algebra and in arithmetic. In clearing an equation of fractions, what is to be done when there is a minus sign before a fraction?

Define binomial. Term. Coëfficient. Exponent. Factor. Prime number. Composite number. What is the reciprocal of a fraction? What are the factors of x^2-1, of x^3-1, of x^3+1, of x^2+3x+2? By how many different methods could you reduce $\frac{1}{2}$, $\frac{1}{3}$, $\frac{5}{8}$, and $\frac{7}{12}$ to a common denominator?

In what cases may cancellation be employed to advantage? What three methods of multiplying a fraction by a whole number? Of dividing a fraction by a whole number? What are infinite series? What the law of a series? How convert $\frac{5}{11}$ into an infinite series?