# III. ALGEBRAIC FRACTIONS.

## DEFINITIONS AND FUNDAMENTAL PROPOSITIONS.

**113.** A **Fraction** is an expression representing one or more of the equal parts into which a unit is supposed to be divided.


Thus, if the line $A\ B$ be supposed to represent 1 foot, and it be divided into 4 equal parts, 1 of those parts, as $Ac$, is called one fourth ($\frac{1}{4}$); 2 of them, as $Ad$, are called two fourths ($\frac{2}{4}$); and 3 of them, as $Ae$, are called three fourths ($\frac{3}{4}$).

In the algebraic fraction $\frac{1}{c}$, if $c=4$ and 1 denotes 1 foot, then $\frac{1}{c}$ denotes one fourth of a foot. In the fraction $\frac{a}{c}$, if $a=3$ and $\frac{1}{c}=\frac{1}{4}$ of a foot, then $\frac{a}{c}$ represents three fourths ($\frac{3}{4}$) of a foot.

**114.** An **Entire Algebraic Quantity** is one not expressed under the form of a fraction.

Thus, $ax+b$ is an entire quantity.

**115.** A **Mixed Quantity** is one composed of an entire quantity and a fraction.

Thus, $a+\frac{b}{x}$ is a mixed quantity.

**116.** An **Improper Algebraic Fraction** is one whose numerator can be divided by the denominator, either with or without a remainder.

Thus, $\frac{ab}{a}$ and $\frac{ax^2+b}{x}$ are improper fractions.

**117.** A **Simple Fraction** is a single fractional expression; as, $\frac{1}{3}$, $\frac{a}{b}$, or $\frac{c}{d}$. It may be either proper or improper.

REVIEW.—112. If the product of two quantities be divided by their G.C.D., what will the quotient be?

113. What is a fraction? 114. An entire algebraic quantity? Example. 115. A mixed quantity? Example. 116. An improper algebraic fraction? Example.

**118.** A **Compound Fraction** is a fraction of a fraction; as, $\frac{1}{2}$ of $\frac{2}{3}$, or $\frac{m}{n}$ of $\frac{a}{b}$.

**119.** A **Complex Fraction** is one that has a fraction either in its numerator or denominator, or in both.

Thus, $\dfrac{2\frac{1}{2}}{4}$, $\dfrac{3\frac{1}{2}}{2\frac{1}{3}}$, $\dfrac{a+\frac{b}{c}}{d}$, and $\dfrac{a+\frac{b}{c}}{e+\frac{m}{n}}$, are complex fractions.

**120.** Algebraic fractions are represented in the same manner as common fractions in Arithmetic.

The **Denominator** is the quantity below the line, and is so called because it *denominates* or shows the number of parts into which the unit is divided.

The **Numerator** is the quantity above the line, and is so called because it *numbers* or shows how many parts are taken.

Thus, in the fraction $\frac{3}{4}$, it is understood that the unit is divided into 4 equal parts, and that three of these parts are taken: $\frac{a}{c}$ denotes that a unit is divided into $c$ equal parts, and that $a$ of these parts are taken.

The numerator and denominator are called the *terms* of a fraction.

**121.** In the preceding definitions of numerator and denominator, reference is had to a *unit* only. This is the simplest method of considering a fraction; but there is another point of view in which it is proper to examine it.

If required to divide 3 apples equally, between 4 boys, it can be effected by dividing each of the 3 apples into 4 equal parts, and

REVIEW.—117. What is a simple fraction? Example. 118. A compound fraction? Example. 119. A complex fraction? Example. 120. In fractions, what is the quantity below the line called? Why? Above the line? Why? Example. What are the terms of a fraction?

giving to each boy 3 parts from 1 apple, or 1 part from each of the 3 apples; that is, $\frac{3}{4}$ of 1 unit is the same as $\frac{1}{4}$ of 3 units.

Thus, $\frac{2}{5}$ may be regarded as expressing *two* fifths of *one* thing, or *one* fifth of *two* things.

So, $\frac{m}{n}$ is either the fraction $\frac{1}{n}$ of one unit taken $m$ times, or it is the $n$th of $m$ units. Hence, the numerator may be regarded as showing the *number of units* to be divided; and the denominator, as showing the divisor, or *what part is taken* from each.

**122. Proposition I.**—*If we multiply the numerator of a fraction without changing the denominator, the value of the fraction is increased as many times as there are units in the multiplier.*

If we multiply the numerator of the fraction $\frac{2}{7}$ by 3, without changing the denominator, it becomes $\frac{6}{7}$.

Now, $\frac{2}{7}$ and $\frac{6}{7}$ have the same denominator, which expresses parts of the same size; but the second fraction, $\frac{6}{7}$, having three times as many of those parts as the first, is three times as large. The same may be shown of any fraction whatever.

**123. Proposition II.**—*If we divide the numerator of a fraction without changing the denominator, the value of the fraction is diminished as many times as there are units in the divisor.*

If we take the fraction $\frac{4}{5}$, and divide the numerator by 2, without changing the denominator, it becomes $\frac{2}{5}$.

Now, $\frac{4}{5}$ and $\frac{2}{5}$ have the same denominator, which expresses parts of the same size; but the second fraction, $\frac{2}{5}$, having only one half as many of those parts as the first, $\frac{4}{5}$, is only one half as large. The same may be shown of other fractions.

**124. Proposition III.**—*If we multiply the denominator of a fraction without changing the numerator, the value of the fraction is diminished as many times as there are units in the multiplier.*

REVIEW.—121. In what two different points of view may every fraction be regarded? Examples. 122. How is the value of a fraction affected by multiplying the numerator only? Give the proof.

123. How is the value of a fraction affected by dividing the numerator only? Give the proof.

If we take the fraction $\frac{3}{4}$, and multiply the denominator by 2, without changing the numerator, it becomes $\frac{3}{8}$.

Now, the fractions $\frac{3}{4}$ and $\frac{3}{8}$ have the same numerator, which expresses the same number of parts; but, in the second, the parts being only one half the size of those in the first, the value of the second fraction is only *one half* that of the first. The same may be shown of any fraction whatever.

**125. Proposition IV.**—*If we divide the denominator of a fraction without changing the numerator, the value of the fraction is increased as many times as there are units in the divisor.*

If we take the fraction $\frac{2}{9}$, and divide the denominator by 3 without changing the numerator, it becomes $\frac{2}{3}$.

Now, the fractions $\frac{2}{9}$ and $\frac{2}{3}$ have the same numerator, which expresses the same number of parts; but, in the second, the parts being three times the size of those of the first, the value of the second fraction is three times that of the first. The same may be shown of other fractions.

**126. Proposition V.**—*Multiplying both terms of a fraction by the same number or quantity, changes the form of a fraction, but does not alter its value.*

If we multiply the numerator of a fraction by any number, its value, Prop. I., is *increased*, as many times as there are units in the multiplier; and, if we multiply the denominator, the value, Prop. III., is *decreased*, as many times as there are units in the multiplier. Hence,

The increase is equal to the decrease, and the value remains unchanged.

**127. Proposition VI.**—*Dividing both terms of a fraction by the same number or quantity, changes the form of the fraction but not its value.*

If we divide the numerator of a fraction by any number, its value, Prop. II., is *decreased*, as many times as there are units in the

REVIEW.—124. How by multiplying only the denominator? How proved? 125. By dividing the denominator only? How proved?

126. How is a fraction affected by multiplying both terms by the same quantity? Why?

divisor; and, if we divide the denominator, the value, Prop. IV., is *increased* as many times. Hence,

The decrease is equal to the increase, and the value remains unchanged.

## CASE I.

### TO REDUCE A FRACTION TO ITS LOWEST TERMS.

**128.** Since the value of a fraction is not changed by dividing both terms by the same quantity, Art. 127, we have the following

**Rule.**—*Divide both terms by their greatest common divisor.*

Or, *Resolve the numerator and denominator into their prime factors, and then cancel those factors common to both terms.*

REMARK.—The last rule is generally most convenient.

1. Reduce $\frac{4ab^2}{6bx^2}$ to its lowest terms.

$$\frac{4ab^2}{6bx^2}=\frac{2ab\times2b}{3x^2\times2b}=\frac{2ab}{3x^2} \text{ Ans.}$$

Reduce the following fractions to their lowest terms:

2. $\frac{4a^3x^2}{6a^4}$. . . . Ans. $\frac{2x^2}{3a}$.

3. $\frac{6a^2x^2}{8ax^3}$. . . . Ans. $\frac{3a}{4x}$.

4. $\frac{9x^4y^3z^5}{12x^3y^4z^5}$. . . Ans. $\frac{3x}{4y}$.

5. $\frac{12x^3y^2z^4}{8x^3z^3}$. . . Ans. $\frac{3y^2z}{2}$.

6. $\frac{2a^2cx^2+2acx}{10ac^2x}$. Ans. $\frac{ax+1}{5c}$.

7. $\frac{5a^2b+5ab^2}{5abc+5abd}$. Ans. $\frac{a+b}{c+d}$.

8. $\frac{12x^2y-18xy^2}{18x^2y+12xy^2}$. . . . . . . . . . Ans. $\frac{2x-3y}{3x+2y}$.

NOTE.—In the preceding examples, the greatest common divisor in each is a monomial; in those which follow, it is a polynomial.

REVIEW.—127. How by dividing both terms by the same quantity? Why? 128. How reduce a fraction to its lowest terms?

9. $\frac{3a^3-3ab^2}{5ab+5b^2}$. This is equal to

$$\frac{3a(a^2-b^2)}{5b(a+b)}=\frac{3a(a+b)(a-b)}{5b(a+b)}=\frac{3a(a-b)}{5b}. \text{ Ans.}$$

10. $\frac{3z^3-24z+9}{4z^3-32z+12}$. Ans. $\frac{3}{4}$.

11. $\frac{n^2-2n+1}{n^2-1}$. Ans. $\frac{n-1}{n+1}$.

12. $\frac{x^3-xy^2}{x^4-y^4}$. Ans. $\frac{x}{x^2+y^2}$.

13. $\frac{x^2-y^2}{x^2-2xy+y^2}$. Ans. $\frac{x+y}{x-y}$.

14. $\frac{x^3-ax^2}{x^2-2ax+a^2}$. Ans. $\frac{x^2}{x-a}$.

15. $\frac{x^2+2x-15}{x^2+8x+15}$. Ans. $\frac{x-3}{x+3}$.

**129.** Exercises in division, Art. 76, in which the quotient is a fraction, and capable of being reduced to lower terms.

1. Divide $5x^2y$ by $3xy^2$. . . . . . . . . . Ans. $\frac{5x}{3y}$.

2. Divide $amn^2$ by $a^2m^2n$. . . . . . . . Ans. $\frac{n}{am}$.

So, also, when one or both of the quantities are polynomials.

3. Divide $3m^2+3n^2$ by $15m^2+15n^2$. . . . . . Ans. $\frac{1}{5}$.

4. Divide $x^3y^2+x^2y^3$ by $ax^2y+axy^2$. . . . . Ans. $\frac{xy}{a}$.

5. Divide $x^2+2x-3$ by $x^2+5x+6$. . . . Ans. $\frac{x-1}{x+2}$.

## CASE II.

### TO REDUCE A FRACTION TO AN ENTIRE OR MIXED QUANTITY.

**130.** Since the numerator of the fraction may be regarded as a dividend, and the denominator as a divisor, this is merely a case of division. Hence,

**Rule.**—*Divide the numerator by the denominator for the entire part; and, if there be a remainder, place it over the denominator for the fractional part.*

NOTE.—The fractional part should be reduced to its lowest terms.

Reduce the following to entire or mixed quantities:

1. $\frac{3ax+b^2}{x}$. . . . . . . . Ans. $\frac{3ax+b^2}{x}=3a+\frac{b^2}{x}$.
2. $\frac{ab+b^2}{a}$. . . . . . . . . . . . . Ans. $b+\frac{b^2}{a}$.
3. $\frac{a^2+x^2}{a-x}$. . . . . . . . . . . Ans. $a+x+\frac{2x^2}{a-x}$.
4. $\frac{2a^2x-x^3}{a}$. . . . . . . . . . . Ans. $2ax-\frac{x^3}{a}$.
5. $\frac{4ax-2x^2-a^2}{2a-x}$. . . . . . . . . . Ans. $2x-\frac{a^2}{2a-x}$.
6. $\frac{a^3+x^3-x^4}{a+x}$. . . . . . Ans. $a^2-ax+x^2-\frac{x^4}{a+x}$.
7. $\frac{12x^3-3x^2}{4x^3-x^2-4x+1}$. . . . . . . Ans. $3+\frac{3}{x^2-1}$.

## CASE III.

### TO REDUCE A MIXED QUANTITY TO THE FORM OF A FRACTION.

**131.**—1. In $2\frac{1}{3}$, there are how many thirds?

In 1 unit there are 3 thirds; hence, in 2 units there are 6 thirds; then, $2\frac{1}{3}$ or $2+\frac{1}{3}=\frac{6}{3}+\frac{1}{3}=\frac{7}{3}$.

So, $a+\frac{b}{c}=\frac{ac}{c}+\frac{b}{c}=\frac{ac+b}{c}$; and $a-\frac{b}{c}=\frac{ac}{c}-\frac{b}{c}=\frac{ac-b}{c}$. Hence,

### TO REDUCE A MIXED QUANTITY TO THE FORM OF A FRACTION,

**Rule.**—*Multiply the entire part by the denominator of the fraction. Add the numerator of the fractional part to this* *product, or subtract it, as the sign may direct, and place the result over the denominator.*

REMARK.—Cases II. and III. are the reverse of, and mutually prove each other.

Before proceeding further, it is important to consider

## THE SIGNS OF FRACTIONS.

**132.** The signs prefixed to the terms of a fraction affect only those terms; but the sign placed before the dividing line of a fraction, affects its whole value.

Thus, in $-\frac{a^2-b^2}{x+y}$, the sign of $a^2$, in the numerator, is plus; of $b^2$, minus; while the sign of each term of the denominator is plus. But the sign of the fraction, taken as a whole, is minus.

By the rule for the signs in division, Art. 75, we have $\frac{+ab}{+a}=+b$; or, changing the signs of both terms, $\frac{-ab}{-a}=+b$.

Changing the sign of the numerator, we have $\frac{-ab}{+a}=-b$.

Changing the sign of the denominator, we have $\frac{+ab}{-a}=-b$. Hence,

*The signs of both terms of a fraction may be changed without altering its value or changing its sign; but, if the sign of either term of a fraction be changed, and not that of the other, the sign of the fraction will be changed.*

Hence, *The signs of either term of a fraction may be changed, without altering its value, if the sign of the fraction be changed at the same time.*

REVIEW.—130. How reduce a fraction to an entire or mixed quantity? 131. A mixed quantity to the form of a fraction?

132. What do the signs prefixed to the terms of a fraction affect? The sign placed before the whole fraction? What is the effect of changing the signs of both terms of a fraction? Of one term, and not the other? The sign of the fraction, and of one of its terms?

Thus, . . . $\frac{ax-x^2}{c}=\frac{ax-x^2}{-c}=-\frac{x^2-ax}{c}$.

And, . . . $a-\frac{a-x}{b}=a+\frac{a-x}{-b}=a+\frac{x-a}{b}$.

**1.** Reduce $3a+\frac{ax-a}{x}$ to a fractional form.

$3a=\frac{3ax}{x}$ and $\frac{3ax}{x}+\frac{ax-a}{x}=\frac{3ax+ax-a}{x}=\frac{4ax-a}{x}$. Ans.

**2.** Reduce $4a-\frac{a-b}{3c}$ to a fractional form.

$4a=\frac{12ac}{3c}$, and $\frac{12ac}{3c}-\frac{a-b}{3c}=\frac{12ac-(a-b)}{3c}=\frac{12ac-a+b}{3c}$. Ans.

Reduce the following quantities to improper fractions:

3. $2+\frac{3}{5}$ and $2-\frac{3}{5}$. . . . . . . . Ans. $\frac{13}{5}$ and $\frac{7}{5}$.

4. $5c+\frac{a-b}{2x}$. . . . . . . . . Ans. $\frac{10cx+a-b}{2x}$.

5. $5c-\frac{a-b}{2x}$. . . . . . . . . Ans. $\frac{10cx-a+b}{2x}$.

6. $3x-\frac{4x^2-5}{5x}$. . . . . . . . . . . Ans. $\frac{11x^2+5}{5x}$.

7. $8y+\frac{3a-y^2}{5y}$. . . . . . . . . . Ans. $\frac{39y^2+3a}{5y}$.

8. $z-1+\frac{1-z}{1+z}$. . . . . . . . . . . Ans. $\frac{z^2-z}{z+1}$.

9. $\frac{4y}{2x+z}-5$. . . . . . . . . Ans. $\frac{4y-10x-5z}{2x+z}$.

10. $\frac{3+5c}{8}-6$. . . . . . . . . . . Ans. $\frac{5c-45}{8}$.

11. $a-x+\frac{a^2+x^2-5}{a+x}$. . . . . . . . . . Ans. $\frac{2a^2-5}{a+x}$.

12. $a^3-a^2x+ax^2-x^3-\frac{a^4+x^4}{a+x}$. . . . . Ans. $-\frac{2x^4}{a+x}$

## CASE IV.

### TO REDUCE FRACTIONS OF DIFFERENT DENOMINATORS TO EQUIVALENT FRACTIONS HAVING A COMMON DENOMINATOR.

**133.**—1. Reduce $\frac{a}{b}$ and $\frac{c}{d}$ to a common denominator.

Multiply both terms of the first fraction by $d$, the denominator of the second, and both terms of the second fraction by $b$, the denominator of the first. We shall then have $\frac{ad}{bd}$ and $\frac{bc}{bd}$.

In this solution, observe; *first*, the values of the fractions are not changed, since, in each, both terms are multiplied by the same quantity; and,

*Second*, the denominators must be the same, since they consist of the product of the same quantities.

2. Reduce $\frac{a}{m}$, $\frac{b}{n}$, and $\frac{c}{r}$, to a common denominator.

Here, we multiply both terms of each fraction by the denominators of the other two fractions. Thus,

$$\frac{a\times n\times r}{m\times n\times r}=\frac{anr}{mnr};\quad \frac{b\times m\times r}{n\times m\times r}=\frac{bmr}{mnr};\quad \frac{c\times m\times n}{r\times m\times n}=\frac{cmn}{mnr}.$$

It is evident that the value of each fraction is not changed, and that they have the same denominators. Hence,

### TO REDUCE FRACTIONS TO A COMMON DENOMINATOR,

**Rule.**—*Multiply both terms of each fraction by the product of all the denominators except its own.*

REMARK.—This is the same as to multiply each numerator by all the denominators except its own, for the new numerators; and all the denominators together, for the common denominator.

REVIEW.—133. How do you reduce fractions of different denominators to equivalent fractions having the same denominator?
Why is the value of each fraction not changed by this process?
Why does this process give to each fraction the same denominator?

Reduce to fractions having a common denominator:

3. $\frac{a}{b}$, $\frac{c}{d}$, and $\frac{1}{2}$. . . . . Ans. $\frac{2ad}{2bd}$, $\frac{2bc}{2bd}$, and $\frac{bd}{2bd}$.

4. $\frac{x}{y}$, and $\frac{x+a}{c}$. . . . . . . Ans. $\frac{cx}{cy}$, and $\frac{xy+ay}{cy}$.

5. $\frac{2}{3}$, $\frac{3a}{4}$, and $\frac{x-y}{b}$. Ans. $\frac{8b}{12b}$, $\frac{9ab}{12b}$, and $\frac{12x-12y}{12b}$.

6. $\frac{2x}{3y}$, $\frac{3x}{5z}$, and $a$. . Ans. $\frac{10xz}{15yz}$, $\frac{9xy}{15yz}$, and $\frac{15ayz}{15yz}$.

7. $\frac{x+y}{x-y}$, and $\frac{x-y}{x+y}$. Ans. $\frac{x^2+2xy+y^2}{x^2-y^2}$, and $\frac{x^2-2xy+y^2}{x^2-y^2}$.

8. $a$, $\frac{3b}{c}$, $d$, and 5. . . . Ans. $\frac{ac}{c}$, $\frac{3b}{c}$, $\frac{cd}{c}$, and $\frac{5c}{c}$.

**134.** When the denominators of the fractions to be reduced contain one or more common factors, the preceding rule does not give the *least* common denominator.

If we find the L.C.M. of all the denominators, and divide it by the denominators severally, it is easy to see that we shall obtain multipliers for each of the fractions, which will, without changing their value, make their denominators the same as the L.C.M.

1. Reduce $\frac{m}{b}$, $\frac{n}{bc}$, and $\frac{r}{cd}$, to equivalent fractions having the least common denominator.

The L.C.M. of the denominators is $bcd$; dividing this by $b$, $bc$, and $cd$, we obtain $cd$, $d$, and $b$. Multiplying both terms by these severally, we have $\frac{mcd}{bcd}$, $\frac{nd}{bcd}$, and $\frac{br}{bcd}$; or thus:

$bcd \div b = cd$, and $\frac{m}{b} \times \frac{cd}{cd} =$ . . . $\frac{mcd}{bcd}$.

$bcd \div bc = d$, and $\frac{n \times d}{bc \times d} =$ . . . . $\frac{nd}{bcd}$

$bcd \div cd = b$, and $\frac{r \times b}{cd \times b} =$ . . $\frac{br}{bcd}$

The process of multiplying the denominators may be omitted, as the product in each case is the same. Hence,

### TO REDUCE FRACTIONS OF DIFFERENT DENOMINATORS TO EQUIVALENT FRACTIONS HAVING THE LEAST COMMON DENOMINATOR,

**Rule.**—1. *Find the least common multiple of all the denominators; this will be the common denominator.*

2. *Divide the least common multiple by the first of the given denominators, and multiply the quotient by the first of the given numerators; the product will be the first of the required numerators.*

3. *Proceed, in a similar manner, to find each of the other numerators.*

NOTE.—Each fraction should first be reduced to its lowest terms.

Reduce the following to equivalent fractions having the least common denominator:

2. $\frac{2a}{3bc}$, $\frac{3x}{cd}$, and $\frac{5y}{6bd}$. . Ans. $\frac{4ad}{6bcd}$, $\frac{18bx}{6bcd}$, and $\frac{5cy}{6bcd}$.

3. $\frac{m}{ac}$, $\frac{n}{b^2c}$, $\frac{r}{c^2d}$. . . . Ans. $\frac{b^2cdm}{ab^2c^2d}$, $\frac{acdn}{ab^2c^2d}$, $\frac{ab^2r}{ab^2c^2d}$.

4. $\frac{x+y}{x-y}$, $\frac{x-y}{x+y}$, $\frac{x^2+y^2}{x^2-y^2}$. Ans. $\frac{(x+y)^2}{x^2-y^2}$, $\frac{(x-y)^2}{x^2-y^2}$, $\frac{x^2+y^2}{x^2-y^2}$.

NOTE.—The two following Art's will be of frequent use, particularly in completing the square, in the solution of equations of the second degree.

**135.** To reduce an entire quantity to the form of a fraction having a given denominator.

1. Let it be required to reduce $a$ to a fraction having $b$ for its denominator.

Since $a=\frac{a}{1}$, if we multiply both terms by $b$, which will not change its value, Art. 126, we have $\frac{a}{1}=\frac{ab}{b}$. Hence,

### TO REDUCE AN ENTIRE QUANTITY TO THE FORM OF A FRACTION HAVING A GIVEN DENOMINATOR,

**Rule.**—*Multiply the entire quantity by the given denominator, and write the product over it.*

2. Reduce $x$ to a fraction whose denominator is 4. Ans. $\frac{4x}{4}$.

3. Reduce $m$ to a fraction whose denominator is $9a^2$. Ans. $\frac{9a^2m}{9a^2}$.

4. Reduce $3c+5$ to a fraction whose denominator is $16c^2$. Ans. $\frac{48c^3+80c^2}{16c^2}$.

5. Reduce $a-b$ to a fraction whose denominator is $a^2-2ab+b^2$. Ans. $\frac{a^3-3a^2b+3ab^2-b^3}{a^2-2ab+b^2}=\frac{(a-b)^3}{(a-b)^2}$.

**136.** To convert a fraction to an equivalent one, having a denominator equal to some multiple of the denominator of the given fraction.

1. Reduce $\frac{a}{b}$ to a fraction whose denominator is $bc$.

It is evident that this will be accomplished without changing the value of the fraction, by multiplying both terms by $c$. This multiplier, $c$, may be found by inspection, or by dividing $bc$ by $b$. Hence,

### TO CONVERT A FRACTION TO AN EQUIVALENT ONE HAVING A GIVEN DENOMINATOR,

**Rule.**—*Divide the given denominator by the denominator of the fraction, and multiply both terms by the quotient.*

Review.—134. How reduce fractions of different denominators to equivalent fractions having the least common denominator?
134. If each fraction is not in its lowest terms before commencing the operation, what is to be done? 135. How reduce an entire quantity to the form of a fraction having a given denominator?

REMARK.—If the required denominator is not a multiple of the given one, the result will be a complex fraction. Thus, if it is required to convert $\frac{1}{2}$ into an equivalent fraction whose denominator is 5, the numerator of the new fraction would be $2\frac{1}{2}$.

2. Convert $\frac{3}{4}$ to an equivalent fraction, having the denominator 16. Ans. $\frac{12}{16}$.

3. Convert $\frac{b}{c}$ to an equivalent fraction, having the denominator $a^2c^2$. Ans. $\frac{a^2bc}{a^2c^2}$.

4. Convert $\frac{m+n}{m-n}$ to an equivalent fraction, having the denominator $m^2-2mn+n^2$. Ans. $\frac{m^2-n^2}{m^2-2mn+n^2}$.

## CASE V.

### ADDITION AND SUBTRACTION OF FRACTIONS.

**137.**—1. Required to find the sum of $\frac{2}{5}$ and $\frac{4}{5}$.

Here, both parts being of the same kind, that is, fifths, we may add them together, and the sum is 6 fifths, ($\frac{6}{5}$).

2. Let it be required to find the sum of $\frac{a}{m}$ and $\frac{b}{m}$.

Here, the parts being the same, that is, $m$ths, we shall have $\frac{a}{m}+\frac{b}{m}=\frac{a+b}{m}$.

3. Let it be required to find the sum of $\frac{a}{m}$ and $\frac{c}{n}$.

Here, the denominators being different, we can not add the numerators, and call them by the same name. We may, however, reduce them to a common denominator, and then add.

Thus, $\frac{a}{m}=\frac{an}{mn}$; $\frac{c}{n}=\frac{cm}{mn}$; and $\frac{an}{mn}+\frac{cm}{mn}=\frac{an+cm}{mn}$. Hence,

TO ADD FRACTIONS,

**Rule.**—*Reduce the fractions, if necessary, to a common denominator; add the numerators together, and write their sum over the common denominator.*

**138.** It is obvious that the same principles would apply in finding the difference between two fractions. Hence,

TO SUBTRACT FRACTIONS,

**Rule.**—*Reduce the fractions, if necessary, to a common denominator; subtract the numerator of the subtrahend from the numerator of the minuend, and write the remainder over the common denominator.*

### EXAMPLES IN ADDITION OF FRACTIONS.

4. Add $\frac{a}{2}$, $\frac{a}{3}$, and $\frac{a}{6}$ together. . . . . . Ans. $a$.

5. Add $\frac{x}{3}$, $\frac{x}{5}$, and $\frac{x}{6}$ together. . . . . . Ans. $\frac{7x}{10}$.

6. Add $\frac{1}{a}$, $\frac{1}{b}$, and $\frac{1}{c}$ together. . . Ans. $\frac{bc+ac+ab}{abc}$.

7. Add $\frac{x}{2}$, $\frac{y}{3}$, and $\frac{z}{4}$ together. . . Ans. $\frac{6x+4y+3z}{12}$.

8. Add $\frac{3x}{4}$, $\frac{4x}{5}$, and $\frac{5x}{6}$ together. Ans. $\frac{143x}{60}=2x+\frac{23x}{60}$.

9. Add $\frac{x+y}{2}$ and $\frac{x-y}{2}$ together. . . . . . Ans. $x$.

10. Add $\frac{1}{a+b}$ and $\frac{1}{a-b}$ together. . . . Ans. $\frac{2a}{a^2-b^2}$.

11. Add $\frac{5+x}{y}$, $\frac{3-ax}{ay}$, and $\frac{b}{3a}$ together.
Ans. $\frac{15a+by+9}{3ay}$.

12. Add $\frac{a-b}{ab}$, $\frac{b-c}{bc}$, and $\frac{c-a}{ac}$ together. . . Ans. 0.

Entire quantities and fractions may be added separately; or, the entire quantities may be put into the form of fractions by making their denominators unity. When mixed quantities occur, it is often better to reduce them to the form of improper fractions.

13. Add $2x$, $3x+\frac{3z}{5}$, and $x+\frac{2z}{9}$ together. Ans. $6x+\frac{37z}{45}$.

14. Add $5x+\frac{x-2}{3}$ and $4x-\frac{2x-3}{5x}$ together.
Ans. $9x+\frac{5x^2-16x+9}{15x}$.

## EXAMPLES IN SUBTRACTION OF FRACTIONS.

1. From $\frac{a}{2}$ take $\frac{a}{3}$. . . . . . . . . . . . Ans. $\frac{a}{6}$.

2. From $\frac{3x}{4}$ take $\frac{2x}{3}$. . . . . . . . . . . Ans. $\frac{x}{12}$.

3. From $\frac{a+b}{2}$ take $\frac{a-b}{2}$. . . . . . . . . . Ans. $b$.

4. From $\frac{2ax}{3}$ take $\frac{5ax}{2}$. . . . . . . . . Ans. $-\frac{11ax}{6}$.

5. From $\frac{3}{4a}$ take $\frac{5}{2x}$. . . . . . . . . Ans. $\frac{3x-10a}{4ax}$.

6. From $\frac{x+y}{x-y}$ take $\frac{x-y}{x+y}$. . . . . . . Ans. $\frac{4xy}{x^2-y^2}$.

7. From $\frac{2a+b}{5c}$ take $\frac{3a-b}{7c}$. . . . . Ans. $\frac{12b-a}{35c}$.

8. From $5x+\frac{x}{b}$ take $2x-\frac{x-b}{c}$. Ans. $3x+\frac{bx+cx-b^2}{bc}$.

9. From $\frac{1}{a-b}$ take $\frac{1}{a+b}$. . . . . . . Ans. $\frac{2b}{a^2-b^2}$.

10. From $a+b$ take $\frac{1}{a}+\frac{1}{b}$. . . Ans. $\frac{a^2b+ab^2-a-b}{ab}$.

REVIEW.—136. How convert a fraction to an equivalent one having a given denominator? Explain the operation by an example.
137. When fractions have the same denominator, how add them together? When fractions have different denominators?
138. If two fractions have the same denominator, how find their difference? When they have different denominators?

11. From $\frac{x^3+y^3}{x-y}$ take $\frac{x^3-y^3}{x+y}$ . . . Ans. $\frac{2x^3y+2xy^3}{x^2-y^2}$.

12. From $x+\frac{1}{x-1}$ take $\frac{2}{x+1}$. . . . Ans. $\frac{x^3-2x+3}{x^2-1}$.

13. From $2a-3x+\frac{a-x}{a}$ take $a-5x+\frac{x-a}{x}$.

Ans. $a+2x+\frac{a^2-x^2}{ax}$

## CASE VI.

### MULTIPLICATION OF FRACTIONS.

**139.** To multiply a fraction by an entire quantity, or an entire quantity by a fraction.

It is evident, from Prop. I., Art. 122, that this may be done by multiplying the numerator.

Thus, $\frac{a}{b}\times2=\frac{2a}{b}$, and $\frac{a}{b}\times m=\frac{am}{b}$.

Again, as either quantity may be made the multiplier, Art. 67, to multiply 4 by $\frac{2}{3}$, is the same as to multiply $\frac{2}{3}$ by 4. Hence,

TO MULTIPLY A FRACTION BY AN ENTIRE QUANTITY, OR AN ENTIRE QUANTITY BY A FRACTION,

**Rule.**—*Multiply the numerator by the entire quantity, and write the product over the denominator.*

From Art. 125, it is evident that a fraction may also be multiplied by dividing its denominator by the entire quantity.

Thus, in multiplying $\frac{5}{8}$ by 2, we may divide the denominator by 2, and the result will be $\frac{5}{4}$, which is the same as to multiply the numerator by 2, and reduce the resulting fraction to its lowest terms. Hence,

REVIEW.—139. How multiply a fraction by an entire quantity, or an entire quantity by a fraction? When the denominator is divisible by the entire quantity, what is the shortest method?

*In multiplying a fraction and an entire quantity together, we should always divide the denominator of the fraction by the entire quantity, when it can be done without a remainder.*

REMARK.—The expression "$\frac{2}{3}$ of 6" is the same as $\frac{2}{3}\times 6$.

1. Multiply $\frac{2a}{bc}$ by $ad$. . . . . . . . . Ans. $\frac{2a^2d}{bc}$.

2. Multiply $\frac{a+b}{c}$ by $xy$. . . . . Ans. $\frac{axy+bxy}{c}$.

3. Multiply $a-2b$ by $\frac{4c}{2a+c}$. . . . Ans. $\frac{4ac-8bc}{2a+c}$.

4. Multiply $a^2-b^2$ by $\frac{3c-a}{2a}$. Ans. $\frac{3a^2c-3b^2c-a^3+ab^2}{2a}$.

5. Multiply $\frac{2a+3xz}{a^2b}$ by $ab$. . . . Ans. $\frac{2a+3xz}{a}$.

6. Multiply $\frac{5bc+3bx}{10x^3y^2-14x^4y^3}$ by $2x^2y^2$. Ans. $\frac{5bc+3bx}{5x-7x^2y}$.

7. Multiply $\frac{ax+by}{4(a+b)(a-b)}$ by $2(a-b)$. Ans. $\frac{ax+by}{2(a+b)}$.

8. Multiply $\frac{5c+4d}{5(a-b)(c^2-d^2)}$ by $5(a-b)(c+d)$.
Ans. $\frac{5c+4d}{c-d}$.

9. Multiply $\frac{a}{c}$ by $c$. . . . . . . Ans. $\frac{ac}{c}=a$, or $\frac{a}{1}$.

REMARK.—If a fraction is multiplied by a quantity equal to its denominator, the product will equal the numerator.

10. Multiply $\frac{a-b}{c+d}$ by $c+d$. . . . . . . . Ans. $a-b$.

11. Multiply $\frac{m^2-n^2}{2x+5y}$ by $2x+5y$. . . . Ans. $m^2-n^2$.

**140.** To multiply a fraction by a fraction.

1. Let it be required to multiply $\frac{4}{5}$ by $\frac{2}{3}$.

Since $\frac{2}{3}$ is the same as 2 multiplied by $\frac{1}{3}$, it is required to multiply $\frac{4}{5}$ by 2, and take $\frac{1}{3}$ of the product.

Now, $\frac{4}{5}\times2=\frac{8}{5}$, and $\frac{1}{3}$ of $\frac{8}{5}$ is $\frac{8}{15}$. Hence, the product of $\frac{4}{5}$ and $\frac{2}{3}$ is $\frac{8}{15}$.

So, to multiply $\frac{a}{c}$ by $\frac{m}{n}$, multiply $\frac{a}{c}$ by $m$, and take $\frac{1}{n}$ of the product. $\frac{a}{c}\times m=\frac{ma}{c}$, and $\frac{1}{n}$ of $\frac{ma}{c}=\frac{ma}{nc}$. Hence,

TO MULTIPLY A FRACTION BY A FRACTION,

**Rule.**—*Multiply the numerators together for a new numerator, and the denominators together for a new denominator.*

REMARKS.—1. The expression "one third of one fourth" has the same meaning as "$\frac{1}{4}$ multiplied by $\frac{1}{3}$;" or, $\frac{1}{3}$ of $\frac{1}{4}=\frac{1}{4}\times\frac{1}{3}$.

2. If either of the factors is a mixed quantity, reduce it to an improper fraction before commencing the operation.

3. When the numerators and denominators have common factors, indicate the multiplication, and cancel such factors.

Thus, $\frac{14a}{15b}\times\frac{5c}{21d}=\frac{2\times7\times5ac}{5\times3\times3\times7bd}=\frac{2ac}{9bd}$.

Also, $\frac{5a}{a^2-b^2}\times\frac{a+b}{2a}=\frac{5a(a+b)}{2a(a+b)(a-b)}=\frac{5}{2(a-b)}$.

1. Multiply $\frac{3a}{4}$ by $\frac{5x}{8}$. . . . . . . . Ans. $\frac{15ax}{32}$.
2. Multiply $\frac{4a}{5x}$ by $\frac{3x}{7a}$. . . . . . . . . Ans. $\frac{12}{35}$.
3. Multiply $\frac{3(a+x)}{2}$ by $\frac{4x}{a+x}$. . . . . Ans. $6x$.
4. Multiply $\frac{2x+3}{5}$ by $\frac{10x}{7}$. . . . . Ans. $\frac{4x^2+6x}{7}$.
5. Multiply $\frac{x^2-y^2}{ab}$ by $\frac{a^2}{x+y}$. . . . . Ans. $\frac{a(x-y)}{b}$.
6. Multiply $\frac{xyz}{x^4+y^3}$ by $\frac{x^4+y^3}{xyz}$. . . . . . . . Ans. 1.

REVIEW.—140. How do you multiply one fraction by another? Explain by analyzing an example. When one factor is a mixed quantity, what ought to be done? When the numerator and denominator have common factors? What the meaning of "one third of one fourth?"

7. Multiply $\frac{a-x}{x^2}$ by $\frac{a^2}{a^2-x^2}$. . . . . Ans. $\frac{a^2}{x^2(a+x)}$.

8. Multiply $\frac{x}{a+x}$, $\frac{a^2-x^2}{x^2}$, and $\frac{a}{a-x}$ together. Ans. $\frac{a}{x}$.

9. Multiply $\frac{a-b}{2}$, $\frac{2}{a^2-b^2}$, and $a+b$ together. Ans. 1.

10. Multiply $\frac{x^2+y^2}{x^2-y^2}$ by $\frac{x-y}{x+y}$. . . Ans. $\frac{x^2+y^2}{x^2+2xy+y^2}$.

11. Multiply $c+\frac{cx}{c-x}$ by $\frac{c^2-x^2}{x+1}$. . . . Ans. $\frac{c^2(c+x)}{x+1}$.

## CASE VII.

### DIVISION OF FRACTIONS.

**141.** To divide a fraction by an entire quantity.

It has been shown, in Arts. 123 and 124, that a fraction is divided by an entire quantity, by dividing its numerator, or multiplying its denominator.

Thus, $\frac{4}{5}$ divided by 2, or $\frac{1}{2}$ of $\frac{4}{5}$, is $\frac{2}{5}$; or, $\frac{4}{5}\div 2=\frac{4}{10}=\frac{2}{5}$.
So, $\frac{ma}{n}$ divided by $m$, or $\frac{1}{m}$ of $\frac{ma}{n}$ is $\frac{a}{n}$. Hence,

### TO DIVIDE A FRACTION BY AN ENTIRE QUANTITY,

**Rule.**—*Divide the numerator by the divisor, if it can be done without a remainder; if not, multiply the denominator.*

To divide a number by 2 is to take $\frac{1}{2}$ of it, or to multiply it by $\frac{1}{2}$; to divide by $m$ is to take $\frac{1}{m}$ of it, or to multiply it by $\frac{1}{m}$. Hence,

*To divide a fraction by an entire quantity, we may write the divisor in the form of a fraction, as $m=\frac{m}{1}$, then invert it, and proceed as in multiplication of fractions.*

REVIEW.—141. How divide a fraction by an entire quantity? Explain the reason of the rule by analyzing an example.

1. Divide $\frac{6a^2b}{7n}$ by $3ab$. . . . . . . . . Ans. $\frac{2a}{7n}$.

2. Divide $\frac{14ac^3m^2}{11xy}$ by $7acm^2$. . . . . Ans. $\frac{2c^2}{11xy}$.

3. Divide $\frac{a^2+ab}{3+2x}$ by $a$. . . . . . . . Ans. $\frac{a+b}{3+2x}$.

4. Divide $\frac{c^2+cd}{5}$ by $c+d$. . . . . . . . Ans. $\frac{c}{5}$.

5. Divide $\frac{x^2+2xy+y^2}{c+d}$ by $x+y$. . . . . Ans. $\frac{x+y}{c+d}$.

6. Divide $\frac{2a}{3c}$ by $b$. . . . . . . . . . Ans. $\frac{2a}{3bc}$.

7. Divide $\frac{3+5a}{a-b}$ by $a+b$. . . . . . Ans. $\frac{3+5a}{a^2-b^2}$.

8. Divide $\frac{3a+5c}{2x+3y}$ by $2x-3y$. . . . Ans. $\frac{3a+5c}{4x^2-9y^2}$.

9. Divide $\frac{b-c}{a^2+ab+b^2}$ by $a-b$. . . . Ans. $\frac{b-c}{a^3-b^3}$.

10. Divide $\frac{x-y}{x^2-xy+y^2}$ by $x+y$. . . . Ans. $\frac{x-y}{x^3+y^3}$.

11. Divide $\frac{a^2+abc}{b+c}$ by $a+bc$. . . . . . Ans. $\frac{a}{b+c}$.

12. Divide $\frac{m^2-n^2}{b+c}$ by $am-an$. . . . . Ans. $\frac{m+n}{ab+ac}$.

13. Divide $\frac{a^3-b^3}{c}$ by $a^2+ab+b^2$. . . . . Ans. $\frac{a-b}{c}$.

**142.** To divide an integral or fractional quantity by a fraction.

1. How many times is $\frac{2}{3}$ contained in 4, or what is the quotient of 4 divided by $\frac{2}{3}$?

**$\frac{1}{3}$ is contained in 4 three times as often as 1 is contained in 4, because 1 is 3 times as great as $\frac{1}{3}$; therefore, $\frac{1}{3}$ is contained in 4, 12 times; $\frac{2}{3}$ is contained in 4 only one half as often as $\frac{1}{3}$, since it is twice as great; therefore, $\frac{2}{3}$ is contained in 4, 6 times.**

2. How many times is $\frac{m}{n}$ contained in $a$?

Reasoning as above, $\frac{1}{n}$ is contained in $a$, $na$ times, and $\frac{m}{n}$ is contained $\frac{na}{m}$ times.

3. How many times is $\frac{2}{3}$ contained in $\frac{3}{4}$?

$\frac{1}{3}$ is contained in $\frac{3}{4}$, three times as often as 1 is contained in $\frac{3}{4}$, that is, $\frac{9}{4}$ times; and $\frac{2}{3}$, half as often as $\frac{1}{3}$, that is, $\frac{9}{8}$ times.

4. How many times is $\frac{m}{n}$ contained in $\frac{a}{c}$?

Reasoning as before, $\frac{1}{n}$ is contained in $\frac{a}{c}$, $\frac{an}{c}$ times; and $\frac{m}{n}$ is contained $\frac{an}{cm}$ times.

An examination of each of these examples will show that the process consists in multiplying the dividend by the denominator of the divisor, and dividing it by the numerator. If, then, the divisor be inverted, the operation will be the same as that in multiplication of fractions. Hence,

### TO DIVIDE AN INTEGRAL OR FRACTIONAL QUANTITY BY A FRACTION,

**Rule.**—*Invert the divisor, and proceed as in multiplication of fractions.*

NOTE.—After inverting the divisor, abbreviate by canceling.

1. Divide 4 by $\frac{a}{3}$. . . . . . . . . . Ans. $\frac{12}{a}$.

2. Divide $a$ by $\frac{1}{4}$. . . . . . . . . . Ans. $4a$

3. Divide $ab^2$ by $\frac{2ab}{5c}$. . . . . . . . . . Ans. $\frac{5bc}{2}$.

4. Divide $\frac{a}{3}$ by $\frac{c}{2}$. . . . . . . . . . . Ans. $\frac{2a}{3c}$.

REVIEW.—142. How divide an integral or fractional quantity by a fraction? Explain the reason of this rule, by analyzing the examples given. When and how can the work be abbreviated?

5. Divide $\frac{x^2y}{3a}$ by $\frac{xy^2}{2b}$. . . . . . . . . Ans. $\frac{2bx}{3ay}$.

6. Divide $\frac{16ax}{5}$ by $\frac{4x}{15}$ . . . . . . . Ans. $12a$.

7. Divide $\frac{6z+4}{5}$ by $\frac{3z+2}{4y}$. . . . . . . . Ans. $\frac{8y}{5}$.

8. Divide $\frac{a^2-b^2}{5}$ by $\frac{a+b}{a}$. . . . . . Ans. $\frac{a(a-b)}{5}$.

9. Divide $\frac{z^2-4}{6}$ by $\frac{z-2}{2}$. . . . . . . . Ans. $\frac{z+2}{3}$.

10. Divide $\frac{x^2-2xy+y^2}{ab}$ by $\frac{x-y}{bc}$. . . Ans. $\frac{cx-cy}{a}$.

11. Divide $\frac{a}{a^2-1}$ by $\frac{a+1}{a-1}$. . . . Ans. $\frac{a}{a^2+2a+1}$.

12. Divide $\frac{2z+3}{x+y}$ by $\frac{10z+15}{x^2-y^2}$. . . . . Ans. $\frac{x-y}{5}$.

13. Divide $\frac{3(a^2-x^2)}{x}$ by $\frac{2(a+x)}{a-x}$. Ans. $\frac{3(a^2-2ax+x^2)}{2x}$.

14. Divide $\frac{2x^2}{a^3+x^3}$ by $\frac{x}{a+x}$. . . . Ans. $\frac{2x}{a^2-ax+x^2}$.

**143.** To reduce a complex to a simple fraction.

This may be regarded as a case of division, in which the dividend and the divisor are either fractions or mixed quantities.

Thus, $\frac{2\frac{1}{3}}{3\frac{1}{2}}$ is the same as $2\frac{1}{3}$ divided by $3\frac{1}{2}$.

Also, $\frac{a+\frac{b}{c}}{m+\frac{n}{r}}$ is the same as $a+\frac{b}{c}$ divided by $m+\frac{n}{r}$.

$$2\tfrac{1}{3}\div 3\tfrac{1}{2}=\frac{7}{3}\div\frac{7}{2}=\frac{7}{3}\times\frac{2}{7}=\frac{2}{3}.$$

$$\left(a+\frac{b}{c}\right)\div\left(m+\frac{n}{r}\right)=\frac{ac+b}{c}\div\frac{mr+n}{r}=\frac{ac+b}{c}\times\frac{r}{mr+n}=\frac{acr+br}{cmr+cn}.$$

In like manner, reduce the following complex to simple fractions:

1. $\dfrac{\frac{a}{b}}{\frac{c}{d}}$. . . Ans. $\dfrac{ad}{bc}$.

2. $\dfrac{3\frac{1}{2}}{\frac{a}{3}}$. . . . Ans. $\dfrac{21}{2a}$.

3. $\dfrac{a+\frac{1}{c}}{m}$. . . Ans. $\dfrac{ac+1}{cm}$.

4. $\dfrac{m}{a+\frac{1}{c}}$. . . Ans. $\dfrac{cm}{ac+1}$.

A complex fraction may also be reduced to a simple one, *by multiplying both terms by the least common multiple of the denominators of the fractional parts of each term.*

Thus, to reduce $\dfrac{4\frac{1}{3}}{5\frac{1}{2}}$, multiply both terms by 6; the result is $\frac{26}{33}$.

**144.** Resolution of fractions into series.

An **Infinite Series** is an unlimited succession of terms, which observe the same law.

The **Law of a Series** is a relation existing between its terms, such as that when some of them are known the others may be found.

Thus, in the infinite series, $1-ax+a^2x^2-a^3x^3+a^4x^4$, etc., any term may be found by multiplying the preceding term by $-ax$.

Any proper algebraic fraction whose denominator is a polynomial can, by division, be resolved into an infinite series; for the process of division never can terminate.

After a few of the terms of the quotient are found, the law of the series will, in general, be easily discovered.

REVIEW.—143. How reduce a complex fraction to a simple one by division? How, by multiplication?

144. What is an infinite series? What is the law of a series? Give an example. Why can any proper algebraic fraction whose denominator is a polynomial, be resolved into an infinite series by division?

1. Convert the fraction $\frac{1}{1-x}$ into an infinite series.

$$\begin{array}{l|l} 1 & 1-x \\ \underline{1-x} & \overline{1+x+x^2+x^3+,\text{ etc.}} \\ \quad +x & \\ \quad \underline{+x-x^2} & \\ \qquad +x^2 & \\ \qquad \underline{+x^2-x^3} & \\ \qquad\quad +x^3 & \end{array}$$

**The law of this series evidently is, that each term is equal to the preceding term multiplied by $+x$.**

**From this, it appears that the fraction $\frac{1}{1-x}$ is equal to the infinite series, $1+x+x^2+x^3+x^4+$, etc.**

Resolve the following into infinite series by division:

2. $\frac{1}{1+x}=1-x+x^2-x^3+x^4-$, etc., to infinity.

3. $\frac{ax}{a-x}=x+\frac{x^2}{a}+\frac{x^3}{a^2}+\frac{x^4}{a^3}+$, etc., to infinity.

4. $\frac{1-x}{1+x}=1-2x+2x^2-2x^3+$, etc., to infinity.

5. $\frac{x+2}{x+1}=1+\frac{1}{x}-\frac{1}{x^2}+\frac{1}{x^3}-$, etc., to infinity.

## GENERAL REVIEW.

**Define a fraction. An entire quantity. A mixed quantity. An improper fraction. Simple. Compound. Complex. Terms of a fraction. Denominator. Numerator. State Proposition I., and illustrate it. Proposition II.; III.; IV.; V.; VI.**

**How reduce a fraction to its lowest terms? To an entire or mixed quantity? How a mixed quantity to a fraction? Rule for the signs of fractions. How reduce to common denominators? To *least* common denominators?**

**How add fractions? Subtract? Multiply? Divide? How reduce a complex to a simple fraction? How resolve a fraction into an infinite series?**

**Define mathematics. Algebra. Theorem. Problem. Factor. Coëfficient. Exponent. Power. Root. Monomial. Binomial. Trinomial. Polynomial. Residual quantity. Reciprocal of a quantity. Prime quantity. Composite. Quadratic trinomial. The G.C.D. The L.C.M.**