UNIT 9 Systems of Linear Equations

UNIT 9 Systems of Linear Equations

List of Objectives

To determine if a given ordered pair is a solution of a system of linear equations

To solve a system of linear equations by graphing

To solve a system of linear equations by the substitution method

To solve a system of linear equations by the addition method

To solve rate-of-wind or current problems

To solve application problems using two variables

SECTION 1 Solving Systems of Linear Equations by Graphing

1.1 Objective

To determine if a given ordered pair is a solution of a system of linear equations

Equations considered together are called a **system of equations.** A system of equations is shown at the right.

$$\begin{aligned} 2x + y &= 3 \\ x + y &= 1 \end{aligned}$$

A **solution of a system of equations** is an ordered pair which is a solution of each equation of the system.

Is $(2, -1)$ a solution of the system

$$\begin{aligned} 2x + y &= 3 \\ x + y &= 1? \end{aligned}$$

$2x + y$	$= 3$
$2(2) + (-1)$	3
$4 + (-1)$	3
$3 = 3$	

$x + y$	$= 1$
$2 + (-1)$	1
$1 = 1$	

Yes, since $(2, -1)$ is a solution of each equation, it is the solution of the system of equations.

Example 1 Is $(1, -3)$ a solution of the system

$$\begin{aligned} 3x + 2y &= -3 \\ x - 3y &= 6? \end{aligned}$$

Solution

$3x + 2y$	$= -3$
$3 \cdot 1 + 2(-3)$	-3
$3 + (-6)$	-3
$-3 = -3$	

$x - 3y$	$= 6$
$1 - 3(-3)$	6
$1 - (-9)$	6
$10 \neq 6$	

No, $(1, -3)$ is not a solution of the system of equations.

Example 2 Is $(-1, -2)$ a solution of the system

$$\begin{aligned} 2x - 5y &= 8 \\ -x + 3y &= -5? \end{aligned}$$

Your solution

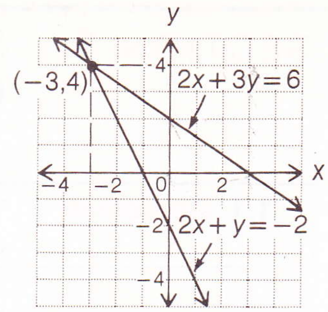
Solution on p. 484

1.2 Objective

To solve a system of linear equations by graphing

The solution of a system of linear equations can be found by graphing the two lines on the same coordinate system. The point of intersection of the lines is the ordered pair which is a solution of each equation of the system. It is the solution of the system of equations.

Solve by graphing: $2x + 3y = 6$
$2x + y = -2$



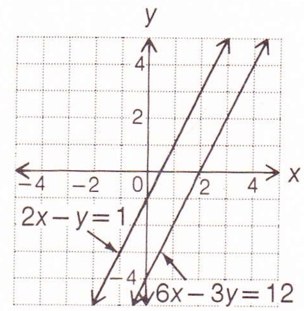
Graph each line.

Find the point of intersection.

$(-3,4)$ is a solution of each equation.

The solution is $(-3,4)$.

Solve by graphing: $2x - y = 1$
$6x - 3y = 12$

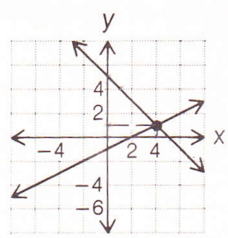


Graph each line.

The lines are parallel and therefore do not intersect. The system of equations has no solution.

Example 3 Solve by graphing:
$x - 2y = 2$
$x + y = 5$

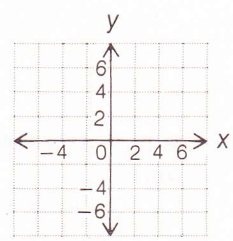
Solution



The solution is $(4,1)$.

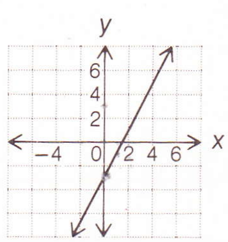
Example 4 Solve by graphing:
$x + 3y = 3$
$-x + y = 5$

Your solution



Example 5 Solve by graphing:
$4x - 2y = 6$
$y = 2x - 3$

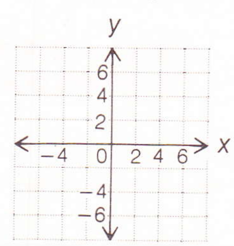
Solution



The two equations represent the same line. Any ordered pair which is a solution of one equation is also a solution of the other equation.

Example 6 Solve by graphing:
$y = 3x - 1$
$6x - 2y = -6$

Your solution



Solutions on p. 484

1.1 Exercises

1. Is (2,3) a solution of the system
$$3x + 4y = 18$$
$$2x - y = 1?$$

2. Is (2, −1) a solution of the system
$$x - 2y = 4$$
$$2x + y = 3?$$

3. Is (1, −2) a solution of the system
$$3x - y = 5$$
$$2x + 5y = -8?$$

4. Is (−1, −1) a solution of the system
$$x - 4y = 3$$
$$3x + y = 2?$$

5. Is (4,3) a solution of the system
$$5x - 2y = 14$$
$$x + y = 8?$$

6. Is (2,5) a solution of the system
$$3x + 2y = 16$$
$$2x - 3y = 4?$$

7. Is (−1,3) a solution of the system
$$4x - y = -5$$
$$2x + 5y = 13?$$

8. Is (4, −1) a solution of the system
$$x - 4y = 9$$
$$2x - 3y = 11?$$

9. Is (0,0) a solution of the system
$$4x + 3y = 0$$
$$2x - y = 1?$$

10. Is (2,0) a solution of the system
$$3x - y = 6$$
$$x + 3y = 2?$$

11. Is (2, −3) a solution of the system
$$y = 2x - 7$$
$$3x - y = 9?$$

12. Is (−1, −2) a solution of the system
$$3x - 4y = 5$$
$$y = x - 1?$$

13. Is (5,2) a solution of the system
$$y = 2x - 8$$
$$y = 3x - 13?$$

14. Is (−4,3) a solution of the system
$$y = 2x + 11$$
$$y = 5x - 19?$$

15. Is (−2, −3) a solution of the system
$$3x - 4y = 6$$
$$2x - 7y = 17?$$

16. Is (0,0) a solution of the system
$$y = 2x$$
$$3x + 5y = 0?$$

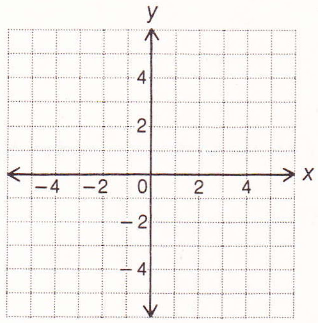
17. Is (0, −3) a solution of the system
$$4x - 3y = 9$$
$$2x + 5y = 15?$$

18. Is (4,0) a solution of the system
$$2x + 3y = 8$$
$$x - 5y = 4?$$

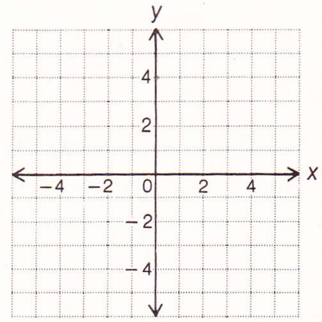
1.2 Exercises

Solve by graphing.

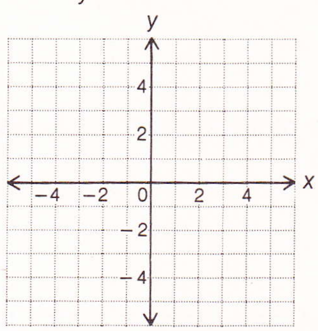
19. $x - y = 3$
$x + y = 5$



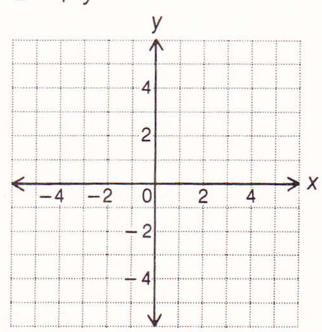
20. $2x - y = 4$
$x + y = 5$



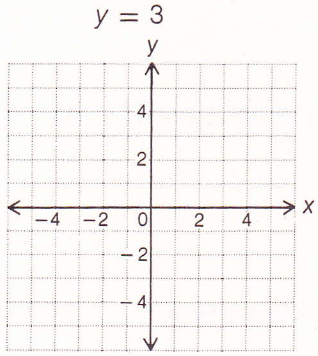
21. $x + 2y = 6$
$x - y = 3$



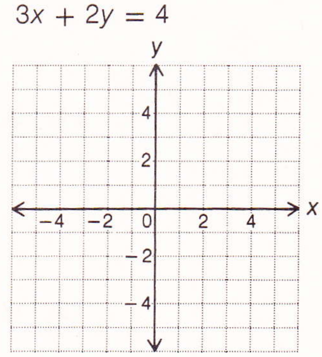
22. $3x - y = 3$
$2x + y = 2$



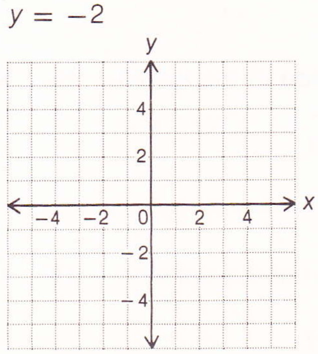
23. $3x - 2y = 6$
$y = 3$



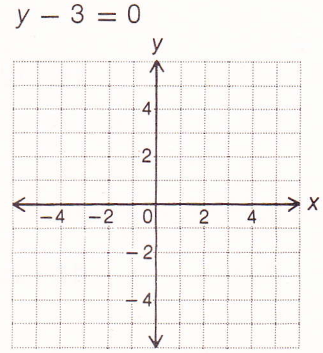
24. $x = 2$
$3x + 2y = 4$



25. $x = 3$
$y = -2$

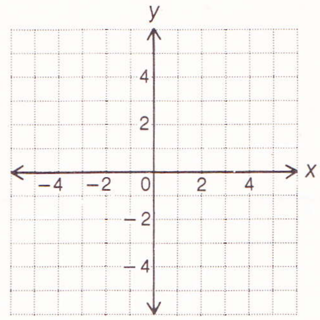


26. $x + 1 = 0$
$y - 3 = 0$

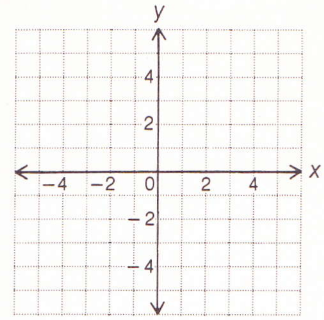


Solve by graphing.

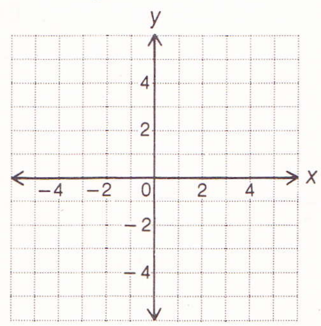
27. $y = 2x - 6$
$x + y = 0$



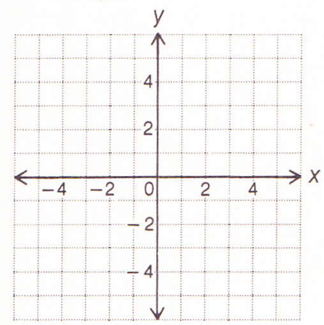
28. $5x - 2y = 11$
$y = 2x - 5$



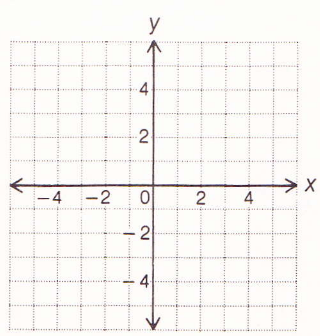
29. $2x + y = -2$
$6x + 3y = 6$



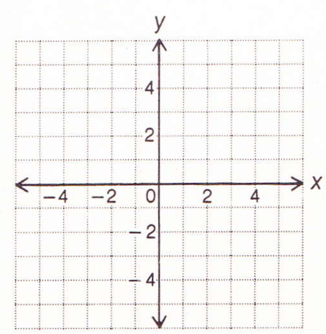
30. $x + y = 5$
$3x + 3y = 6$



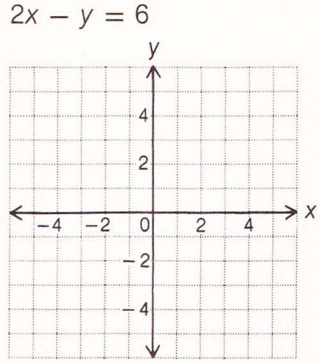
31. $4x - 2y = 4$
$y = 2x - 2$



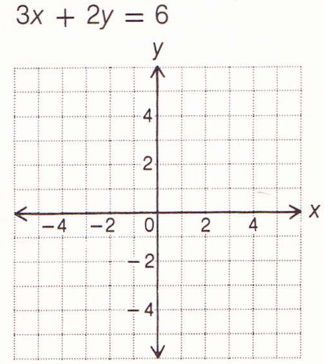
32. $2x + 6y = 6$
$y = -\frac{1}{3}x + 1$



33. $x - y = 5$
$2x - y = 6$



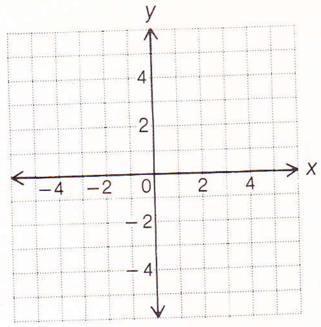
34. $5x - 2y = 10$
$3x + 2y = 6$



Solve by graphing.

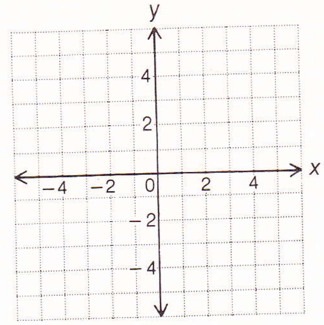
35. $3x + 4y = 0$

$2x - 5y = 0$



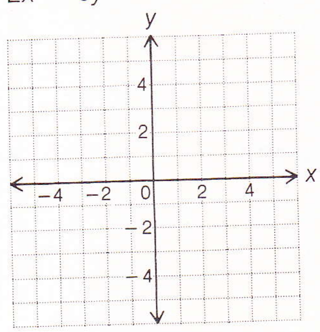
36. $2x - 3y = 0$

$y = -\frac{1}{3}x$



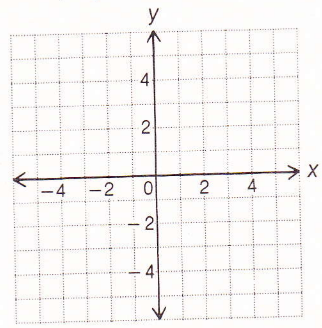
37. $x - 3y = 3$

$2x - 6y = 12$



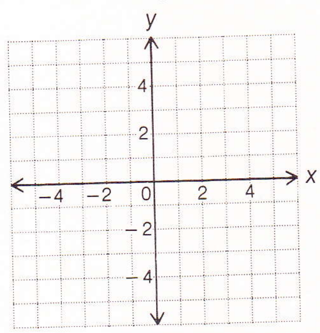
38. $4x + 6y = 12$

$6x + 9y = 18$



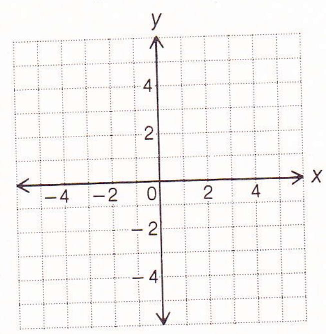
39. $3x + 2y = -4$

$x = 2y + 4$



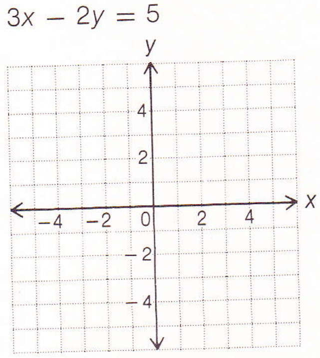
40. $5x + 2y = -14$

$3x - 4y = 2$



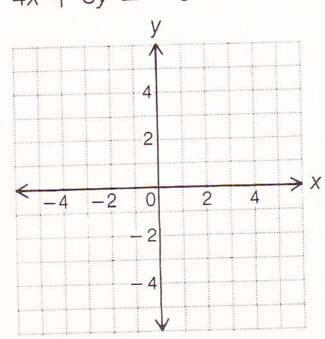
41. $4x - y = 5$

$3x - 2y = 5$



42. $2x - 3y = 9$

$4x + 3y = -9$



SECTION 2 Solving Systems of Linear Equations by the Substitution Method

2.1 Objective To solve a system of linear equations by the substitution method

A graphical solution of a system of equations may give only an approximate solution of the system. For example, the point $\left(\frac{1}{4}, \frac{1}{2}\right)$ would be difficult to read from the graph. An algebraic method, called the **substitution method,** can be used to find an exact solution of a system.

In the system of equations at the right, equation (2) states that $y = 3x - 9$. Substitute $3x - 9$ for y in equation (1).

$$(1) \quad 2x + 5y = -11$$
$$(2) \quad y = 3x - 9$$

$$2x + 5(3x - 9) = -11$$

Solve for x.

$$2x + 15x - 45 = -11$$
$$17x - 45 = -11$$
$$17x = 34$$
$$x = 2$$

Substitute the value of x into equation (2) and solve for y.

$$(2) \quad y = 3x - 9$$
$$y = 3 \cdot 2 - 9$$
$$y = 6 - 9$$
$$y = -3$$

The solution is $(2, -3)$.

Solve: $5x + y = 4$
$2x - 3y = 5$

$$(1) \quad 5x + y = 4$$
$$(2) \quad 2x - 3y = 5$$

Solve equation (1) for y. Equation (1) is chosen because it is the easier equation to solve for one variable in terms of the other.

$$5x + y = 4$$
$$y = -5x + 4$$

Substitute $-5x + 4$ for y in equation (2).

$$2x - 3(-5x + 4) = 5$$

Solve for x.

$$2x + 15x - 12 = 5$$
$$17x - 12 = 5$$
$$17x = 17$$
$$x = 1$$

Substitute the value of x in equation (1) and solve for y.

$$5x + y = 4$$
$$5(1) + y = 4$$
$$5 + y = 4$$
$$y = -1$$

The solution is $(1, -1)$.

Example 1 Solve by substitution:

$$3x + 4y = -2$$
$$-x + 2y = 4$$

Solution Solve equation (2) for x.

$$-x + 2y = 4$$
$$-x = -2y + 4$$
$$x = 2y - 4$$

Substitute in equation (1).

$$3(2y - 4) + 4y = -2$$
$$6y - 12 + 4y = -2$$
$$10y - 12 = -2$$
$$10y = 10$$
$$y = 1$$

Substitute in equation (2).

$$-x + 2y = 4$$
$$-x + 2(1) = 4$$
$$-x + 2 = 4$$
$$-x = 2$$
$$x = -2$$

The solution is $(-2,1)$.

Example 2 Solve by substitution:

$$7x - y = 4$$
$$3x + 2y = 9$$

Your solution

Example 3 Solve by substitution:

$$4x + 2y = 5$$
$$y = -2x + 1$$

Solution

$$4x + 2y = 5$$
$$4x + 2(-2x + 1) = 5$$
$$4x - 4x + 2 = 5$$
$$2 = 5$$

This is not a true equation. The lines are parallel and therefore do not intersect. The system does not have a solution.

Example 4 Solve by substitution:

$$3x - y = 4$$
$$y = 3x + 2$$

Your solution

Example 5 Solve by substitution:

$$6x - 2y = 4$$
$$y = 3x - 2$$

Solution

$$6x - 2y = 4$$
$$6x - 2(3x - 2) = 4$$
$$6x - 6x + 4 = 4$$
$$4 = 4$$

This is a true equation. The two equations represent the same line. Any ordered pair that is a solution of one equation is also a solution of the other equation.

Example 6 Solve by substitution:

$$y = -2x + 1$$
$$6x + 3y = 3$$

Your solution

Solutions on p. 485

2.1 Exercises

Solve by substitution.

1. $2x + 3y = 7$, $x = 2$

2. $y = 3$, $3x - 2y = 6$

3. $y = x - 3$, $x + y = 5$

4. $y = x + 2$, $x + y = 6$

5. $x = y - 2$, $x + 3y = 2$

6. $x = y + 1$, $x + 2y = 7$

7. $2x + 3y = 9$, $y = x - 2$

8. $3x + 2y = 11$, $y = x + 3$

9. $3x - y = 2$, $y = 2x - 1$

10. $2x - y = -5$, $y = x + 4$

11. $x = 2y - 3$, $2x - 3y = -5$

12. $x = 3y - 1$, $3x + 4y = 10$

13. $y = 4 - 3x$, $3x + y = 5$

14. $y = 2 - 3x$, $6x + 2y = 7$

15. $x = 3y + 3$, $2x - 6y = 12$

16. $x = 2 - y$, $3x + 3y = 6$

17. $3x + 5y = -6$, $x = 5y + 3$

18. $y = 2x + 3$, $4x - 3y = 1$

19. $4x - 3y = -1$, $y = 2x - 3$

20. $3x - 7y = 28$, $x = 3 - 4y$

21. $7x + y = 14$, $2x - 5y = -33$

Solve by substitution.

22. $3x + y = 4$
$4x - 3y = 1$

23. $x - 4y = 9$
$2x - 3y = 11$

24. $3x - y = 6$
$x + 3y = 2$

25. $4x - y = -5$
$2x + 5y = 13$

26. $3x - y = 5$
$2x + 5y = -8$

27. $3x + 4y = 18$
$2x - y = 1$

28. $4x + 3y = 0$
$2x - y = 0$

29. $5x + 2y = 0$
$x - 3y = 0$

30. $6x - 3y = 6$
$2x - y = 2$

31. $3x + y = 4$
$9x + 3y = 12$

32. $x - 5y = 6$
$2x - 7y = 9$

33. $x + 7y = -5$
$2x - 3y = 5$

34. $y = 2x + 11$
$y = 5x - 19$

35. $y = 2x - 8$
$y = 3x - 13$

36. $y = -4x + 2$
$y = -3x - 1$

37. $x = 3y + 7$
$x = 2y - 1$

38. $x = 4y - 2$
$x = 6y + 8$

39. $x = 3 - 2y$
$x = 5y - 10$

40. $y = 2x - 7$
$y = 4x + 5$

41. $3x - y = 11$
$2x + 5y = -4$

42. $-x + 6y = 8$
$2x + 5y = 1$

SECTION 3 Solving Systems of Linear Equations by the Addition Method

3.1 Objective To solve a system of linear equations by the addition method

Another algebraic method for solving a system of equations is called the **addition method.** It is based on the Addition Property of Equations.

Note, for the system of equations at the right, the effect of adding equation (2) to equation (1). Since $2y$ and $-2y$ are opposites, adding the equations results in an equation with only one variable.

$$\begin{aligned} (1)\quad 3x + 2y &= 4 \\ (2)\quad 4x - 2y &= 10 \\ 7x + 0y &= 14 \\ 7x &= 14 \end{aligned}$$

The solution of the resulting equation is the first component of the ordered pair solution of the system.

$$\begin{aligned} 7x &= 14 \\ x &= 2 \end{aligned}$$

The second component is found by substituting the value of x into equation (1) or (2) and then solving for y. Equation (1) is used here.

$$\begin{aligned} (1)\quad 3x + 2y &= 4 \\ 3 \cdot 2 + 2y &= 4 \\ 6 + 2y &= 4 \\ 2y &= -2 \\ y &= -1 \end{aligned}$$

The solution is $(2, -1)$.

Sometimes adding the two equations does not eliminate one of the variables. In this case, use the Multiplication Property of Equations to rewrite one or both of the equations, so that when the equations are added, one of the variables is eliminated.

To do this, first choose which variable to eliminate. The coefficients of that variable must be opposites. Multiply each equation by a constant which will produce coefficients which are opposites.

Solve: $3x + 2y = 7$
$5x - 4y = 19$

$$\begin{aligned} (1)\quad 3x + 2y &= 7 \\ (2)\quad 5x - 4y &= 19 \end{aligned}$$

Eliminate y. Multiply equation (1) by 2.

$$\begin{aligned} 2(3x + 2y) &= 2 \cdot 7 \\ 5x - 4y &= 19 \end{aligned}$$

Now the coefficients of the y terms are opposites.

$$\begin{aligned} 6x + 4y &= 14 \\ 5x - 4y &= 19 \end{aligned}$$

Add the equations.
Solve for x.

$$\begin{aligned} 11x + 0y &= 33 \\ 11x &= 33 \\ x &= 3 \end{aligned}$$

Substitute the value of x into one of the equations and solve for y. Equation (2) is used here.

$$\begin{aligned} (2)\quad 5x - 4y &= 19 \\ 5 \cdot 3 - 4y &= 19 \\ 15 - 4y &= 19 \\ -4y &= 4 \\ y &= -1 \end{aligned}$$

The solution is $(3, -1)$.

Solve: $5x + 6y = 3$
$2x - 5y = 16$

(1) $5x + 6y = 3$
(2) $2x - 5y = 16$

Eliminate x. Multiply equation (1) by 2 and equation (2) by -5. Note how the constants are selected.

$2(5x + 6y) = 2 \cdot 3$
$-5(2x - 5y) = -5 \cdot 16$

The negative is used so that the coefficients will be opposites.

Now the coefficients of the x terms are opposites.

$10x + 12y = 6$
$-10x + 25y = -80$

Add the equations.
Solve for y.

$0x + 37y = -74$
$37y = -74$
$y = -2$

Substitute the value of y into one of the equations and solve for x. Equation (1) is used here.

(1) $5x + 6y = 3$
$5x + 6(-2) = 3$
$5x - 12 = 3$
$5x = 15$
$x = 3$

The solution is $(3, -2)$.

Solve: $5x = 2y + 19$
$3x + 4y = 1$

(1) $5x = 2y + 19$
(2) $3x + 4y = 1$

Write equation (1) in the form $Ax + By = C$.

$5x - 2y = 19$
$3x + 4y = 1$

Eliminate y. Multiply equation (1) by 2.

$2(5x - 2y) = 2 \cdot 19$
$3x + 4y = 1$

Now the coefficients of the y terms are opposites.

$10x - 4y = 38$
$3x + 4y = 1$

Add the equations.
Solve for x.

$13x + 0y = 39$
$13x = 39$
$x = 3$

Substitute the value of x into one of the equations and solve for y. Equation (1) is used here.

$5x = 2y + 19$
$5 \cdot 3 = 2y + 19$
$15 = 2y + 19$
$-4 = 2y$
$-2 = y$

The solution is $(3, -2)$.

Solve: $2x + y = 2$
$4x + 2y = 5$

(1) $2x + y = 2$
(2) $4x + 2y = 5$

Eliminate y. Multiply equation (1) by -2

$$-2(2x + y) = -2 \cdot 2$$
$$4x + 2y = 5$$

$$-4x - 2y = -4$$
$$4x + 2y = 5$$

Add the equations.

$$0x + 0y = 1$$
$$0 = 1$$

This is not a true equation. The lines are parallel and therefore do not intersect. The system does not have a solution.

Example 1 Solve by the addition method:
$2x + 4y = 7$
$5x - 3y = -2$

Solution Eliminate x.

$$5(2x + 4y) = 5 \cdot 7$$
$$-2(5x - 3y) = -2 \cdot (-2)$$

$$10x + 20y = 35$$
$$-10x + 6y = 4$$

Add the equations.

$$26y = 39$$
$$y = \frac{39}{26} = \frac{3}{2}$$

Replace y in equation (1).

$$2x + 4\left(\frac{3}{2}\right) = 7$$
$$2x + 6 = 7$$
$$2x = 1$$
$$x = \frac{1}{2}$$

The solution is $\left(\frac{1}{2}, \frac{3}{2}\right)$.

Example 2 Solve by the addition method:
$x - 2y = 1$
$2x + 4y = 0$

Your solution

Solution on p. 486

Example 3 Solve by the addition method:

$$6x + 9y = 15$$
$$4x + 6y = 10$$

Solution Eliminate x.

$$4(6x + 9y) = 4 \cdot 15$$
$$-6(4x + 6y) = -6 \cdot 10$$

$$24x + 36y = 60$$
$$-24x - 36y = -60$$

Add the equations.

$$0x + 0y = 0$$
$$0 = 0$$

This is a true equation. The two equations represent the same line. Any ordered pair that is a solution of one equation is also a solution of the other equation.

Example 4 Solve by the addition method:

$$2x - 3y = 4$$
$$-4x + 6y = -8$$

Your solution

Example 5 Solve by the addition method:

$$2x = y + 8$$
$$3x + 2y = 5$$

Solution Write equation (1) in the form $Ax + By = C$.

$$2x = y + 8$$
$$2x - y = 8$$

Eliminate y.

$$2(2x - y) = 2 \cdot 8$$
$$3x + 2y = 5$$

$$4x - 2y = 16$$
$$3x + 2y = 5$$

Add the equations.

$$7x = 21$$
$$x = 3$$

Replace x in equation (1).

$$2 \cdot 3 = y + 8$$
$$6 = y + 8$$
$$-2 = y$$

The solution is $(3, -2)$.

Example 6 Solve by the addition method:

$$4x + 5y = 11$$
$$3y = x + 10$$

Your solution

Solutions on p. 486

3.1 Exercises

Solve by the addition method.

1. $x + y = 4$
   $x - y = 6$

2. $2x + y = 3$
   $x - y = 3$

3. $x + y = 4$
   $2x + y = 5$

4. $x - 3y = 2$
   $x + 2y = -3$

5. $2x - y = 1$
   $x + 3y = 4$

6. $x - 2y = 4$
   $3x + 4y = 2$

7. $4x - 5y = 22$
   $x + 2y = -1$

8. $3x - y = 11$
   $2x + 5y = 13$

9. $2x - y = 1$
   $4x - 2y = 2$

10. $x + 3y = 2$
    $3x + 9y = 6$

11. $4x + 3y = 15$
    $2x - 5y = 1$

12. $3x - 7y = 13$
    $6x + 5y = 7$

13. $2x - 3y = 1$
    $4x - 6y = 2$

14. $2x + 4y = 6$
    $3x + 6y = 9$

15. $5x - 2y = -1$
    $x + 3y = -5$

16. $4x - 3y = 1$
    $8x + 5y = 13$

17. $5x + 7y = 10$
    $3x - 14y = 6$

18. $7x + 10y = 13$
    $4x + 5y = 6$

19. $3x - 2y = 0$
    $6x + 5y = 0$

20. $5x + 2y = 0$
    $3x + 5y = 0$

21. $2x - 3y = 16$
    $3x + 4y = 7$

Solve by the addition method.

22. $3x + 4y = 10$
$4x + 3y = 11$

23. $5x + 3y = 7$
$2x + 5y = 1$

24. $-2x + 7y = 9$
$3x + 2y = -1$

25. $7x - 2y = 13$
$5x + 3y = 27$

26. $12x + 5y = 23$
$2x - 7y = 39$

27. $8x - 3y = 11$
$6x - 5y = 11$

28. $4x - 8y = 36$
$3x - 6y = 27$

29. $5x + 15y = 20$
$2x + 6y = 8$

30. $y = 2x - 3$
$3x + 4y = -1$

31. $3x = 2y + 7$
$5x - 2y = 13$

32. $2y = 4 - 9x$
$9x - y = 25$

33. $2x + 9y = 16$
$5x = 1 - 3y$

34. $3x - 4 = y + 18$
$4x + 5y = -21$

35. $2x + 3y = 7 - 2x$
$7x + 2y = 9$

36. $5x - 3y = 3y + 4$
$4x + 3y = 11$

37. $3x + y = 1$
$5x + y = 2$

38. $2x - y = 1$
$2x - 5y = -1$

39. $4x + 3y = 3$
$x + 3y = 1$

40. $2x - 5y = 4$
$x + 5y = 1$

41. $3x - 4y = 1$
$4x + 3y = 1$

42. $2x - 7y = -17$
$3x + 5y = 17$

SECTION 4 Application Problems in Two Variables

4.1 Objective To solve rate-of-wind or current problems

Motion problems which involve an object moving with or against a wind or current normally require two variables to solve.

Flying with the wind, a small plane can fly 600 mi in 3 h. Against the wind, the plane can fly the same distance in 4 h. Find the rate of the plane in calm air and the rate of the wind.

STRATEGY FOR SOLVING RATE-OF-WIND OR CURRENT PROBLEMS

▷ Choose one variable to represent the rate of the object in calm conditions and a second variable to represent the rate of the wind or current. Using these variables, express the rate of the object with and against the wind or current. Use the equation $d = rt$ to write expressions for the distance traveled by the object. The results can be recorded in a table.

Rate of plane in calm air: p
Rate of wind: w

	Rate	·	Time	=	Distance
With the wind	$p + w$	·	3	=	$3(p + w)$
Against the wind	$p - w$	·	4	=	$4(p - w)$

▷ Determine how the expressions for distance are related.

The distance traveled with the wind is 600 mi. $3(p + w) = 600$
The distance traveled against the wind is 600 mi. $4(p - w) = 600$

Solve the system of equations.

$3(p + w) = 600$ $\quad \frac{1}{3} \cdot 3(p + w) = \frac{1}{3} \cdot 600$ $\quad p + w = 200$

$4(p - w) = 600$ $\quad \frac{1}{4} \cdot 4(p - w) = \frac{1}{4} \cdot 600$ $\quad p - w = 150$

$2p = 350$
$p = 175$

$p + w = 200$
$175 + w = 200$
$w = 25$

The rate of the plane in calm air is 175 mph.
The rate of the wind is 25 mph.

Example 1

A 450-mile trip from one city to another takes 3 h when a plane is flying with the wind. The return trip, against the wind, takes 5 h. Find the rate of the plane in still air and the rate of the wind.

Strategy

▷ Rate of the plane in still air: p
Rate of the wind: w

	Rate	Time	Distance
With wind	$p + w$	3	$3(p + w)$
Against wind	$p - w$	5	$5(p - w)$

▷ The distance traveled with the wind is 450 mi.
The distance traveled against the wind is 450 mi.

Solution

$3(p + w) = 450 \qquad \frac{1}{3} \cdot 3(p + w) = \frac{1}{3} \cdot 450$

$5(p - w) = 450 \qquad \frac{1}{5} \cdot 5(p - w) = \frac{1}{5} \cdot 450$

$$p + w = 150$$
$$p - w = 90$$
$$2p = 240$$
$$p = 120$$

$$p + w = 150$$
$$120 + w = 150$$
$$w = 30$$

The rate of the plane in still air is 120 mph.
The rate of the wind is 30 mph.

Example 2

A canoeist paddling with the current can travel 15 mi in 3 h. Against the current it takes 5 h to travel the same distance. Find the rate of the current and the rate of the canoeist in calm water.

Your strategy

Your solution

Solution on p. 487

4.2 Objective — To solve application problems using two variables

The application problems in this section are varieties of those problems solved earlier in the text. Each of the strategies for the problems in this section will result in a system of equations.

A jeweler purchased 5 oz of a gold alloy and 20 oz of a silver alloy for a total cost of \$540. The next day, at the same prices per ounce, the jeweler purchased 4 oz of the gold alloy and 25 oz of the silver alloy for a total cost of \$450. Find the cost per ounce of the gold and silver alloys.

STRATEGY FOR SOLVING AN APPLICATION PROBLEM IN TWO VARIABLES

▷ Choose one variable to represent one of the unknown quantities and a second variable to represent the other unknown quantity. Write numerical or variable expressions for all the remaining quantities. These results can be recorded in two tables, one for each of the conditions.

Cost per ounce of gold: g
Cost per ounce of silver: s

First Day

	Amount	·	Unit Cost	=	Value
Gold	5	·	g	=	$5g$
Silver	20	·	s	=	$20s$

Second Day

	Amount	·	Unit Cost	=	Value
Gold	4	·	g	=	$4g$
Silver	25	·	s	=	$25s$

▷ Determine a system of equations. The strategies presented in Unit 4 can be used to determine the relationships between the expressions in the tables. Each table will give one equation of the system.

The total value of the purchase on the first day was \$540. $5g + 20s = 540$
The total value of the purchase on the second day was \$450. $4g + 25s = 450$

Solve the system of equations.

$$5g + 20s = 540 \qquad 4(5g + 20s) = 4 \cdot 540 \qquad 20g + 80s = 2160$$
$$4g + 25s = 450 \qquad -5(4g + 25s) = -5 \cdot 450 \qquad -20g - 125s = -2250$$

$$-45s = -90$$
$$s = 2$$

$$5g + 20s = 540$$
$$5g + 20(2) = 540$$
$$5g + 40 = 540$$
$$5g = 500$$
$$g = 100$$

The cost per ounce of the gold alloy was \$100.
The cost per ounce of the silver alloy was \$2.

Example 3

In five years, an oil painting will be twice as old as a water color painting will be then. Five years ago, the oil painting was three times as old as the water color was then. Find the present age of each painting.

Strategy

▷ Present age of oil painting: x
Present age of water color: y

	Present	Future
Oil	x	$x + 5$
Water Color	y	$y + 5$

	Present	Past
Oil	x	$x - 5$
Water Color	y	$y - 5$

▷ In five years, twice the age of the water color will be the age of the oil painting. Five years ago, three times the age of the water color was the age of the oil painting.

Solution

$$2(y + 5) = x + 5 \qquad 2y + 10 = x + 5$$
$$3(y - 5) = x - 5 \qquad 3y - 15 = x - 5$$

$$2y + 5 = x$$
$$3y - 15 = x - 5$$

$$3y - 15 = (2y + 5) - 5$$
$$3y - 15 = 2y$$
$$y - 15 = 0$$
$$y = 15$$

$$2y + 5 = x$$
$$2(15) + 5 = x$$
$$30 + 5 = x$$
$$35 = x$$

The present age of the oil painting is 35 years.
The present age of the water color is 15 years.

Example 4

Two coin banks contain only dimes and quarters. In the first bank, the total value of the coins is \$4.80. In the second bank, there are twice as many quarters as in the first bank and one half the number of dimes. The total value of the coins in the second bank is \$8.40. Find the number of dimes and the number of quarters in the first bank.

Your strategy

Your solution

Solution on p. 487

4.1 Application Problems

Solve.

1. Paddling with the current, a canoeist can go 24 mi in 3 h. Against the current it takes 4 h to go the same distance. Find the rate of the canoeist in calm water and the rate of the current.

2. A pilot flying with the wind flew the 750 mi between two cities in 3 h. The return trip against the wind took 5 h. Find the rate of the plane in calm air and the rate of the wind.

3. A motorboat traveling with the current can go 100 mi in 4 h. Against the current it takes 5 h to go the same distance. Find the rate of the motorboat in still water and the rate of the current.

4. A plane flying with a tailwind flew 360 mi in 2 h. Against the wind, it took 3 h to fly the same distance. Find the rate of the plane in calm air and the rate of the wind.

5. A rowing team rowing with the current traveled 16 mi in 2 h. Against the current, the team rowed 8 mi in 2 h. Find the rate of the rowing team in calm water and the rate of the current.

6. A small plane flew 300 mi with the wind in 2 h. Against the wind, it took 3 h to travel the same distance. Find the rate of the plane in calm air and the rate of the wind.

7. A small plane flew 260 mi in 2 h with the wind. Flying against the wind, the plane flew 180 mi in 2 h. Find the rate of the plane in calm air and the rate of the wind.

8. A motorboat traveling with the current went 30 mi in 3 h. Traveling against the current the boat went 12 mi in 3 h. Find the rate of the boat in calm water and the rate of the current.

9. A crew can row 60 km downstream in 3 h. Rowing upstream, against the current, the crew traveled 24 km in 3 h. Find the rowing rate of the crew in calm water and the rate of the current.

10. A plane flew 2000 km in 5 h traveling with the wind. Against the wind, the plane could fly only 1500 km in the same amount of time. Find the rate of the plane in calm air and the rate of the wind.

4.2 Application Problems

Solve.

11. A business manager had two reports photocopied. The first report, which cost \$3 to photocopy, included 50 black-and-white pages and 10 color pages. The total cost for photocopying the 75 black-and-white and the 20 color pages in the second report was \$5. Find the cost per copy for a black-and-white page and for a color page.

12. A computer store received two shipments of calculators. The value of the first shipment, which contained 10 scientific and 15 business calculators, was \$425. The value of the second shipment, which contained 8 scientific and 20 business calculators, was \$460. Find the cost of a scientific and the cost of a business calculator.

13. A metallurgist made two purchases. The first purchase, which cost \$110, included 20 kg of a tin alloy and 25 kg of an aluminum alloy. The second purchase, which cost \$60, included 10 kg of the tin alloy and 15 kg of the aluminum alloy. Find the cost per kilogram of the tin and the aluminum alloys.

14. For \$28, a customer purchased 2 lb of kona-blend coffee and 3 lb of a mocha-blend coffee. A second customer purchased 4 lb of the kona coffee and 2 lb of the mocha coffee for a total of \$32. Find the cost per pound of the kona coffee and the mocha coffee.

15. Two coin banks contain only nickels and quarters. The total value of the coins in the first bank is \$3.30. In the second bank there are two fewer quarters than in the first bank and twice as many nickels. The total value of the coins in the second bank is \$3.10. Find the number of nickels and the number of quarters in the first bank.

16. Two coin banks contain only nickels and dimes. The total value of the coins in the first bank is \$4. In the second bank there are 10 more nickels than in the first bank and one half as many dimes. The total value of the coins in the second bank is \$3.50. Find the number of nickels and the number of dimes in the first bank.

17. The total value of the dimes and quarters in a coin bank is \$3.70. If the quarters were dimes and the dimes were quarters, the total value of the coins would be \$4. Find the number of dimes and the number of quarters in the bank.

18. The total value of the nickels and dimes in a coin bank is \$5. If the nickels were dimes and the dimes were nickels, the total value of the coins would be \$4. Find the number of nickels and the number of dimes in the bank.

19. One year ago, an adult was five times the age a child was then. One year from now the adult will be four times the age the child will be then. Find the present ages of the adult and the child.

20. If twice the age of a stamp is added to three times the age of a coin, the result is 100. The difference between five times the age of the stamp and twice the age of the coin is three. Find the age of each.

Review/Test

SECTION 1

1.1a Is $(-2,3)$ a solution of the system

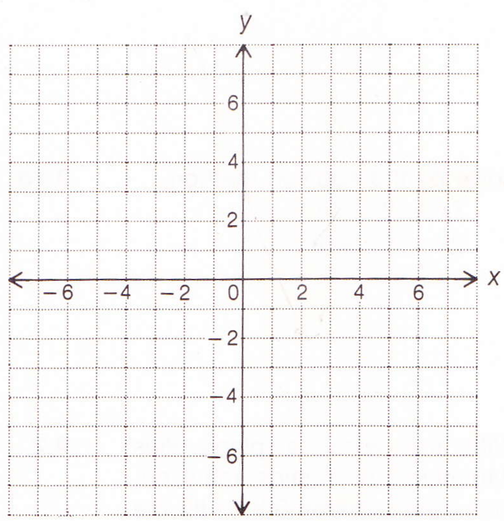
$$\begin{aligned} 2x + 5y &= 11 \\ x + 3y &= 7? \end{aligned}$$

1.1b Is $(1,-3)$ a solution of the system

$$\begin{aligned} 3x - 2y &= 9 \\ 4x + y &= 1? \end{aligned}$$

1.2 Solve by graphing:

$$\begin{aligned} 3x + 2y &= 6 \\ 5x + 2y &= 2 \end{aligned}$$



Review/Test

SECTION 2

2.1a Solve by substitution.

$$4x - y = 11$$
$$y = 2x - 5$$

2.1b Solve by substitution.

$$x = 2y + 3$$
$$3x - 2y = 5$$

2.1c Solve by substitution.

$$3x + 5y = 1$$
$$2x - y = 5$$

2.1d Solve by substitution.

$$3x - 5y = 13$$
$$x + 3y = 1$$

SECTION 3

3.1a Solve by the addition method.

$$4x + 3y = 11$$
$$5x - 3y = 7$$

3.1b Solve by the addition method.

$$2x - 5y = 6$$
$$4x + 3y = -1$$

3.1c Solve by the addition method.

$$7x + 3y = 11$$
$$2x - 5y = 9$$

3.1d Solve by the addition method.

$$5x + 6y = -7$$
$$3x + 4y = -5$$

SECTION 4

4.1 With the wind, a plane flies 240 mi in 2 h. Against the wind, the plane requires 3 h to fly the same distance. Find the rate of the plane in calm air and the rate of the wind.

4.2 For the first performance of a play in a community theater, 50 reserved-seat tickets and 80 general-admission tickets were sold. The total receipts were $980. For the second performance, 60 reserved-seat tickets and 90 general-admission tickets were sold. The total receipts were $1140. Find the price of a reserved-seat ticket and the price of a general-admission ticket.

Review/Test

SECTION 1

1.1a Which ordered pair is a solution of the system

$$3x - 2y = 8$$
$$4x + 5y = 3?$$

a) (2,1)
b) (4,2)
c) (−2,−7)
d) (2,−1)

1.1b Which ordered pair is a solution of the system

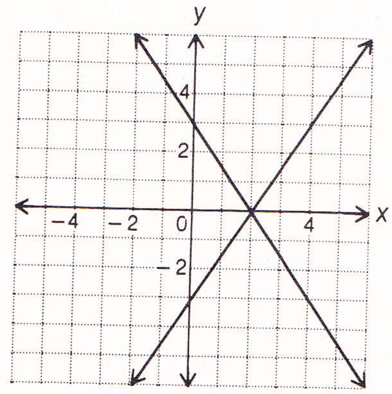
$$5x - 3y = 10$$
$$4x + 7y = 8?$$

a) (2,3)
b) (2,0)
c) (−1,2)
d) (1,−2)

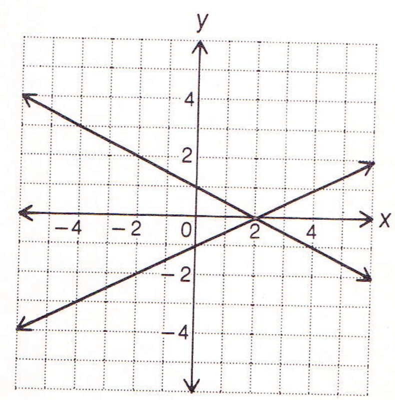
1.2 Solve by graphing:

$$3x + 2y = 6$$
$$3x - 2y = 6$$

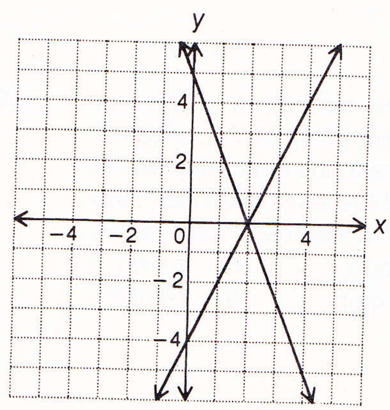
a)



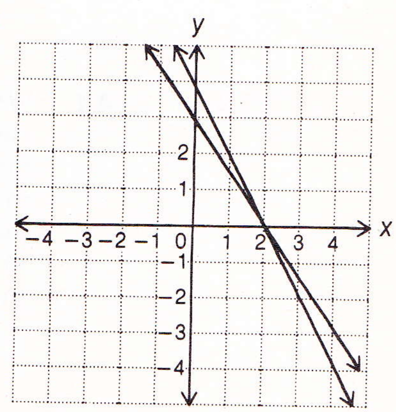
b)



c)



d)



Review/Test

SECTION 2

2.1a Solve by substitution.

$$3x - y = 5$$
$$y = 2x - 3$$

a) (8,13) b) (2,1)
c) (2,−1) d) (1,2)

2.1b Solve by substitution.

$$x = 3y + 1$$
$$2x + 5y = 13$$

a) (13,4) b) (4,1)
c) (1,4) d) (4,13)

2.1c Solve by substitution.

$$4x - 3y = 1$$
$$2x + y = 3$$

a) (−5,−7) b) (−1,5)
c) (1,1) d) (2,1)

2.1d Solve by substitution.

$$3x - 5y = -23$$
$$x + 2y = -4$$

a) (−10,3) b) (−6,1)
c) (2,−1) d) (4,−4)

SECTION 3

3.1a Solve by the addition method.

$$3x + 2y = 2$$
$$5x - 2y = 14$$

a) (2,−2) b) (0,1)
c) (6,−8) d) (−2,4)

3.1b Solve by the addition method.

$$5x + 4y = 7$$
$$3x - 2y = 13$$

a) (3,2) b) (3,−2)
c) (5,1) d) (−2,4)

3.1c Solve by the addition method.

$$5x - 3y = 29$$
$$4x + 7y = -5$$

a) (4,−3) b) (7,11)
c) (0,−5) d) (−3,4)

3.1d Solve by the addition method.

$$9x - 2y = 17$$
$$5x + 3y = -7$$

a) (3,5) b) (−4,1)
c) (1,−4) d) (−1,−13)

SECTION 4

4.1 With the current, a motorboat can travel 48 mi in 3 h. Against the current, the boat requires 4 h to travel the same distance. Find the rate of the boat in calm water.

a) 2 mph b) 4 mph
c) 14 mph d) 16 mph

4.2 Two coin banks contain only dimes and nickels. In the first bank, the total value of the coins is $5.50. In the second bank, there are one half as many dimes as in the first bank and 10 less nickels. The total value of the coins in the second bank is $3. Find the number of dimes in the first bank.

a) 15 b) 40
c) 30 d) 50