Answers to Unit 6

SECTION 1 Pages 201–202

Example 2

$12x^3y^6 = 2 \cdot 2 \cdot 3 \cdot x^3y^6$

$15x^2y^3 = 3 \cdot 5 \cdot x^2 \cdot y^3$

The GCF is $3x^2y^3$.

Example 4

$14a^2 = 2 \cdot 7 \cdot a^2$

$21a^4b = 3 \cdot 7 \cdot a^4b$

The GCF is $7a^2$.

$14a^2 - 21a^4b = 7a^2(2) + 7a^2(-3a^2b) =$
$7a^2(2 - 3a^2b)$

Example 6

$6x^4y^2 = 2 \cdot 3 \cdot x^4 \cdot y^2$

$9x^3y^2 = 3 \cdot 3 \cdot x^3 \cdot y^2$

$12x^2y^4 = 2 \cdot 2 \cdot 3 \cdot x^2 \cdot y^4$

The GCF is $3x^2y^2$.

$6x^4y^2 - 9x^3y^2 + 12x^2y^4 =$
$3x^2y^2(2x^2) + 3x^2y^2(-3x) + 3x^2y^2(4y^2) =$
$3x^2y^2(2x^2 - 3x + 4y^2)$

Pages 203–204

1. x^3 **3.** xy^4 **5.** xy^4z^2 **7.** ab^2c^3 **9.** $3x^2$ **11.** $2a$ **13.** $7a^3$ **15.** There is no common factor other than 1. **17.** $3a^2b^2$ **19.** ab **21.** $2x$ **23.** $3x$ **25.** $5(a + 1)$ **27.** $8(2 - a^2)$ **29.** $4(2x + 3)$ **31.** $6(5a - 1)$ **33.** $x(7x - 3)$ **35.** $a^2(3 + 5a^3)$ **37.** $y(14y + 11)$ **39.** $2x(x^3 - 2)$ **41.** $2x^2(5x^2 - 6)$ **43.** $4a^5(2a^3 - 1)$ **45.** $xy(xy - 1)$ **47.** $3xy(xy^3 - 2)$ **49.** $xy(x - y^2)$ **51.** There is no common factor other than 1. **53.** $6b^2(a^2b - 2)$ **55.** $a(a^2 - 3a + 5)$ **57.** $5(x^2 - 3x + 7)$ **59.** $3x(x^2 + 2x + 3)$ **61.** $2x^2(x^2 - 2x + 3)$ **63.** $2x(x^2 + 3x - 7)$ **65.** $y^3(2y^2 - 3y + 7)$ **67.** $xy(x^2 - 3xy + 7y^2)$ **69.** $5y(y^2 + 2y - 5)$ **71.** $3b^2(a^2 - 3a + 5)$

SECTION 2 Pages 205–208

Example 2

$(x - \square)(x - \square)$

Factors	Sums
$-1, -20$	-21
$-2, -10$	-12
$-4, -5$	-9

$(x - 4)(x - 5)$

$x^2 - 9x + 20 = (x - 4)(x - 5)$

Example 4

$(x + \square)(x - \square)$

Factors	Sums
$+1, -18$	-17
$-1, +18$	17
$+2, -9$	-7
$-2, +9$	7
$+3, -6$	-3
$-3, +6$	3

$(x + 6)(x - 3)$

$x^2 + 3x - 18 = (x + 6)(x - 3)$

Example 6

The GCF is $3b$.

$3a^2b - 18ab - 81b = 3b(a^2 - 6a - 27)$

Factor the trinomial.

$3b(a + \square)(a - \square)$

Factors	Sums
$+1, -27$	-26
$-1, +27$	26
$+3, -9$	-6
$-3, +9$	6

$3b(a + 3)(a - 9)$

$3a^2b - 18ab - 81ab = 3b(a + 3)(a - 9)$

Example 8

The GCF is 3.

$3x^2 - 9xy - 12y^2 = 3(x^2 - 3xy - 4y^2)$

Factor the trinomial.

$3(x + \square y)(x - \square y)$

Factors	Sums
$+1, -4$	-3
$-1, +4$	3
$+2, -2$	0

$3(x + y)(x - 4y)$

$3x^2 - 9xy - 12y^2 = 3(x + y)(x - 4y)$

Pages 209–212

1. $(x + 1)(x + 2)$ **3.** $(x + 1)(x - 2)$ **5.** $(a + 4)(a - 3)$ **7.** $(a - 2)(a - 1)$ **9.** $(a + 2)(a - 1)$
11. $(b - 3)(b - 3)$ **13.** $(b + 8)(b - 1)$ **15.** $(y + 11)(y - 5)$ **17.** $(y - 2)(y - 3)$ **19.** $(z - 5)(z - 9)$
21. $(z + 8)(z - 20)$ **23.** $(p + 3)(p + 9)$ **25.** $(x + 10)(x + 10)$ **27.** $(b + 4)(b + 5)$ **29.** $(x + 3)(x - 14)$
31. $(b + 4)(b - 5)$ **33.** $(y + 3)(y - 17)$ **35.** $(p + 3)(p - 7)$ **37.** Irreducible over the integers.
39. $(x - 5)(x - 15)$ **41.** $(x - 7)(x - 8)$ **43.** $(x + 8)(x - 7)$ **45.** $(a + 3)(a - 24)$ **47.** $(a - 3)(a - 12)$
49. $(z + 8)(z - 17)$ **51.** $(c + 9)(c - 10)$ **53.** $(z + 4)(z + 11)$ **55.** $(c + 2)(c + 17)$ **57.** $(x + 8)(x - 12)$
59. $(x - 8)(x - 14)$ **61.** $(b + 15)(b - 7)$ **63.** $(a + 3)(a - 12)$ **65.** $(b - 6)(b - 17)$ **67.** $(a + 3)(a + 24)$
69. $(x + 12)(x + 13)$ **71.** $(x + 6)(x - 16)$ **73.** $2(x + 1)(x + 2)$ **75.** $3(a + 3)(a - 2)$ **77.** $a(b + 5)(b - 3)$
79. $x(y - 2)(y - 3)$ **81.** $z(z - 3)(z - 4)$ **83.** $3y(y - 2)(y - 3)$ **85.** $3(x + 4)(x - 3)$ **87.** $5(z + 4)(z - 7)$
89. $2a(a + 8)(a - 4)$ **91.** $(x - 2y)(x - 3y)$ **93.** $(a - 4b)(a - 5b)$ **95.** $(x + 4y)(x - 7y)$
97. Irreducible over the integers. **99.** $z^2(z - 5)(z - 7)$ **101.** $b^2(b - 10)(b - 12)$ **103.** $2y^2(y + 3)(y - 16)$
105. $x^2(x + 8)(x - 1)$ **107.** $4y(x + 7)(x - 2)$ **109.** $8(y - 1)(y - 3)$ **111.** $c(c + 3)(c + 10)$
113. $3x(x - 3)(x - 9)$ **115.** $(x - 3y)(x - 5y)$ **117.** $(a - 6b)(a - 7b)$ **119.** $(y + z)(y + 7z)$
121. $3y(x + 21)(x - 1)$ **123.** $3x(x - 3)(x + 4)$ **125.** $4z(z + 11)(z - 3)$ **127.** $4x(x + 3)(x - 1)$
129. $5(p + 12)(p - 7)$ **131.** $p^2(p + 12)(p - 3)$ **133.** $(t - 5s)(t - 7s)$ **135.** $(a + 3b)(a - 11b)$
137. $5x^2(x - 2)(x - 4)$ **139.** $15a(b + 4)(b - 1)$ **141.** $3y(x + 15)(x - 3)$

SECTION 3 Pages 213–216

Example 2

($x +$)($x -$) or ($x -$)($x +$)

Factors of 2: 1,2 Factors of −3: +1, −3
−1, +3

Trial Factors	Middle Term
$(1x + 1)(2x - 3)$	$-3x + 2x = -x$
$(1x - 3)(2x + 1)$	$x - 6x = -5x$
$(1x - 1)(2x + 3)$	$3x - 2x = x$
$(1x + 3)(2x - 1)$	$-x + 6x = 5x$

$(x + 1)(2x - 3)$

$2x^2 - x - 3 = (x + 1)(2x - 3)$

Example 4

The GCF is $2a^2$.

$4a^2b^2 + 26a^2b - 14a^2 = 2a^2(2b^2 + 13b - 7)$

Factor the trinomial.

$2a^2($ $b +$)($b -$) or $2a^2($ $b -$)($b +$)

Factors of 2: 1,2 Factors of −7: +1, −7
−1, +7

Trial Factors	Middle Term
$(1b + 1)(2b - 7)$	$-7b + 2b = -5b$
$(1b - 7)(2b + 1)$	$b - 14b = -13b$
$(1b - 1)(2b + 7)$	$7b - 2b = 5b$
$(1b + 7)(2b - 1)$	$-b + 14b = 13b$

$2a^2(b + 7)(2b - 1)$

$4a^2b^2 + 26a^2b - 14a^2 = 2a^2(b + 7)(2b - 1)$

Example 6

The GCF is $3y$.

$12y + 12y^2 - 45y^3 = 3y(4 + 4y - 15y^2)$

Factor the trinomial.

$3y($ $+$ $y)($ $-$ $y)$ or $3y($ $-$ $y)($ $+$ $y)$

Factors of 4: 1,4 Factors of −15: +1, −15
2,2 −1, +15
+3, −5
−3, +5

Trial Factors	Middle Term
$(1 + 1y)(4 - 15y)$	$-15y + 4y = -11y$
$(1 - 15y)(4 + 1y)$	$y - 60y = -59y$
$(1 - 1y)(4 + 15y)$	$15y - 4y = 11y$
$(1 + 15y)(4 - 1y)$	$-y + 60y = 59y$
$(1 + 3y)(4 - 5y)$	$-5y + 12y = 7y$
$(1 - 5y)(4 + 3y)$	$3y - 20y = -17y$
$(1 - 3y)(4 + 5y)$	$5y - 12y = -7y$
$(1 + 5y)(4 - 3y)$	$-3y + 20y = 17y$
$(2 + 1y)(2 - 15y)$	$-30y + 2y = -28y$
$(2 - 1y)(2 + 15y)$	$30y - 2y = 28y$
$(2 + 3y)(2 - 5y)$	$-10y + 6y = -4y$
$(2 - 3y)(2 + 5y)$	$10y - 6y = 4y$

$3y(2 - 3y)(2 + 5y)$

$12y + 12y^2 - 45y^3 = 3y(2 - 3y)(2 + 5y)$

Pages 217–220

1. $(x + 1)(2x + 1)$ **3.** $(y + 3)(2y + 1)$ **5.** $(a - 1)(2a - 1)$ **7.** $(b - 5)(2b - 1)$ **9.** $(x + 1)(2x - 1)$
11. $(x - 3)(2x + 1)$ **13.** $(t + 2)(2t - 5)$ **15.** $(p - 5)(3p - 1)$ **17.** $(3y - 1)(4y - 1)$ **19.** Irreducible over the integers. **21.** $(2t - 1)(3t - 4)$ **23.** $(x + 4)(8x + 1)$ **25.** Irreducible over the integers.
27. $(3y + 1)(4y + 5)$ **29.** $(a + 7)(7a - 2)$ **31.** $(b - 4)(3b - 4)$ **33.** $(z - 14)(2z + 1)$
35. $(p + 8)(3p - 2)$ **37.** $(2x - 3)(3x - 4)$ **39.** $(b + 7)(5b - 2)$ **41.** $(2a - 3)(3a + 8)$
43. $(z + 2)(4z + 3)$ **45.** $(2p + 5)(11p - 2)$ **47.** $(y + 1)(8y + 9)$ **49.** $(3t + 1)(6t - 5)$
51. $(b + 12)(6b - 1)$ **53.** $(3x + 2)(3x + 2)$ **55.** $(2b - 3)(3b - 2)$ **57.** $(3b + 5)(11b - 7)$
59. $(3y - 4)(6y - 5)$ **61.** $(3a + 7)(5a - 3)$ **63.** $(2y - 5)(4y - 3)$ **65.** $(2z + 3)(4z - 5)$ **67.** Irreducible over the integers. **69.** $(2z - 5)(5z - 2)$ **71.** $(6z + 5)(6z + 7)$ **73.** $2(x + 1)(2x + 1)$ **75.** $5(y - 1)(3y - 7)$
77. $x(x - 5)(2x - 1)$ **79.** $b(a - 4)(3a - 4)$ **81.** Irreducible over the integers. **83.** $(x + y)(3x - 2y)$
85. $(a + 2b)(3a - b)$ **87.** $(y - 2z)(4y - 3z)$ **89.** $(3 - x)(4 + x)$ **91.** $(4 + z)(7 - z)$ **93.** $(8 + x)(1 - x)$
95. $3(x + 5)(3x - 4)$ **97.** $4(4y - 1)(5y - 1)$ **99.** $z(2z + 3)(4z + 1)$ **101.** $y(2x - 5)(3x + 2)$
103. $4(2x - 3)(3x - 2)$ **105.** $a^2(7a - 1)(5a + 2)$ **107.** $5(3b - 2)(b - 7)$ **109.** $(x - 7y)(3x - 5y)$
111. $3(8y - 1)(9y + 1)$ **113.** $(1 - x)(21 + x)$ **115.** $(3a - 2b)(5a + 7b)$ **117.** $z(3 - z)(11 + z)$
119. $2x(x + 1)(5x + 1)$ **121.** $5(t + 2)(2t - 5)$ **123.** $p(p - 5)(3p - 1)$ **125.** $2(z + 4)(13z - 3)$
127. $2y(y - 4)(5y - 2)$ **129.** $yz(z + 2)(4z - 3)$ **131.** $b^2(4b + 5)(5b + 4)$ **133.** $2a(2a - 3)(3a + 8)$
135. $p^2(3 - p)(12 + p)$ **137.** $y(2x - 7y)(4x - 5y)$ **139.** $xy(3x + 2)(3x + 2)$ **141.** $ab(2a - b)(a - 5b)$

SECTION 4 Pages 221–224

Example 2
$25a^2 - b^2 = (5a)^2 - b^2 = (5a + b)(5a - b)$

Example 4
$n^8 - 36 = (n^4)^2 - 6^2 = (n^4 + 6)(n^4 - 6)$

Example 6
$\sqrt{a^2} = a$
$\sqrt{100} = 10$ $\quad 2(10a) = 20a$

The trinomial is a perfect square.

$a^2 + 20a + 100 = (a + 10)^2$

Example 8
$\sqrt{25a^2} = 5a$
$\sqrt{9b^2} = 3b$ $\quad 2(5a \cdot 3b) = 30ab$

The trinomial is a perfect square.

$25a^2 - 30ab + 9b^2 = (5a - 3b)^2$

Example 10
$5x(2x + 3) - 4(2x + 3) = (2x + 3)(5x - 4)$

Example 12
$2y(5x - 2) - 3(2 - 5x) =$
$2y(5x - 2) + 3(5x - 2) = (5x - 2)(2y + 3)$

Example 14
The GCF is $3x$.

$12x^3 - 75x = 3x(4x^2 - 25)$

Factor the difference of two squares.

$3x(2x + 5)(2x - 5)$

Example 16
The common binomial factor is $b - 7$.

$a^2(b - 7) + (7 - b) =$
$a^2(b - 7) - (b - 7) = (b - 7)(a^2 - 1)$

Factor the difference of two squares.

$(b - 7)(a + 1)(a - 1)$

Example 18
The GCF is $4x$.

$4x^3 + 28x^2 - 120x = 4x(x^2 + 7x - 30)$

Factor the trinomial.

$4x(x + 10)(x - 3)$

Pages 225–228

1. $(x + 2)(x - 2)$ **3.** $(a + 9)(a - 9)$ **5.** $(2x + 1)(2x - 1)$ **7.** $(x^3 + 3)(x^3 - 3)$ **9.** $(5x + 1)(5x - 1)$
11. $(1 + 7x)(1 - 7x)$ **13.** Irreducible over the integers. **15.** $(x^2 + y)(x^2 - y)$ **17.** $(3x + 4y)(3x - 4y)$
19. $(xy + 2)(xy - 2)$ **21.** $(4 + xy)(4 - xy)$ **23.** $(y + 7)^2$ **25.** Irreducible over the integers. **27.** $(x - 6)^2$
29. $(x + 3y)^2$ **31.** $(5x + 1)^2$ **33.** $(3a + 1)^2$ **35.** $(2a - 5)^2$ **37.** $(3a - 7)^2$ **39.** $(2a - 3b)^2$
41. $(2y - 9z)^2$ **43.** $(a + b)(x + 2)$ **45.** $(b + 2)(x - y)$ **47.** $(x - 3)(z - 1)$ **49.** $(b - 2c)(x + y)$
51. $(x - 2)(a - 5)$ **53.** $(y - 2)(b - 2a)$ **55.** $(y - 3)(b - 3)$ **57.** $(x - y)(a + 2)$ **59.** $5(x - 1)(x + 1)$
61. $x(x + 2)^2$ **63.** $x^2(x + 7)(x - 5)$ **65.** $5(b + 3)(b + 12)$ **67.** Irreducible over the integers.
69. $2y(x - 3)(x + 11)$ **71.** $x(x^2 - 6x - 5)$ **73.** $3(y^2 - 12)$ **75.** $(2a + 1)(10a + 1)$ **77.** $y^2(x - 8)(x + 1)$
79. $5(2a - 3b)(a + b)$ **81.** $2(5 - x)(5 + x)$ **83.** $b^2(a - 5)^2$ **85.** $ab(3a - b)(4a + b)$ **87.** $3a(2a - 1)^2$
89. $3(81 + a^2)$ **91.** $2a(2a - 5)(3a - 4)$ **93.** $a(2a + 5)^2$ **95.** $3b(3a - 1)^2$ **97.** $6(4 + x)(2 - x)$
99. $x^2(x - y)(x + y)$ **101.** $2a(3a + 2)^2$ **103.** $b(2 - 3a)(1 + 2a)$ **105.** $8x(3y + 1)^2$
107. $y^2(5 + x)(3 - x)$ **109.** $3(x - 3y)(x + 3y)$ **111.** $y(y - 3)(y + 3)$ **113.** $x^2y^2(3x - 5y)(5x + 4y)$
115. $2(x - 1)(a + b)$ **117.** $(x - 2)(x - 1)(x + 1)$ **119.** $(x - 2)(x + 2)(a + b)$ **121.** $(2 - x)(2 + x)(x - 5)$
123. $(x + 2)(x - 2)^2$

SECTION 5

Pages 229–232

Example 2

$2x(x + 7) = 0$

$2x = 0$ $\qquad$ $x + 7 = 0$
$x = 0$ $\qquad$ $x = -7$

The solutions are 0 and -7.

Example 4

$4x^2 - 9 = 0$
$(2x - 3)(2x + 3) = 0$

$2x - 3 = 0$ $\qquad$ $2x + 3 = 0$
$2x = 3$ $\qquad$ $2x = -3$
$x = \frac{3}{2}$ $\qquad$ $x = -\frac{3}{2}$

The solutions are $\frac{3}{2}$ and $-\frac{3}{2}$.

Example 6

$(x + 2)(x - 7) = 52$
$x^2 - 5x - 14 = 52$
$x^2 - 5x - 66 = 0$
$(x + 6)(x - 11) = 0$

$x + 6 = 0$ $\qquad$ $x - 11 = 0$
$x = -6$ $\qquad$ $x = 11$

The solutions are -6 and 11.

Example 8

Strategy
First positive consecutive integer: n
Second positive consecutive integer: $n + 1$

The sum of the squares of two positive consecutive integers is 61.

Solution
$n^2 + (n + 1)^2 = 61$
$n^2 + n^2 + 2n + 1 = 61$
$2n^2 + 2n + 1 = 61$
$2n^2 + 2n - 60 = 0$
$2(n^2 + n - 30) = 0$
$2(n - 5)(n + 6) = 0$

$n - 5 = 0$ $\qquad$ $n + 6 = 0$
$n = 5$ $\qquad$ $n = -6$

Since -6 is not a positive integer, it is not a solution.

$n = 5$
$n + 1 = 5 + 1 = 6$

The two integers are 5 and 6.

Example 10

Strategy
Width $= x$
Length $= 2x + 4$

The area of a rectangle is 96 in.^{2}.
Use the equation $A = l \cdot w$.

Solution
$A = l \cdot w$
$96 = (2x + 4)x$
$96 = 2x^2 + 4x$
$0 = 2x^2 + 4x - 96$
$0 = 2(x^2 + 2x - 48)$
$0 = 2(x + 8)(x - 6)$

$x + 8 = 0$ $\qquad$ $x - 6 = 0$
$x = -8$ $\qquad$ $x = 6$

Since the width cannot be a negative number, -8 is not a solution.

$x = 6$
$2x + 4 = 2(6) + 4 = 12 + 4 = 16$

The width is 6 in.
The length is 16 in.

Pages 233–234

1. The solutions are -3 and -2. **3.** The solutions are 7 and 3. **5.** The solutions are 0 and 5. **7.** The solutions are 0 and 9. **9.** The solutions are 0 and $-\frac{3}{2}$. **11.** The solutions are 0 and $\frac{2}{3}$. **13.** The solutions are -2 and 5. **15.** The solutions are 9 and -9. **17.** The solutions are $\frac{7}{2}$ and $-\frac{7}{2}$. **19.** The solutions are $\frac{1}{3}$ and $-\frac{1}{3}$. **21.** The solutions are -2 and -4. **23.** The solutions are -7 and 2. **25.** The solutions are 2 and 3. **27.** The solutions are -7 and 3. **29.** The solutions are $-\frac{1}{2}$ and 5. **31.** The solutions are $-\frac{1}{3}$ and $-\frac{1}{2}$. **33.** The solutions are 0 and 3. **35.** The solutions are 0 and 7. **37.** The solutions are -1 and -4. **39.** The solutions are 2 and 3. **41.** The solutions are $\frac{1}{2}$ and -4. **43.** The solutions are $\frac{1}{3}$ and 4. **45.** The solutions are 3 and 9. **47.** The solutions are 9 and -2. **49.** The solutions are -1 and -2. **51.** The solutions are -9 and 5. **53.** The solutions are -7 and 4. **55.** The solutions are -2 and -3. **57.** The solutions are -5 and -8. **59.** The solutions are 1 and 3. **61.** The solutions are 5 and -12. **63.** The solutions are $-\frac{3}{2}$ and -2. **65.** The solutions are $-\frac{1}{2}$ and -4.

Pages 235–236

1. The number is 6. **3.** The numbers are 2 and 4. **5.** The numbers are 6 and 7. **7.** The numbers are 3 and 7. **9.** The number is 9 or -15. **11.** The numbers are 14 and 15. **13.** The height is 5 ft and the length is 20 ft. **15.** The width is 10 in. and the length is 30 in. **17.** The width is 5 in. and the length is 15 in. **19.** The length of a side of the original square is 10 in. **21.** The time is 4 s. **23.** The radius of the original circle is 3.8078556 in.

REVIEW / TESTS
Pages 237–238

1.1 $4ab^3$ **1.2** $2x(3x^2 - 4x + 5)$ **2.1a** $(p + 2)(p + 3)$ **2.1b** $(a - 3)(a - 16)$ **2.1c** $(x - 3)(x + 5)$
2.1d $(x + 3)(x - 12)$ **2.2a** $5(x^2 - 9x - 3)$ **2.2b** $2y^2(y - 8)(y + 1)$ **3.1a** Irreducible over the integers.
3.1b $(2x + 1)(3x + 8)$ **3.2a** $4(2x - 3)(x + 4)$ **3.2b** $3y^2(2x^2 + 3x + 4)$ **4.1a** $(b + 4)(b - 4)$
4.1b $(2x - 7y)(2x + 7y)$ **4.2a** $(p + 6)^2$ **4.2b** $(2a - 3b)^2$ **4.3a** $(x - 2)(a + b)$ **4.3b** $(p + 1)(x - 1)$
4.4a $3(a - 5)(a + 5)$ **4.4b** $3(x + 2y)^2$ **5.1a** The solutions are -7 and $\frac{3}{2}$. **5.1b** The solutions are 3 and 5.
5.2 The width is 6 cm and the length is 15 cm.

Pages 239–240

1.1 b **1.2** a **2.1a** b **2.1b** d **2.1c** a **2.1d** c **2.2a** d **2.2b** b **3.1a** c **3.1b** a **3.2a** c
3.2b a **4.1a** d **4.1b** b **4.2a** b **4.2b** c **4.3a** a **4.3b** c **4.4a** d **4.4b** b **5.1a** a **5.1b** c
5.2 a