UNIT 4

Solving Equations: Applications

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List of Objectives

To write a percent as a fraction or a decimal

To write a fraction or a decimal as a percent

To solve the basic percent equation

To solve application problems

To solve markup problems

To solve discount problems

To solve investment problems

To solve value mixture problems

To solve percent mixture problems

To solve uniform motion problems

To solve perimeter problems

To solve problems involving the angles of a triangle

To solve consecutive integer problems

To solve coin and stamp problems

To solve age problems

SECTION 1 Introduction to Percent

1.1 Objective To write a percent as a fraction or a decimal

"A population growth rate of 3%," "a manufacturer's discount of 25%," and "an 8% increase in pay" are typical examples of the many ways in which percent is used in applied problems. **Percent** means "parts of 100." Thus, 27% means 27 parts of 100.

In applied problems involving a percent, it is usually necessary either to rewrite the percent as a fraction or a decimal, or to rewrite a fraction or a decimal as a percent.

To write a percent as a fraction, drop the percent sign and multiply by $\frac{1}{100}$.

Write 27% as a fraction.

Drop the percent sign and multiply by $\frac{1}{100}$. $27\% = 27\left(\frac{1}{100}\right) = \frac{27}{100}$

To write a percent as a decimal, drop the percent sign and multiply by 0.01.

Write 33% as a decimal.

Drop the percent sign.
Then multiply by 0.01.

$33\% = 33(0.01) = 0.33$

Move the decimal point two places to the left. Then drop the percent sign.

Example 1 Write 130% as a fraction and as a decimal.

Solution $130\% = 130\left(\frac{1}{100}\right) = \frac{130}{100} = 1\frac{3}{10}$

$130\% = 130(0.01) = 1.30$

Example 2 Write 125% as a fraction and as a decimal.

Your solution

Example 3 Write $33\frac{1}{3}\%$ as a fraction.

Solution $33\frac{1}{3}\% = 33\frac{1}{3}\left(\frac{1}{100}\right) = \frac{100}{3}\left(\frac{1}{100}\right) = \frac{1}{3}$

Example 4 Write $16\frac{2}{3}\%$ as a fraction.

Your solution

Example 5 Write 0.25% as a decimal.

Solution $0.25\% = 0.25(0.01) = 0.0025$

Example 6 Write 0.5% as a decimal.

Your solution

Solutions on p. 454

1.2 Objective To write a fraction or a decimal as a percent

A fraction or decimal can be written as a percent by multiplying by 100%.

Write $\frac{5}{8}$ as a percent.

Multiply by 100%. $\frac{5}{8} = \frac{5}{8}(100\%) = \frac{500}{8}\% = 62.5\%$ or $62\frac{1}{2}\%$

Write 0.82 as a percent.

Multiply by 100%. $0.82 = 0.82(100\%) = 82\%$

Move the decimal point two places to the right. Then write the percent sign.

Example 7 Write 0.027 as a percent.

Solution $0.027 = 0.027(100\%) = 2.7\%$

Example 8 Write 0.043 as a percent.

Your solution

Example 9 Write 1.34 as a percent.

Solution $1.34 = 1.34(100\%) = 134\%$

Example 10 Write 2.57 as a percent.

Your solution

Example 11 Write $\frac{5}{6}$ as a percent. Round to the nearest tenth of a percent.

Solution $\frac{5}{6} = \frac{5}{6}(100\%) = \frac{500}{6}\% \approx 83.3\%$

Example 12 Write $\frac{5}{9}$ as a percent. Round to the nearest tenth of a percent.

Your solution

Example 13 Write $\frac{7}{16}$ as a percent. Write the remainder in fractional form.

Solution $\frac{7}{16} = \frac{7}{16}(100\%) = \frac{700}{16}\% = 43\frac{3}{4}\%$

Example 14 Write $\frac{9}{16}$ as a percent. Write the remainder in fractional form.

Your solution

Solutions on p. 454

1.1 Exercises

Write as a fraction and a decimal.

1. 75% **2.** 40% **3.** 50% **4.** 10%

5. 64% **6.** 88% **7.** 125% **8.** 160%

9. 19% **10.** 87% **11.** 5% **12.** 2%

13. 450% **14.** 380% **15.** 8% **16.** 4%

Write as a fraction.

17. $11\frac{1}{9}\%$ **18.** $4\frac{2}{7}\%$ **19.** $12\frac{1}{2}\%$ **20.** $37\frac{1}{2}\%$

21. $31\frac{1}{4}\%$ **22.** $66\frac{2}{3}\%$ **23.** $\frac{1}{4}\%$ **24.** $\frac{1}{2}\%$

25. $5\frac{3}{4}\%$ **26.** $68\frac{3}{4}\%$ **27.** $6\frac{1}{4}\%$ **28.** $83\frac{1}{3}\%$

Write as a decimal.

29. 7.3% **30.** 9.1% **31.** 15.8% **32.** 16.7%

33. 0.3% **34.** 0.9% **35.** 9.15% **36.** 121.2%

37. 18.23% **38.** 62.14% **39.** 0.15% **40.** 0.27%

1.2 Exercises

Write as a percent.

41. 0.15 **42.** 0.37 **43.** 0.05 **44.** 0.02

45. 0.175 **46.** 0.125 **47.** 1.15 **48.** 1.36

49. 0.62 **50.** 0.96 **51.** 3.165 **52.** 2.142

53. 0.008 **54.** 0.004 **55.** 0.065 **56.** 0.083

Write as a percent. Round to the nearest tenth of a percent.

57. $\frac{27}{50}$ **58.** $\frac{83}{100}$ **59.** $\frac{1}{3}$ **60.** $\frac{3}{8}$

61. $\frac{5}{11}$ **62.** $\frac{4}{9}$ **63.** $\frac{7}{8}$ **64.** $\frac{9}{20}$

65. $1\frac{2}{3}$ **66.** $2\frac{1}{2}$ **67.** $1\frac{2}{7}$ **68.** $1\frac{11}{12}$

Write as a percent. Write the remainder in fractional form.

69. $\frac{17}{50}$ **70.** $\frac{17}{25}$ **71.** $\frac{3}{8}$ **72.** $\frac{7}{16}$

73. $\frac{5}{14}$ **74.** $\frac{3}{19}$ **75.** $\frac{3}{16}$ **76.** $\frac{4}{7}$

77. $1\frac{1}{4}$ **78.** $2\frac{5}{8}$ **79.** $1\frac{5}{9}$ **80.** $1\frac{13}{16}$

SECTION 2 The Percent Equation

2.1 Objective To solve the basic percent equation

The solution of a problem which involves a percent requires solving the basic percent equation shown at the right.

BASIC PERCENT EQUATION

Percent × Base = Amount

$P \times B = A$

In any percent problem, two parts of the equation are given, and one is unknown.

To translate a problem involving a percent into an equation, remember that the word "of" translates to "multiply" and the word "is" translates to "=". The base usually follows the word "of."

20% of what number is 30?

Given: $P = 20\% = 0.20$
$A = 30$
Unknown: Base

$$P \times B = A$$
$$(0.20)B = 30$$
$$\frac{1}{0.20}(0.20)B = \frac{1}{0.20}(30)$$
$$B = 150$$

The number is 150.

Find 25% of 200.

Given: $P = 25\% = 0.25$
$B = 200$
Unknown: Amount

$$P \times B = A$$
$$0.25(200) = A$$
$$50 = A$$

25% of 200 is 50.

In most cases, the percent is written as a decimal before solving the basic percent equation. However, some percents are more easily written as a fraction. For example,

$33\frac{1}{3}\% = \frac{1}{3}$ $\quad 66\frac{2}{3}\% = \frac{2}{3}$ $\quad 16\frac{2}{3}\% = \frac{1}{6}$ $\quad 83\frac{1}{3}\% = \frac{5}{6}$

Example 1 12 is $33\frac{1}{3}\%$ of what number?

Solution

$$12 = \frac{1}{3}B \qquad \left(33\tfrac{1}{3}\% = \tfrac{1}{3}\right)$$
$$3 \cdot 12 = 3 \cdot \frac{1}{3}B$$
$$36 = B$$

The number is 36.

Example 2 27 is what percent of 60?

Your solution

Solution on p. 454

2.2 Objective To solve application problems

The key to solving a percent problem is identifying the percent, the base, and the amount. The base usually follows the word "of."

Example 3 A student answered 76 of the 80 questions on a test correctly. What percent of the questions were answered correctly?

Strategy To find the percent of the questions answered correctly, solve the basic percent equation using $B = 80$ and $A = 76$. The percent is unknown.

Solution

$$P \times B = A$$
$$P(80) = 76$$
$$P(80)\left(\frac{1}{80}\right) = 76\left(\frac{1}{80}\right)$$
$$P = 0.95$$

95% of the questions were answered correctly.

Example 4 A quality control inspector found that 6 out of 200 wheel bearings inspected were defective. What percent of the wheel bearings were defective?

Your strategy

Your solution

Example 5 A new labor contract increased an employee's hourly wage by 5%. What is the amount of increase for an employee who was making $9.60 an hour?

Strategy To find the amount of increase, solve the basic percent equation using $B = 9.60$ and $P = 5\% = 0.05$. The amount is unknown.

Solution

$$P \times B = A$$
$$(0.05)(9.60) = A$$
$$0.48 = A$$

The amount of increase is $.48.

Example 6 A company was producing 2500 gal of paint each week. Due to a decrease in demand for the paint, the company reduced its weekly production by 500 gal. What percent decrease does this represent?

Your strategy

Your solution

Solutions on p. 454

2.1 Exercises

Solve.

1. 12 is what percent of 50?

2. What percent of 125 is 50?

3. Find 18% of 40.

4. What is 25% of 60?

5. 12% of what is 48?

6. 45% of what is 9?

7. What is $33\frac{1}{3}\%$ of 27?

8. Find $16\frac{2}{3}\%$ of 30.

9. What percent of 12 is 3?

10. 10 is what percent of 15?

11. 60% of what is 3?

12. 75% of what is 6?

13. 12 is what percent of 6?

14. 20 is what percent of 16?

15. $5\frac{1}{4}\%$ of what is 21?

16. $37\frac{1}{2}\%$ of what is 15?

17. Find 15.4% of 50.

18. What is 18.5% of 46?

19. 1 is 0.5% of what?

20. 3 is 1.5% of what?

21. $\frac{3}{4}\%$ of what is 3?

22. $\frac{1}{2}\%$ of what is 3?

23. Find 125% of 16.

24. What is 250% of 12?

25. 16.43 is what percent of 20.45? Round to the nearest hundredth of a percent.

26. Find 18.37% of 625.43. Round to the nearest hundredth.

2.2 Application Problems

Solve.

1. A company's budget for the development of a new product is $250,000. Of this amount, $50,000 is for materials. What percent of the total budget is for materials?

2. Fifteen years ago, a painting was priced at $6000. Today the painting has a value of $15,000. What percent of the price 15 years ago is its value today?

3. An engineer estimates that 32% of the gasoline used by a car is used efficiently. Using this estimate, how many gallons out of 20 gal of gasoline are used efficiently?

4. Approximately 80% of the air in the atmosphere is nitrogen. Using this estimate, how many liters of nitrogen are in a room which contains 500 L of air?

5. The value of a car today is $66\frac{2}{3}\%$ of its value two years ago. The value of the car two years ago was $6000. What is the value of the car today?

6. An appliance store estimates that 15% of the washing machines it sells will require service within one year. Using this estimate, how many washing machines were sold in a year in which 27 new machines were serviced?

7. The normal underwater visibility off the coast of an island is 30 ft. Unusual turbulence reduced the visibility by 12 ft. What percent decrease does this represent?

8. The seating capacity of a baseball stadium, which had seated 50,000, was expanded by 6250 seats. What percent increase does this represent?

9. The number of take-offs and landings at a municipal airport this year was 303,750. This represents 112.5% of last year's take-offs and landings. How many take-offs and landings were there last year?

10. The annual license fee on a car is 1.4% of the value of the car. What is the value of a car during a year in which the license fee was $91.00?

11. In a recent city election, 14,375 people out of 50,000 registered voters voted. What percent of the people voted in the election?

12. There are approximately 8760 h in one year. An adult sleeps approximately 2700 h during a year. What percent of the year does an adult spend sleeping? Round to the nearest hundredth of a percent.

SECTION 3 Markup and Discount

3.1 Objective To solve markup problems

Cost is the price which a business pays for a product. **Selling price** is the price for which a business sells a product to a customer. The difference between selling price and cost is called **markup.** Markup is added to a retailer's cost to cover the expenses of operating a business. Markup is usually expressed as a percent of the retailer's cost. This percent is called the **markup rate.**

The basic markup equations used by a business are:

Selling price = cost + markup
$S = C + M$

Markup = markup rate × cost
$M = r \times C$

Substituting $r \times C$ for M in the first equation, selling price can also be written as $S = C + (r \times C) = C + rC$

The manager of a clothing store buys a suit for \$80 and sells the suit for \$116. Find the markup rate.

Given: $C = \$80$
$S = \$116$
Unknown: r
Use the equation $S = C + rC$.

$$S = C + rC$$
$$116 = 80 + 80r$$
$$116 - 80 = 80 - 80 + 80r$$ Do this step mentally.
$$36 = 80r$$
$$\frac{1}{80} \cdot 36 = \frac{1}{80} \cdot 80r$$ Do this step mentally.
$$\frac{36}{80} = r$$
$$0.45 = r$$

The markup rate is 45%.

Example 1 A hardware store employee uses a markup rate of 40% on all items. The selling price of a lawn mower is \$105. Find the cost.

Strategy
Given: $r = 40\% = 0.40$
$S = \$105$
Unknown: C
Use the equation $S = C + rC$.

Solution
$S = C + rC$
$105 = C + 0.40C$
$105 = 1.40C$
$75 = C$

The cost is \$75.

Example 2 The cost to the manager of a sporting goods store for a tennis racket is \$40. The selling price of the racket is \$60. Find the markup rate.

Your strategy

Your solution

Solution on p. 455

3.2 Objective To solve discount problems

Discount is the amount by which a retailer reduces the regular price of a product for a promotional sale. Discount is usually expressed as a percent of the regular price. This percent is called the **discount rate.**

The basic discount equations used by a business are:

Sale Price = regular price − discount Discount = discount rate × regular price

$S = R - D$ $D = r \times R$

Substituting $r \times R$ for D in the first equation, sale price can also be written as $S = R - (r \times R) = R - rR$

In a garden supply store, the regular price of a 100-foot garden hose is \$32. During an "after-summer sale," the hose is being sold for \$24. Find the discount rate.

Given: $R = \$32$
$S = \$24$
Unknown: r
Use the equation $S = R - rR$.

$$S = R - rR$$
$$24 = 32 - 32r$$
$$24 - 32 = 32 - 32 - 32r$$ Do this step mentally.
$$-8 = -32r$$
$$\left(-\tfrac{1}{32}\right)(-8) = \left(-\tfrac{1}{32}\right)(-32r)$$ Do this step mentally.
$$\tfrac{1}{4} = r$$
$$0.25 = r$$

The discount rate is 25%.

Example 3 The sale price for a chemical sprayer is \$23.40. This price is 35% off the regular price. Find the regular price.

Strategy

Given: $S = \$23.40$
$r = 35\% = 0.35$
Unknown: R
Use the equation $S = R - rR$.

Solution

$$S = R - rR$$
$$23.40 = R - 0.35R$$
$$23.40 = 0.65R$$
$$36 = R$$

The regular price is \$36.

Example 4 A case of motor oil which regularly sells for \$27.60 is on sale for \$20.70. What is the discount rate?

Your strategy

Your solution

Solution on p. 455

3.1 Application Problems

Solve.

1. The manager of a garden supply store buys a 50-foot rubber hose for $12. A markup rate of 35% is used. What is the selling price?

2. A bookstore buys a paperback book for $3 and uses a markup rate of 20%. Find the selling price of the book.

3. A shoe store uses a markup rate of 45%. One pair of shoes has a selling price of $72.50. Find the cost of the shoes.

4. The selling price for a gallon of ice cream is $6.57. The markup rate used by the ice cream store is 46%. Find the cost of the ice cream.

5. The cost to a furniture manufacturer for making a kitchen chair is $15. The manufacturer then sells the chair for $24. What is the markup rate?

6. The manufacturer's cost for a car air conditioner is $300. The manufacturer then sells the air conditioner for $450. What is the markup rate?

7. The meat manager at a market uses a markup rate of 22%. What is the selling price for a steak which costs $2.50?

8. A pen and pencil set costs a retailer $15. Find the selling price when the markup rate is $66\frac{2}{3}\%$.

9. A restaurant serves a breakfast for $1.75. The restaurant's cost to make the breakfast is $1.00. What is the markup rate?

10. The selling price for a silk jacket is $200. The cost to the clothing store for the jacket is $125. What is the markup rate?

11. The cost to a car dealer for a car is $5696. The selling price of the car is $8900. What is the markup rate?

12. The cost to an appliance dealer for a refrigerator is $483. The dealer sells the refrigerator for $920. What is the markup rate? Round to the nearest hundredth of a percent.

3.2 Application Problems

Solve.

13. A hair dryer which regularly sells for $28 is on sale for 20% off the regular price. Find the sale price.

14. A briefcase is on sale for 25% off the regular price of $85. Find the sale price.

15. The sale price of a desk lamp is $26, which is 20% off the regular price. Find the regular price.

16. The sale price for a car tire is $35.75, which is 45% off the regular price. Find the regular price.

17. A box of stationery which regularly sells for $4 is on sale for $3. Find the discount rate.

18. An art print which regularly sells for $50 is on sale for $40. Find the discount rate.

19. A gas barbecue is on sale for $200. This is $33\frac{1}{3}$% off the regular price. Find the regular price.

20. During a "Going-Out-of-Business" sale, a clothing store reduced all items 60%. What was the regular price of an umbrella which was on sale for $7.20?

21. A children's gym set, which regularly sells for $130, is on sale for $96.20. Find the discount rate.

22. A radio with cassette recorder, which regularly sells for $60, is on sale for $37.80. Find the discount rate.

23. A cabin tent, which regularly sells for $129, is on sale for $99. Find the discount rate to the nearest tenth of a percent.

24. A portable television set, which regularly sells for $139, is on sale for $89.90. Find the discount rate to the nearest tenth of a percent.

SECTION 4 Investment Problems

4.1 Objective To solve investment problems

The annual simple interest which an investment earns is given by the equation $I = Pr$, where I is the simple interest, P is the principal, or the amount invested, and r is the simple interest rate.

The annual interest rate on a \$2500 investment is 14%. Find the annual simple interest earned on the investment.

Given: $P = \$2500$
$r = 14\% = 0.14$
Unknown: I

$I = Pr$
$I = 2500(0.14)$
$I = 350$

The annual simple interest is \$350.

An investor has a total of \$10,000 to deposit into two simple interest accounts. On one account, the annual simple interest rate is 7%. On the second account, the annual simple interest rate is 11%. How much should be invested in each account so that the total annual interest earned is \$1000?

STRATEGY FOR SOLVING A PROBLEM INVOLVING MONEY DEPOSITED IN TWO SIMPLE INTEREST ACCOUNTS

▷ For each amount invested, write a numerical or variable expression for the principal, the interest rate, and the interest earned. The results can be recorded in a table.

The sum of the amounts at each interest rate is \$10,000.

Amount invested at 7%: x
Amount invested 11%: $\$10{,}000 - x$

	Principal, P	·	Interest rate, r	=	Interest earned, I
Amount at 7%	x	·	0.07	=	$0.07x$
Amount at 11%	$10{,}000 - x$	·	0.11	=	$0.11(10{,}000 - x)$

▷ Determine how the amounts of interest earned on each amount are related. For example, the total interest earned by both accounts may be known or it may be known that the interest earned on one account is equal to the interest earned by the other account.

The total annual interest earned is \$1000.

$$0.07x + 0.11(10{,}000 - x) = 1000$$
$$0.07x + 1100 - 0.11x = 1000$$
$$-0.04x + 1100 = 1000$$
$$-0.04x = -100$$
$$x = 2500$$
$$10{,}000 - x = 10{,}000 - 2500 = 7500$$

The amount invested at 7% is \$2500.
The amount invested at 11% is \$7500.

Example 1

An investment counselor invested 75% of a client's money into a 13% annual simple interest money market fund. The remainder was invested in 9% annual simple interest government securities. Find the amount invested in each if the total annual interest earned is $5400.

Strategy

▷ Amount invested: x
Amount invested at 9%: $0.25x$
Amount invested at 13%: $0.75x$

	Principal	Rate	Interest
Amount at 9%	$0.25x$	0.09	$0.0225x$
Amount at 13%	$0.75x$	0.13	$0.0975x$

▷ The sum of the interest earned by the two investments equals the total annual interest earned ($5400).

Solution

$$0.0225x + 0.0975x = 5400$$
$$0.12x = 5400$$
$$x = 45{,}000$$

$0.25x = 0.25(45{,}000) = 11{,}250$

$0.75x = 0.75(45{,}000) = 33{,}750$

The amount invested at 9% is $11,250.
The amount invested at 13% is $33,750.

Example 2

An investment of $5000 is made at an annual simple interest rate of 8%. How much additional money must be invested at 14% so that the total interest earned will be 11% of the total investment?

Your strategy

Your solution

Solution on p. 455

4.1 Application Problems

Solve.

1. A total of $5000 is deposited into two simple interest accounts. On one account the annual simple interest rate is 8%, while on the second account the annual simple interest rate is 12%. How much should be invested in each account so that the total annual interest earned is $520?

2. An investment club invested a part of $10,000 in a 9.5% annual simple interest account and the remainder in a 14% annual simple interest account. The amount of interest earned for one year was $1085. How much was invested in each account?

3. An investment of $4000 is made at an annual simple interest rate of 15%. How much additional money must be invested at an annual simple interest rate of 10% so that the total interest earned is 12% of the total investment?

4. An investment of $3500 is made at an annual simple interest rate of 12%. How much additional money must be invested at an annual simple interest rate of 16% so that the total interest earned is 14% of the total investment?

5. A total of $4000 is invested into two simple interest accounts. On one account the annual simple interest rate is 12%, while on the second account the annual simple interest rate is 20%. How much should be invested in each account so that the interest earned by each account is the same?

6. An investment advisor deposited a total of $6000 into two money market funds. One fund earns 14% annual simple interest, while a second tax-free fund earns 7% annual simple interest. How much must be invested in each fund so that the interest earned by each is the same?

7. An accountant deposited an amount of money into a 12% annual simple interest account. Another deposit, $2000 more than the first, was placed in a 15% annual simple interest account. The total interest earned on both investments for one year was $975. How much money was deposited in the 12% account?

8. A deposit was made into a 6% annual simple interest savings account. Another deposit, $3500 less than the first, was placed in a 16% annual simple interest bond market account. The total interest earned on both accounts for one year was $540. How much money was deposited in the 6% account?

Solve.

9. An investment of $12,000 is made into a 10.5% simple interest account. How much additional money must be deposited into an 8% simple interest account so that the total interest earned on both accounts is 9.5% of the total investment?

10. To provide for retirement income, an engineer purchases a $10,000 bond. The simple interest rate on the bond is 8.5%. How much money must be invested in additional bonds which have an interest rate of 9.25% so that the total interest earned each year is $2700?

11. A stock broker's client has $25,000 to invest. The broker recommends that part of the $25,000 be placed in 7.5% tax-free municipal bonds and the remainder in 11.25% commercial bonds. How much should be invested in each type of bond so that the total interest earned is $2250?

12. A corporation gave a university $250,000 to support research assistants in science. The university deposited some of the money in a 12% simple interest account and the remainder in a 15% simple interest account. How much should be deposited in each so that the total interest earned is $32,250?

13. The manager of a trust decided to invest 60% of a client's account in stocks which earn 6% simple interest. The remainder was invested in a tax shelter which earns 8% simple interest. The annual interest earned from the investments was $1632. What was the total amount invested?

14. The portfolio manager for a corporation invested 75% of the company's investment account in 12.5% short term certificates. The remainder was invested in 10% corporate bonds. The annual interest earned from the two investments was $47,500. What was the total amount invested?

15. A financial manager recommended an investment plan in which 40% of a client's cash be placed in a 7% simple interest account, 35% be placed in 10% high grade bonds, and the remainder in a 20% high-risk investment. The total interest earned from the investments would be $5650. What is the total amount to be invested?

16. An investment company deposited 70% of its investment capital in a 12.5% simple interest account. The remainder was deposited in a 14.5% simple interest account. The total interest earned from the investments was $58,950. How much was invested in each account?

SECTION 5 Mixture Problems

5.1 Objective To solve value mixture problems

A value mixture problem involves combining two ingredients which have different prices into a single blend. For example, a coffee merchant may blend two types of coffee into a single blend, or a candy manufacturer may combine two types of candy to sell as a "variety pack."

The solution of a value mixture problem is based upon the equation $V = AC$, where V is the value of an ingredient, A is the amount of the ingredient, and C is the cost per unit of the ingredient.

A coffee merchant wants to make 6 lb of a blend of coffee to sell for \$5 per pound. The blend is made using a \$6 grade and a \$3 grade of coffee. How many pounds of each of these grades should be used?

STRATEGY FOR SOLVING A VALUE MIXTURE PROBLEM

▷ For each ingredient in the mixture, write a numerical or variable expression for the amount of the ingredient used, the unit cost of the ingredient, and the value of the amount used. For the blend, write a numerical or variable expression for the amount, the unit cost of the blend, and the value of the amount. The results can be recorded in a table.

The sum of the amounts is 6 lb.

Amount of \$6 coffee: x
Amount of \$3 coffee: $6 - x$

	Amount, A	·	Unit cost, C	=	Value, V
\$6 grade	x	·	\$6	=	$6x$
\$3 grade	$6 - x$	·	\$3	=	$3(6 - x)$
\$5 blend	6	·	\$5	=	5(6)

▷ Determine how the values of each ingredient are related. Use the fact that the sum of the values of each ingredient is equal to the value of the blend.

The sum of the values of the \$6 grade and the \$3 grade is equal to the value of the \$5 blend.

$$6x + 3(6 - x) = 5(6)$$
$$6x + 18 - 3x = 30$$
$$3x + 18 = 30$$
$$3x = 12$$
$$x = 4$$
$$6 - x = 6 - 4 = 2$$

The merchant must use 4 lb of the \$6 coffee and 2 lb of the \$3 coffee.

Example 1

How many ounces of a silver alloy which costs \$4 an ounce must be mixed with 10 oz of an alloy which costs \$6 an ounce to make a mixture which costs \$4.32 an ounce?

Strategy

▷ Ounces of \$4 alloy: x

	Amount	Cost	Value
\$4 alloy	x	\$4	$4x$
\$6 alloy	10	\$6	$6(10)$
\$4.32 mixture	$10 + x$	\$4.32	$4.32(10 + x)$

▷ The sum of the values before mixing equals the value after mixing.

Solution

$$\begin{aligned} 4x + 6(10) &= 4.32(10 + x) \\ 4x + 60 &= 43.2 + 4.32x \\ -0.32x + 60 &= 43.2 \\ -0.32x &= -16.8 \\ x &= 52.5 \end{aligned}$$

52.5 oz of the \$4 silver alloy must be used.

Example 2

A gardener has 20 lb of a lawn fertilizer which costs \$.80 per pound. How many pounds of a fertilizer which costs \$.55 per pound should be mixed with the 20 lb of lawn fertilizer to produce a mixture which costs \$.75 per pound?

Your strategy

Your solution

Solution on p. 456

5.2 Objective To solve percent mixture problems

The amount of a substance in a solution can be given as a percent of the total solution. For example, a 5% salt water solution means that 5% of the total solution is salt. The remaining 95% is water.

The solution of a percent mixture problem is based upon the equation $Q = Ar$, where Q is the quantity of a substance in the solution, r is the percent of concentration, and A is the amount of solution.

A 500-milliliter bottle contains a 4% solution of hydrogen peroxide. Find the amount of hydrogen peroxide in the solution.

Given: $A = 500$
$r = 4\% = 0.04$
Unknown: Q

$Q = Ar$
$Q = 500(0.04)$
$Q = 20$

The bottle contains 20 ml of hydrogen peroxide.

How many gallons of a 20% salt solution must be mixed with 6 gal of a 30% salt solution to make a 22% salt solution?

STRATEGY FOR SOLVING A PERCENT MIXTURE PROBLEM

▷ For each solution, write a numerical or variable expression for the amount of solution, percent of concentration, and the quantity of the substance in the solution. The results can be recorded in a table.

The unknown quantity of 20% solution: x

	Amount of solution, A	·	Percent of concentration, r	=	Quantity of substance, Q
20% solution	x	·	0.20	=	$0.20x$
30% solution	6	·	0.30	=	$0.30(6)$
22% solution	$x + 6$	·	0.22	=	$0.22(x + 6)$

▷ Determine how the quantities of the substance in each solution are related. Use the fact that the sum of the quantities of the substances being mixed is equal to the quantity of the substance after mixing.

The sum of the quantities of the substances in the 20% solution and the 30% solution is equal to the quantity of the substance in the 22% solution.

$$0.20x + 0.30(6) = 0.22(x + 6)$$
$$0.20x + 1.80 = 0.22x + 1.32$$
$$-0.02x + 1.80 = 1.32$$
$$-0.02x = -0.48$$
$$x = 24$$

24 gal of the 20% solution are required.

Example 3

A chemist wishes to make 2 L of an 8% acid solution by mixing a 10% acid solution and a 5% acid solution. How many liters of each solution should the chemist use?

Strategy

▷ Liters of 10% solution: x
Liters of 5% solution: $2 - x$

	Amount	Percent	Quantity
10%	x	0.10	$0.10x$
5%	$2 - x$	0.05	$0.05(2 - x)$
8%	2	0.08	0.08(2)

▷ The sum of the quantities before mixing is equal to the quantity after mixing.

Solution

$$0.10x + 0.05(2 - x) = 0.08(2)$$
$$0.10x + 0.10 - 0.05x = 0.16$$
$$0.05x + 0.10 = 0.16$$
$$0.05x = 0.06$$
$$x = 1.2$$
$$2 - x = 2 - 1.2 = 0.8$$

The chemist needs 1.2 L of 10% solution and 0.8 L of the 5% solution.

Example 4

A pharmacist dilutes 5 L of a 12% solution by adding water. How many liters of water are added to make an 8% solution?

Your strategy

Your solution

Solution on p. 456

5.1 Application Problems

Solve.

1. A butcher combined hamburger which cost $2.50 per pound with hamburger which cost $3.10 per pound. How many pounds of each were used to make an 80-pound mixture to sell for $2.65 per pound?

2. A butcher combined hamburger which cost $4.40 per kilogram with hamburger which cost $8.40 per kilogram. How many kilograms of each were used to make a mixture of 50 kg to sell for $6.00 per kilogram?

3. How many ounces of pure gold which cost $400 an ounce must be mixed with 20 oz of an alloy which cost $220 an ounce to make an alloy which would cost $300 an ounce?

4. A goldsmith combined pure gold which cost $675 per ounce with an alloy costing $325 per ounce. How many ounces of each should be used to make 5 oz of a gold alloy which sells for $465 per ounce?

5. Find the selling price per pound of a mixture made from 12 lb of chocolate which cost $4.00 per pound and 30 lb of chocolate which cost $2.25 per pound.

6. How many kilograms of chocolates which cost $7.00 per kilogram must be mixed with 20 kg of chocolates which cost $3.50 per kilogram to make a box of mixed chocolates to sell for $4.50 per kilogram?

7. A grocer combined cranberry juice which cost $3.25 per gallon with apple juice which cost $2.25 per gallon. How many gallons of each should be used to make 100 gal of cranapple juice to sell for $2.50 per gallon?

8. Find the selling price per liter of a mixture made from 40 L of cranberry juice which cost $1.00 per liter and 120 L of apple juice which cost $.60 per liter.

9. How many pounds of walnuts which cost $1.60 per pound must be mixed with 18 lb of cashews which cost $2.50 per pound to make a mixture which costs $1.90 per pound?

10. A grocer combined peanuts which cost $2.50 per kilogram with walnuts which cost $4.50 per kilogram. How many kilograms of each were used to make a 100-kilogram mixture to sell for $3.24 per kilogram?

Solve.

11. Find the selling price per pound of a mixture of coffee made from 25 lb of coffee which cost $4.82 per pound and 40 lb of coffee which cost $3.00 per pound.

12. How many kilograms of coffee which cost $9 per kilogram must be mixed with 16 kg of coffee which cost $5 per kilogram to make a mixture which costs $6.50 per kilogram?

13. How many pounds of cheese which cost $4.20 per pound must be mixed with 12 lb of cheese which cost $2.25 per pound to make a grated cheese topping which costs $3.40 per pound?

14. Find the selling price per kilogram of a grated cheese mixture made from 8 kg of cheese which cost $9.20 per kilogram and 12 kg of cheese which cost $5.50 per kilogram.

15. To make a feed for cattle, a feed store operation combined soybeans which cost $8 per bushel with corn which cost $3 per bushel. How many bushels of each were used to make a mixture of 5000 bushels to sell for $4.50 per bushel?

16. How many bushels of soybeans which cost $7.50 per bushel must be mixed with 2400 bushels of corn which cost $3.25 per bushel to make a mixture which costs $4.50 per bushel?

17. Find the selling price per ounce of a mixture of 200 oz of silver which cost $5.50 per ounce and 500 oz of an alloy which cost $2.00 per ounce.

18. A silversmith combined a silver alloy which cost $4.30 per ounce with an alloy which cost $1.80 per ounce. How many ounces of each were used to make a mixture of 200 oz to sell for $2.50 per ounce?

19. How many liters of face cream which cost $80 per liter must be mixed with 6 L of face cream which cost $25 per liter to make a face cream which sells for $36 per liter?

20. Find the selling price per ounce of a face cream mixture made from 40 oz of face cream which cost $4.40 per ounce and 100 oz of face cream which cost $2.30 per ounce.

5.2 Application Problems

Solve.

21. A farmer has some cream which is 21% butterfat and some which is 15% butterfat. How many gallons of each must be mixed to produce 60 gal of cream which is 19% butterfat?

22. A chemist has some 8% hydrogen peroxide solution and some 5% hydrogen peroxide solution. How many milliliters of each should be mixed to make a 300-milliliter solution which is 6% hydrogen peroxide?

23. How many grams of pure acid must be added to 40 g of a 20% acid solution to make a solution which is 36% acid?

24. How many ounces of pure water must be added to 60 oz of a 15% salt solution to make a salt solution which is 10% salt?

25. A hygenist mixed 50 L of a 36% disinfectant solution with 40 L of water. What is the percent concentration of the resulting solution?

26. A researcher mixed 80 lb of a 30% aluminum alloy with 120 lb of a 25% aluminum alloy. What is the percent concentration of the resulting alloy?

27. A syrup manufacturer has some pure maple syrup and some which is 85% maple syrup. How many liters of each should be mixed to make 150 L which is 96% maple syrup?

28. A butcher has some hamburger which is 20% fat and some which is 15% fat. How many pounds of each should be mixed to make 50 lb of hamburger which is 18% fat?

29. A 100-pound bag of animal feed is 40% oats. How many pounds of oats must be added to this feed to produce a mixture which is 50% oats?

30. A goldsmith has 10 g of a 50% gold alloy. How many grams of pure gold should be added to the alloy to make an alloy which is 75% gold?

Solve.

31. Ten grams of sugar are added to a 40-gram serving of a breakfast cereal which is 30% sugar. What is the percent concentration of sugar in the resulting mixture?

32. Thirty ounces of pure grapefruit juice is added to 50 oz of a fruit punch which is 20% grapefruit juice. What is the percent concentration of grapefruit juice in the resulting mixture?

33. A clothing manufacturer has some fiber which is 20% polyester and some which is 50% polyester. How many pounds of each fiber should be woven together to produce 600 lb of a fabric which is 35% polyester?

34. A nurse wants to make 50 ml of a 16% salt solution. How many milliliters each of a 13% salt solution and an 18% salt solution should be mixed to produce the desired solution?

35. A baker mixed some flour which was 40% wheat with 80 lb of flour which was 30% wheat to make a mixture which is 32% wheat. How many pounds of the 40% wheat flour were used?

36. A manufacturer mixed a chemical which was 60% fire retardant with 70 lb of a chemical which was 80% fire retardant to make a mixture which is 74% fire retardant. How much of the 60% mixture was used?

37. A 200-pound alloy of tin, which is 35% tin, is mixed with 300 lb of another tin alloy. The resulting alloy is 20% tin. Find the percent of tin in the 300-pound alloy.

38. A silversmith mixes 50 g of one alloy which is 50% silver with 150 g of another silver alloy. The resulting alloy is 68% silver. Find the percent of silver in the 150-gram alloy.

39. A chemist mixes a 5% silver nitrate solution with an 8% silver nitrate solution. How many ounces of each should be used to make 50 oz of a 7% silver nitrate solution? Round to the nearest hundredth.

40. How many grams of salt must be added to 180 g of a solution which is 15% salt to make a solution which is 23% salt? Round to the nearest tenth.

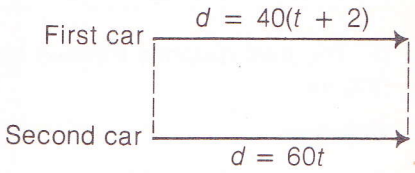
SECTION 6 Uniform Motion Problems

6.1 Objective To solve uniform motion problems

A train which travels constantly in a straight line at 50 mph is in *uniform motion*. **Uniform motion** means that the speed of an object does not change.

The solution of a uniform motion problem is based upon the equation $d = rt$, where d is the distance traveled, r is the rate of travel, and t is the time spent traveling.

A car leaves a town traveling at 40 mph. Two hours later, a second car leaves the same town, on the same road, traveling at 60 mph. In how many hours will the second car be passing the first car?



STRATEGY FOR SOLVING A UNIFORM MOTION PROBLEM

▷ For each object, write a numerical or variable expression for the distance, rate, and time. The results can be recorded in a table.

The first car traveled 2 h longer than the second car.

Unknown time for the second car: t
Time for the first car: $t + 2$

	Rate, r	·	Time, t	=	Distance, d
First car	40	·	$t + 2$	=	$40(t + 2)$
Second car	60	·	t	=	$60t$

▷ Determine how the distances traveled by each object are related. For example, the total distance traveled by both objects may be known or it may be known that the two objects traveled the same distance.

The two cars travel the same distance.

$$40(t + 2) = 60t$$
$$40t + 80 = 60t$$
$$80 = 20t$$
$$4 = t$$

The second car will be passing the first car in 4 h.

Example 1

Two cars, one traveling 10 mph faster than the second car, start at the same time from the same point and travel in opposite directions. In 3 h they are 300 mi apart. Find the rate of each car.

Strategy

▷ Rate of 1st car: r
Rate of 2nd car: $r + 10$

	Rate	Time	Distance
1st car	r	3	$3r$
2nd car	$r + 10$	3	$3(r + 10)$

▷ The total distance traveled by the two cars is 300 mi.

Solution

$$3r + 3(r + 10) = 300$$
$$3r + 3r + 30 = 300$$
$$6r + 30 = 300$$
$$6r = 270$$
$$r = 45$$
$$r + 10 = 45 + 10 = 55$$

The first car is traveling 45 mph.
The second car is traveling 55 mph.

Example 2

Two trains, one traveling at twice the speed of the other, start at the same time from stations which are 288 mi apart and travel toward each other. In 3 h, the trains pass each other. Find the rate of each train.

Your strategy

Your solution

Example 3

How far can a bicycling club ride out into the country at a speed of 12 mph and return over the same road at 8 mph if they travel a total of 10 h?

Strategy

▷ Time spent riding out: t
Time spent riding back: $10 - t$

	Rate	Time	Distance
Out	12	t	$12t$
Back	8	$10 - t$	$8(10 - t)$

▷ The distance out equals the distance back.

Solution

$$12t = 8(10 - t)$$
$$12t = 80 - 8t$$
$$20t = 80$$
$$t = 4 \text{ (The time is 4 h.)}$$

The distance out $= 12t = 12(4) = 48$ mi.

The club can ride 48 mi into the country.

Example 4

On a survey mission, a pilot flew out to a parcel of land and back in 5 h. The rate out was 150 mph and the rate returning was 100 mph. How far was the parcel of land?

Your strategy

Your solution

Solutions on p. 457

6.1 Application Problems

Solve.

1. Two cyclists start from the same point and ride in opposite directions. One cyclist rides twice as fast as the other. In three hours they are 72 mi apart. Find the rate of each cyclist.

2. Two small planes start from the same point and fly in opposite directions. The first plane is flying 25 mph slower than the second plane. In two hours the planes are 430 mi apart. Find the rate of each plane.

3. A motorboat leaves a harbor and travels at an average speed of 8 mph toward a small island. Two hours later a cabin cruiser leaves the same harbor and travels at an average speed of 16 mph toward the same island. In how many hours after the cabin cruiser leaves will the cabin cruiser be alongside the motorboat?

4. A long-distance runner started on a course running at an average speed of 6 mph. One hour later, a second runner began the same course at an average speed of 8 mph. How long after the second runner started will the second runner overtake the first runner?

5. A family drove to a resort at an average speed of 30 mph and later returned over the same road at an average speed of 50 mph. Find the distance to the resort if the total driving time was 8 h.

6. As part of flight training, a student pilot was required to fly to an airport and then return. The average speed to the airport was 90 mph, and the average speed returning was 120 mph. Find the distance between the two airports if the total flying time was 7 h.

7. Running at an average rate of 8 m/s, a sprinter ran to the end of a track and then jogged back to the starting point at an average rate of 3 m/s. The sprinter took 55 s to run to the end of the track and jog back. Find the length of the track.

8. Three campers left their campsite by canoe and paddled downstream at an average rate of 8 mph. They then turned around and paddled back upstream at an average rate of 4 mph to return to their campsite. How long did it take the campers to canoe downstream if the total trip took 1 h?

9. A car traveling at 48 mph overtakes a cyclist who, riding at 12 mph, has had a 3 h head start. How far from the starting point does the car overtake the cyclist?

10. A jet plane traveling at 600 mph overtakes a propeller-driven plane which has had a 2 h head start. The propeller-driven plane is traveling at 200 mph. How far from the starting point does the jet overtake the propeller-driven plane?

Solve.

11. On a 195-mile trip, a car traveled at an average speed of 45 mph and then reduced the speed to 30 mph for the remainder of the trip. The trip took a total of 5 h. For how long did the car travel at each speed?

12. A 555-mile, 5-hour plane trip was flown at two speeds. For the first part of the trip, the average speed was 105 mph. For the remainder of the trip, the average speed was 115 mph. For how long did the plane fly at each speed?

13. A bus traveled on a level road for 2 h at an average speed of 20 mph faster than it traveled on a winding road. The time spent on the winding road was 3 h. Find the average speed on the winding road if the total trip was 200 mi.

14. After a sailboat had been on the water for 3 h, a change in wind direction reduced the average speed of the boat by 5 mph. The entire distance sailed was 57 mi. The total time spent sailing was 6 h. How far did the sailboat travel in the first 3 h?

15. An executive drove from home at an average speed of 30 mph to an airport where a helicopter was waiting. The executive boarded the helicopter and flew to the corporate offices at an average speed of 60 mph. The entire distance was 150 mi. The entire trip took 3 h. Find the distance from the airport to the corporate offices.

16. A passenger train leaves a train depot two hours after a freight train leaves the same depot. The freight train is traveling 20 mph slower than the passenger train. Find the rate of each train if the passenger train overtakes the freight train in 3 h.

17. A car and a bus set out at 2 p.m. from the same point headed in the same direction. The average speed of the car is 30 mph slower than twice the speed of the bus. In two hours, the car is 20 mi ahead of the bus. Find the rate of the car.

18. A cyclist and a jogger set out at 11 a.m. from the same point headed in the same direction. The average speed of the cyclist is twice the speed of the jogger. In one hour, the cyclist is 8 mi ahead of the jogger. Find the rate of the cyclist.

19. Two cyclists start at the same time from opposite ends of a course which is 45 mi long. One cyclist is riding at 14 mph and the second cyclist is riding at 16 mph. How long after they begin will they meet?

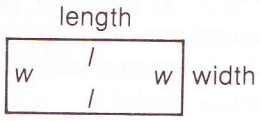
20. Two joggers start at the same time from opposite ends of a 10-mile course. One jogger is running at 4 mph and the other is running at 6 mph. How long after they begin will they meet?

SECTION 7 Geometry Problems

7.1 Objective To solve perimeter problems

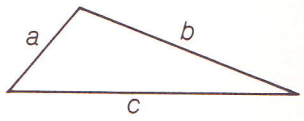
The **perimeter** of a geometric figure is a measure of the distance around the figure. The equations for the perimeters of a rectangle and a triangle are shown at the right.

Rectangle



Perimeter = $2l + 2w$

Triangle



Perimeter = $a + b + c$

The perimeter of a rectangle is 26 ft. The length of the rectangle is 1 ft more than twice the width. Find the width of the rectangle.

STRATEGY FOR SOLVING A PERIMETER PROBLEM

▷ Let a variable represent the measure of one of the unknown sides of the figure. Express the measures of the remaining sides in terms of that variable.

Width: w
Length: $2w + 1$

▷ Determine which perimeter equation to use.

Use the equation for the perimeter of a rectangle.

$$2l + 2w = P$$
$$2(2w + 1) + 2w = 26$$
$$4w + 2 + 2w = 26$$
$$6w + 2 = 26$$
$$6w = 24$$
$$w = 4$$

The width is 4 ft.

Example 1

The perimeter of a triangle is 25 ft. Two sides of the triangle are equal. The third side is 2 ft less than the length of one of the equal sides. Find the measure of one of the equal sides.

Strategy

▷ Each equal side: x
The third side: $x - 2$
▷ Use the equation for the perimeter of a triangle.

Solution

$$a + b + c = P$$
$$x + x + (x - 2) = 25$$
$$3x - 2 = 25$$
$$3x = 27$$
$$x = 9$$

Each of the equal sides measures 9 ft.

Example 2

The perimeter of a rectangle is 34 m. The width of the rectangle is 3 m less than the length. Find the measure of the width.

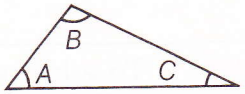
Your strategy

Your solution

Solution on p. 457

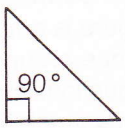
7.2 Objective To solve problems involving the angles of a triangle

In a triangle, the sum of the measures of all the angles is 180°.

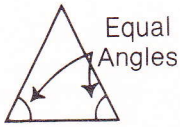


$A + B + C = 180°$

Two special types of triangles are shown at the right. A **right triangle** has one right angle (90°). An **isosceles triangle** has two equal angles.



Right triangle



Isosceles triangle

In a right triangle, the measure of one angle is twice the measure of the smallest angle. Find the measure of the smallest angle.

STRATEGY FOR SOLVING A PROBLEM INVOLVING THE ANGLES OF A TRIANGLE

▷ Let a variable represent one of the unknown angles. Express the other angles in terms of that variable.

Measure of smallest angle: x
Measure of second angle: $2x$
Measure of right angle: 90°

▷ Use the equation $A + B + C = 180°$.

$$x + 2x + 90 = 180$$
$$3x + 90 = 180$$
$$3x = 90$$
$$x = 30$$

The measure of the smallest angle is 30°.

Example 3

In an isosceles triangle, the measure of one angle is 20° more than twice the measure of one of the equal angles. Find the measure of one of the equal angles.

Strategy

▷ Measure of one of the equal angles: x
Measure of the second equal angle: x
Measure of the third angle: $2x + 20$
▷ Use the equation $A + B + C = 180°$.

Solution

$$x + x + (2x + 20) = 180$$
$$4x + 20 = 180$$
$$4x = 160$$
$$x = 40$$

The measure of one of the equal angles is 40°.

Example 4

In a triangle, the measure of one angle is twice the measure of the second angle. The measure of the third angle is 4° less than the measure of the second angle. Find the measure of each angle.

Your strategy

Your solution

Solution on p. 457

7.1 Application Problems

Solve.

1. The perimeter of a rectangle is 50 m. The width of the rectangle is 5 m less than the length. Find the length and width of the rectangle.

2. The perimeter of a rectangle is 120 ft. The length of the rectangle is twice the width. Find the length and width of the rectangle.

3. The width of a rectangle is 25% of the length. The perimeter is 250 cm. Find the length and width of the rectangle.

4. The width of a rectangle is 30% of the length. The perimeter of the rectangle is 338 ft. Find the length and width of the rectangle.

5. In an isosceles triangle, two sides are equal. The third side is 2 m less than one of the equal sides. The perimeter is 10 m. Find the length of each side.

6. The perimeter of a triangle is 33 ft. One side of the triangle is 1 ft longer than the second side. The third side is 2 ft longer than the second side. Find the measure of each side.

7. The perimeter of a rectangle is 42 m. The length of the rectangle is 3 m less than twice the width. Find the length and width of the rectangle.

8. The perimeter of a triangle is 20 ft. The first side is 1 ft less than twice the second side. The third side is 1 ft more than twice the second side. Find the measure of each side.

9. The perimeter of a triangle is 110 cm. One side is twice the second side. The third side is 30 cm more than the second side. Find the measure of each side.

10. In an isosceles triangle, two sides are equal. The third side is 50% of the length of one of the equal sides. Find the length of each side when the perimeter is 125 ft.

11. The perimeter of a rectangle is 56.24 m. The width of the rectangle is 0.84 m less than the length. Find the length and width of the rectangle.

12. In an isosceles triangle, two sides are equal. The length of one of the equal sides is 3.52 times the length of the third side. The perimeter is 10.43 m. Find the length of each side. Round to the nearest hundredth.

7.2 Application Problems

Solve.

13. In an equiangular triangle, all three angles are equal. Find the measures of the equal angles.

14. In an isosceles triangle, one angle is three times the measure of one of the equal angles. Find the measure of each angle.

15. In an isosceles right triangle, two angles are equal and the third angle is 90°. Find the measures of the equal angles.

16. One angle of a right triangle is 3° less than twice the measure of the smallest angle. Find the measure of each angle.

17. In an isosceles triangle, one angle is 12° more than twice the measure of one of the equal angles. Find the measure of each angle.

18. In an isosceles triangle, one angle is 5° less than three times the measure of one of the equal angles. Find the measure of each angle.

19. In a triangle, one angle is twice the measure of the second angle. The third angle is three times the measure of the second angle. Find the measure of each angle.

20. In a triangle, one angle is 5° more than the measure of the second angle. The third angle is 10° more than the measure of the second angle. Find the measure of each angle.

21. One angle of a triangle is three times the measure of the third angle. The second angle is 5° less than the measure of the third angle. Find the measure of each angle.

22. One angle of a triangle is twice the measure of the second angle. The third angle is three times the measure of the first angle. Find the measure of each angle.

23. The first angle of a triangle is twice the measure of the second angle. The third angle is 10° less than the measure of the first angle. Find the measure of each angle.

24. The first angle of a triangle is three times the measure of the second angle. The third angle is 33° more than the measure of the first angle. Find the measure of each angle.

SECTION 8 Puzzle Problems

8.1 Objective To solve consecutive integer problems

Recall that the integers are the numbers . . . −4, −3, −2, −1, 0, 1, 2, 3, 4, . . . An **even integer** is an integer that is divisible by 2. Examples of even integers are −8, 0, and 22. An **odd integer** is an integer that is not divisible by 2. Examples of odd integers are −17, 1, and 39.

Consecutive integers are integers which follow one another in order. Examples of consecutive integers are shown at the right. (Assume that the variable n represents an integer.)

11, 12, 13
−8, −7, −6
$n, n + 1, n + 2$

Examples of **consecutive even integers** are shown at the right. (Assume that the variable n represents an even integer.)

24, 26, 28
−10, −8, −6
$n, n + 2, n + 4$

Examples of **consecutive odd integers** are shown at the right. (Assume that the variable n represents an odd integer.)

19, 21, 23
−1, 1, 3
$n, n + 2, n + 4$

The sum of three consecutive odd integers is 45. Find the integers.

STRATEGY FOR SOLVING A CONSECUTIVE INTEGER PROBLEM

▷ Let a variable represent one of the integers. Express each of the other integers in terms of that variable. Remember that consecutive integers will differ by 1. Consecutive even or consecutive odd integers differ by 2.

Represent three consecutive odd integers.

First odd integer: n
Second odd integer: $n + 2$
Third odd integer: $n + 4$

▷ Determine the relationship among the integers.

The sum of the three odd integers is 45.

$$n + (n + 2) + (n + 4) = 45$$
$$3n + 6 = 45$$
$$3n = 39$$
$$n = 13$$
$$n + 2 = 13 + 2 = 15$$
$$n + 4 = 13 + 4 = 17$$

The three consecutive odd integers are 13, 15, and 17.

Example 1

Find three consecutive even integers such that three times the second is four more than the sum of the first and third.

Strategy

▷ First even integer: n
Second even integer: $n + 2$
Third even integer: $n + 4$
▷ Three times the second equals four more than the sum of the first and third.

Solution

$$3(n + 2) = n + (n + 4) + 4$$
$$3n + 6 = 2n + 8$$
$$n + 6 = 8$$
$$n = 2$$

$n + 2 = 2 + 2 = 4$

$n + 4 = 2 + 4 = 6$

The three even integers are 2, 4, and 6.

Example 2

Find three consecutive integers whose sum is -6.

Your strategy

Your solution

Solution on p. 458

8.2 Objective To solve coin and stamp problems

In solving problems dealing with coins or stamps of different values, it is necessary to represent the value of the coins or stamps in the same unit of money. The unit of money is frequently cents. For example:

The value of 3 quarters in cents is $3 \cdot 25$, or 75 cents.
The value of 4 nickels in cents is $4 \cdot 5$, or 20 cents.
The value of d dimes in cents is $d \cdot 10$, or $10d$ cents.

A coin bank contains \$1.35 in dimes and quarters. In all, there are nine coins in the bank. Find the number of dimes and the number of quarters in the bank.

STRATEGY FOR SOLVING A COIN PROBLEM

▷ For each denomination of coin, write a numerical or variable expression for the number of coins, the value of the coin, and the total value of the coins in cents. The results can be recorded in a table.

The total number of coins is 9.

Number of quarters: x
Number of dimes: $9 - x$

Coin	Number of coins	·	Value of coin in cents	=	Total value in cents
Quarter	x	·	25	=	$25x$
Dime	$9 - x$	·	10	=	$10(9 - x)$

▷ Determine the relationship between the total values of the coins. Use the fact that the sum of the total values of each denomination of coin is equal to the total value of all the coins.

The sum of the total values of each denomination of coin is equal to the total value of all the coins (135 cents).

$$25x + 10(9 - x) = 135$$
$$25x + 90 - 10x = 135$$
$$15x + 90 = 135$$
$$15x = 45$$
$$x = 3$$
$$9 - x = 9 - 3 = 6$$

There are 3 quarters and 6 dimes in the bank.

Example 3

A collection of stamps consists of 3¢ stamps and 8¢ stamps. The number of 8¢ stamps is five more than three times the number of 3¢ stamps. The total value of all the stamps is $1.48. Find the number of 3¢ stamps.

Strategy

▷ Number of 3¢ stamps: x
Number of 8¢ stamps: $3x + 5$

Stamp	Number	Value	Total value
3¢	x	3	$3x$
8¢	$3x + 5$	8	$8(3x + 5)$

▷ The sum of the total values of each type of stamp equals the total value of all the stamps (148 cents).

Solution

$$3x + 8(3x + 5) = 148$$
$$3x + 24x + 40 = 148$$
$$27x + 40 = 148$$
$$27x = 108$$
$$x = 4$$

There are four 3¢ stamps in the collection.

Example 4

A coin bank contains nickels, dimes, and quarters. There are four times as many nickels as dimes, and five more quarters than dimes. The total value of all the coins is $6.75. Find the number of each kind of coin in the bank.

Your strategy

Your solution

Solution on p. 458

8.3 Objective To solve age problems

The goal of an age problem is to determine the age of a person or an object.

A painting is 20 years old and a sculpture is 10 years old. How many years ago was the painting three times as old as the sculpture was then?

STRATEGY FOR SOLVING AN AGE PROBLEM

▷ Represent the ages in terms of numerical or variable expressions. To represent a past age, subtract from the present age. To represent a future age, add to the present age. The results can be recorded in a table.

The number of years ago: x

	Present age	Past age
Painting	20	$20 - x$
Sculpture	10	$10 - x$

▷ Determine the relationship among the ages.

At a past age, the painting was three times as old as the sculpture was then.

$$20 - x = 3(10 - x)$$
$$20 - x = 30 - 3x$$
$$20 + 2x = 30$$
$$2x = 10$$
$$x = 5$$

Five years ago the painting was three times as old as the sculpture.

Example 5

A stamp collector has a 3¢ stamp which is 25 years older than a 5¢ stamp. In 18 years, the 3¢ stamp will be twice as old as the 5¢ stamp will be then. How old are the stamps now?

Strategy

▷ Present age of 5¢ stamp: x

	Present	Future
3¢ stamp	$x + 25$	$x + 43$
5¢ stamp	x	$x + 18$

▷ At a future age, the 3¢ stamp will be twice as old as the 5¢ stamp.

Solution

$$2(x + 18) = x + 43$$
$$2x + 36 = x + 43$$
$$x + 36 = 43$$
$$x = 7$$
$$x + 25 = 7 + 25 = 32$$

The 3¢ stamp is 32 years old and the 5¢ stamp is 7 years old.

Example 6

A half-dollar is now 25 years old. A dime is 15 years old. How many years ago was the half-dollar twice as old as the dime?

Your strategy

Your solution

Solution on p. 459

8.1 Application Problems

Solve.

1. The sum of three consecutive integers is 48. Find the integers.

2. The sum of three consecutive integers is 60. Find the integers.

3. The sum of three consecutive even integers is 66. Find the integers.

4. The sum of three consecutive even integers is 42. Find the integers.

5. The sum of three consecutive odd integers is 51. Find the integers.

6. The sum of three consecutive odd integers is 75. Find the integers.

7. Find two consecutive even integers such that three times the first equals twice the second.

8. Find two consecutive even integers such that four times the first is three times the second.

9. Five times the first of two consecutive odd integers equals three times the second. Find the integers.

10. Seven times the first of two consecutive odd integers is five times the second. Find the integers.

11. Find three consecutive integers whose sum is negative twenty-one.

12. Find three consecutive even integers whose sum is negative eighteen.

13. Twice the smallest of three consecutive odd integers is seven more than the largest. Find the integers.

14. Three times the smallest of three consecutive even integers is four more than twice the largest. Find the integers.

Solve.

15. Find three consecutive odd integers such that three times the middle integer is one more than the sum of the first and third.

16. Find three consecutive even integers such that three times the middle integer is four more than the sum of the first and third.

8.2 Application Problems

Solve.

17. A coin purse contains 16 coins in nickels and dimes. The coins have a total value of $1. Find the number of nickels and dimes in the coin purse.

18. A bank contains 30 coins in dimes and quarters. The coins have a total value of $5.40. Find the number of dimes and quarters in the bank.

19. A postal clerk sold some 15¢ stamps and some 25¢ stamps. Altogether 10 stamps were sold for a total cost of $1.70. How many of each type of stamp were sold?

20. A business executive purchased 40 stamps for $7.70. The purchase included 15¢ stamps and 20¢ stamps. How many of each type of stamp were purchased?

21. The total value of the dimes and quarters in a bank is $6.50. There are five more quarters than dimes. Find the number of each type of coin in the bank.

22. A drawer contains 15¢ stamps and 18¢ stamps. The number of 15¢ stamps is two less than three times the number of 18¢ stamps. The total value of all the stamps is $.96. How many 15¢ stamps are in the drawer?

23. A bank teller cashed a check for $150 using twenty-dollar bills and ten-dollar bills. In all, nine bills were handed to the customer. Find the number of twenty-dollar bills and ten-dollar bills.

24. A total of 30 bills are in a cash box. Some of the bills are one-dollar bills and the rest are five-dollar bills. The total amount of cash in the box is $50. Find the number of each type of bill in the cash box.

Solve.

25. A coin bank contains pennies, nickels, and quarters. There are six times as many nickels as pennies and four times as many quarters as pennies. The total amount of money in the bank is \$6.55. Find the number of pennies in the bank.

26. A coin bank contains pennies, nickels, and dimes. There are five times as many nickels as pennies and three times as many dimes as pennies. The total amount of money in the bank is \$6.72. Find the number of pennies in the bank.

27. A collection of stamps consists of 2¢ stamps, 5¢ stamps, and 7¢ stamps. There are eight more 2¢ stamps than 5¢ stamps, and twice as many 7¢ stamps as 5¢ stamps. The total value of the stamps is \$1.63. Find the number of each type of stamp in the collection.

28. A collection of stamps consists of 3¢ stamps, 7¢ stamps, and 12¢ stamps. The number of 3¢ stamps is six less than the number of 7¢ stamps. The number of 12¢ stamps is one half the number of 7¢ stamps. The total value of all the stamps is \$3.02. Find the number of each type of stamp in the collection.

29. A child's piggy bank contains nickels, dimes, and quarters. There are twice as many nickels as dimes and three more quarters than nickels. The total value of all the coins is \$11.25. Find the number of each type of coin.

30. A collection of stamps consists of 6¢ stamps, 8¢ stamps, and 15¢ stamps. The number of 6¢ stamps is three times the number of 8¢ stamps. There are seven more 15¢ stamps than there are 6¢ stamps. The total value of all the stamps is \$4.60. Find the number of each type of stamp.

8.3 Application Problems

Solve.

31. A book dealer has an autographed, first-edition book which is 35 years old and a reprint of the book which is 7 years old. In how many years will the autographed first edition be three times as old as the reprint will be then?

32. A collector of hand woven rugs has an oval rug which is 42 years old and a circular rug which is 14 years old. How many years ago was the oval rug five times as old as the circular rug was then?

33. A coin collector has a dime which is 24 years older than a nickel. In 8 years the dime will be twice as old as the nickel will be then. Find the present age of the dime and nickel.

34. An oil painting is 10 years older than a lithograph. Five years ago the painting was twice as old as the lithograph was then. Find the present age of each.

Solve.

35. An art collector has a porcelain vase which is 15 years old and a crystal vase which is 95 years old. In how many years will the crystal vase be three times as old as the porcelain vase will be then?

36. A stamp collector has a 2¢ stamp which is 20 years old and a 3¢ stamp which is 16 years old. How many years ago was the 2¢ stamp twice as old as the 3¢ stamp was then?

37. An antique butterchurn is 85 years old and an antique ice box is 75 years old. How many years ago was the butterchurn twice the age the ice box was then?

38. A diamond ring is 2 years old and a ruby ring is 22 years old. In how many years will the ruby ring be twice the age the diamond ring will be then?

39. An antique car is 45 years older than a replica of the car. In 13 years, the antique car will be four times as old as the replica will be then. Find the present ages of the two cars.

40. A gold coin is 84 years older than a silver coin. Twenty years ago, the gold coin was three times as old as the silver coin was then. Find the present ages of the two coins.

41. The sum of the ages of an oil painting and a watercolor is 20 years. The oil painting one year from now will be nine times the age of the watercolor one year ago. Find the present age of each painting.

42. The sum of the ages of two cars is 8 years. Two years ago the age of the older car was three times the age the younger car was then. Find the present age of each car.

43. The sum of the ages of a 5¢ coin and a 10¢ coin is 12. Two years from now the age of the 5¢ coin will equal the age of the 10¢ coin two years ago. Find the present age of each coin.

44. The sum of the ages of two children is 18. Six years from now the age of the older child will be twice the age of the younger child. Find the present ages of the two children.

Review/Test

SECTION 1

1.1a Write 60% as a fraction and a decimal.

1.1b Write $62\frac{1}{2}\%$ as a fraction.

1.2a Write 0.375 as a percent.

1.2b Write $\frac{7}{8}$ as a percent. Write the remainder in fractional form.

SECTION 2

2.1a Find 16% of 40.

2.1b 20 is what percent of 16?

2.2 The value of a personal computer today is \$2400. This is 80% of the computer's value last year. Find the value of the computer last year.

SECTION 3

3.1 The manager of a sports shop uses a markup rate of 50%. The selling price for a set of golf clubs is \$300. Find the cost of the golf clubs.

3.2 A portable typewriter which regularly sells for \$100 is on sale for \$80. Find the discount rate.

SECTION 4

4.1 A total of \$7000 is deposited into two simple interest accounts. On one account, the annual simple interest rate is 10%, while on the second account the simple interest rate is 15%. How much should be invested in each account so that the total annual interest earned is \$800?

Review/Test

SECTION 5

5.1 A coffee merchant wants to make 12 lb of a blend of coffee to sell for $6 per pound. The blend is made using a $7 grade and a $4 grade of coffee. How many pounds of each of these grades should be used?

5.2 How many gallons of a 15% acid solution must be mixed with 5 gal of a 20% acid solution to make a 16% acid solution?

SECTION 6

6.1 Two planes start at the same time from the same point and fly in opposite directions. The first plane is flying 100 mph faster than the second plane. In three hours the two planes are 1050 mi apart. Find the rate of each plane.

SECTION 7

7.1 The perimeter of a rectangle is 38 m. The length of the rectangle is 1 m less than three times the width. Find the length and width of the rectangle.

7.2 In a triangle, the first angle is 15° more than the second angle. The third angle is three times the second angle. Find the measure of each angle.

SECTION 8

8.1 Find three consecutive odd integers such that three times the first integer is one less than the sum of the second and third integers.

8.2 A coin bank contains 50 coins in nickels and quarters. The total amount of money in the bank is $9.50. Find the number of nickels and the number of quarters in the bank.

8.3 The age of a 20¢ stamp is 5 years, and the age of a 5¢ stamp is 35 years. In how many years will the 5¢ stamp be three times the age the 20¢ stamp will be then?

Review/Test

SECTION 1

1.1a Write 80% as a fraction.

a) $\frac{4}{5}$ b) $\frac{4}{50}$

c) $\frac{2}{25}$ d) $\frac{8}{25}$

1.1b Write $16\frac{2}{3}\%$ as a fraction.

a) $\frac{2}{3}$ b) $\frac{1}{6}$

c) $\frac{4}{25}$ d) $\frac{50}{3}$

1.2a Write 0.075 as a percent.

a) 0.075% b) 0.75%

c) 7.5% d) 75%

1.2b Write $\frac{2}{25}$ as a percent.

a) 80% b) 8%

c) 0.8% d) 0.08%

SECTION 2

2.1a Find $83\frac{1}{3}\%$ of 24.

a) 20 b) 16

c) 8 d) 3

2.1b 30% of what is 12?

a) 40 b) 30

c) 12 d) 3.6

2.2 A survey of 250 librarians showed that 50 of the libraries had a particular reference book on their shelves. What percent of the libraries had the reference book?

a) 80% b) 75% c) 50% d) 20%

SECTION 3

3.1 A department store buys a chain necklace for $8 and sells it for $14. Find the markup rate.

a) 25% b) 43% c) 57% d) 75%

3.2 A file cabinet which normally sells for $99 is on sale for 20% off. Find the sale price.

a) $4.95 b) $19.80 c) $79.20 d) $94.05

SECTION 4

4.1 A deposit of $4000 is made into an account which earns 11% simple interest. How much additional money must be deposited into an account which pays 14% simple interest so that the total interest earned is 12% of the total investment?

a) $5000 b) $4000 c) $3000 d) $2000

Review/Test

SECTION 5

5.1 How many grams of a gold alloy which costs \$4 a gram must be mixed with 30 g of a gold alloy which costs \$7 a gram to make an alloy which sells for \$5 a gram?

a) 40 g b) 50 g
c) 60 g d) 70 g

5.2 How many ounces of pure water must be added to 70 oz of a 10% salt solution to make a 7% salt solution?

a) 30 oz b) 35 oz
c) 40 oz d) 45 oz

SECTION 6

6.1 A car traveling at 50 mph overtakes a cyclist who, riding at 10 mph, has had a 2 h head start. How far from the starting point does the car overtake the cyclist?

a) 0.5 mi b) 25 mi c) 40 mi d) 80 mi

SECTION 7

7.1 The perimeter of a triangle is 49 ft. The first side is twice the length of the third side, and the second side is 5 ft more than the length of the third side. Find the measure of the first side.

a) 11 ft b) 16 ft
c) 22 ft d) 49 ft

7.2 In an isosceles triangle, two angles are equal. The third angle is 8° less than twice the measure of one of the equal angles. Find the measure of one of the equal angles.

a) 40° b) 47°
c) 43° d) 94°

SECTION 8

8.1 Three times the second of three consecutive even integers is 14 more than the sum of the first and third integers. Find the middle even integer.

a) 10 b) 12
c) 14 d) 16

8.2 A coin bank contains dimes and quarters. The number of quarters is five less than four times the number of dimes. The total amount in the bank is \$6.45. Find the number of dimes in the bank.

a) 7 b) 21
c) 43 d) 53

8.3 The age of a gold coin is 60 years, and the age of a silver coin is 40 years. How many years ago was the gold coin twice the age the silver coin was then?

a) 20 b) 30 c) 40 d) 50