MATH MILESTONE # A5

DIVISION

The word, **milestone,** means "a point at which a significant change occurs." A Math Milestone refers to a significant point in the understanding of mathematics.

To reach this milestone one should be able to divide numbers with single and double-digit divisors mentally without effort.

A calculator shall be required to check the answers arrived at through mental math.

Please consult the **Glossary** supplied with this Milestone for mathematical terms. Consult a regular dictionary at **www.dictionary.com** for general English words that one does not understand fully.

You may start with the Diagnostic Test on the next page to assess your proficiency on this milestone. Then continue with the lessons with special attention to those, which address the weak areas.

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DIAGNOSTIC TEST

1. Mangoes cost $3 each. How many mangoes can you buy for $24?
2. How many times can you take 7 away from 56?
3. Divide the following mentally.
   (a) 12 ÷ 3 (b) 63 ÷ 7 (c) 72 ÷ 9 (d) 102 ÷ 17
4. Use the basic rules of division to answer the following.
   (a) 5 ÷ 5 (b) 5 ÷ 1 (c) 0 ÷ 5 (d) 5 ÷ 0
5. Fill in the blank using the knowledge of multiplication
   (a) 3 x __ = 12 (b) 7 x __ = 63 (c) 9 x __ = 72 (d) 17 x __ = 102
6. State the quotient and the remainder.
   (a) 8 ÷ 3 (b) 16 ÷ 5 (c) 18 ÷ 6 (d) 23 ÷ 7
7. State if the division is exact or inexact.
   (a) 9 ÷ 3 (b) 17 ÷ 5 (c) 20 ÷ 4 (d) 9 ÷ 2
8. State if the divisor is a factor of the dividend.
   (a) 17 ÷ 3 (b) 63 ÷ 7 (c) 72 ÷ 9 (d) 100 ÷ 17
9. Divide and convert remainder into fraction in the quotient.
   (a) 9 ÷ 4 (b) 19 ÷ 6 (c) 24 ÷ 5 (d) 31 ÷ 7
10. Divide using the long form of division.
    (a) 725 ÷ 6 (b) 6243 ÷ 7 (c) 84425 ÷ 5 (d) 243618 ÷ 3
11. Divide using the short form of division.
    (a) 612 ÷ 6 (b) 3437 ÷ 7 (c) 54321 ÷ 5 (d) 700312 ÷ 3
12. Divide the following.
    (a) 563 ÷ 16 (b) 396 ÷ 23 (c) 7611 ÷ 62 (d) 630526 ÷ 87
13. If you cut a 56-inch long string into eight equal parts, what is the length of each part?
14. If 6 pens cost $16, how much will 9 pens cost?

Answer: **1.** 8 **2.** 8 **3.** (a) 4 (b) 9 (c) 8 (d) 6 **4.** (a) 1 (b) 5 (c) 0 (d) undefined **5.** (a) 4 (b) 9 (c) 8 (d) 6 **6.** (a) 2 R2 (b) 3 R1 (c) 3 R0 (d) 3 R2 **7.** (a) exact (b) inexact (c) exact (d) inexact **8.** (a) no (b) yes (c) yes (d) no **9.** (a) 2 and ¼ (b) 3 and 1/6 (c) 4 and 4/5 (d) 4 and 3/7 **10.** (a) 120 R5 (b) 891 R6 (c) 16885 (d) 81206 **11.** (a) 102 (b) 491 (c) 10864 R1 (d) 233437 R1 **12.** (a) 35 R3 (b) 17 R5 (c) 122 R47 (d) 7247 R37 **13.** 7 inches **14.** $24

LESSONS

Lesson A5.1 Repeated Subtraction

Division is taking a number out of another as many times as possible. The number of times it can be taken out is called the QUOTIENT. The quantity remaining is called the REMAINDER.

1. To find out how many books, at $6 each, you can buy for $30, start subtracting $6 from $30 repeatedly until you cannot subtract any further. You find out that you can subtract $6 from $30 five times, and the **remainder** is 0. Thus, you can buy 5 books. This is the **quotient**.

   $30 – $6 – $6 – $6 – $6 – $6 = 0

2. There are 90 cookies in a jar. You eat 10 cookies a day. To find out how many days those cookies will last, subtract the largest multiple of 10 from 90. 10 times 9 (90) is the largest multiple that you can subtract from 90. This gives you a **quotient** of 9, and a **remainder** of 0. Therefore, at this rate the cookies will last 9 days.

   90 – (10 + 10 + 10 + 10 + 10 + 10 + 10 + 10 + 10) = 0

3. You are 100 months old. There are 12 months in a year. To find out your age in years, find the largest multiple of 12, which you can take out of 100. The largest multiple is 8 (12 x 8 = 96), and after you take it out a quantity of 4 remains. Therefore, the **quotient** is 8, and the **remainder** is 4. Your age is 8 years and 4 months.

   100 – (12 + 12 + 12 + 12 + 12 + 12 + 12 + 12) = 4

☺ Exercise A5.1

1. Mangoes cost $3 each. How many mangoes can you buy for $24? Remainder?
2. There are 62 apples. You need 6 apples to create one fruit basket. How many fruit baskets can you make? Remainder?
3. A table is 64 inches long. There are 12 inches in a foot. What is the length of that table in feet and inches?
4. Solve.
   (a) How many times can you take 5 away from 50? Remainder?
   (b) How many times can you take 6 away from 50? Remainder?
   (c) How many times can you take 7 away from 50? Remainder?

Answer: **1.** 8 mangoes **2.** 10 baskets, remainder 2 apples **3.** 5 feet, 4 inches **4.** (a) 10, remainder 0 (b) 8, remainder 2 (c) 7, remainder 1

Lesson A5.2 Terms and Definitions

Division is "repeated subtraction" as many times as possible.

1. We write division with a division (÷) sign between the starting number and the number to be subtracted repeatedly. If we are repeatedly subtracting 6 from 30, we write

$$30 \div 6 = 5$$

(a) **"÷"** is the **sign** for division**; "="** is the **equals sign**.
(b) We read the above expression as, **"30 divided by 6 equals 5."**
(c) **"30"** is the first number called the **dividend** (number to be divided).
(d) **"6"** is the second number called the **divisor** (number by which divided).
(e) "5" is the outcome of division called the **quotient.**

2. A division may also be written as "dividend over divisor."

$$18 \div 6 = \frac{18}{6} = 3$$

3. To divide, we find the largest multiple of divisor that can be taken out of the dividend. Some simple word problems require division only:

(a) A man divided 28 cents among 4 boys. How many did he give each?
This is clearly a division problem. We find the largest multiple of 4 that can be taken out of 28. 4 x 7 = 28, which can be taken out of 28. The quotient is 7. Therefore, he gave each boy 7 cents.

(b) If a man travels 3 miles in an hour, how many hours will it take him to travel 27 miles?
This is also a division problem. We divide 27 by 3. The quotient is 9. Therefore, it will take him 9 hours.

☺ Exercise A5.2

1. Write down "24 divided by 3 equals 8" using mathematical symbols in two different ways.
2. Identify the dividend, divisor and the quotient in the following divisions.
   (a) 27 ÷ 3 = 9 (b) 56 ÷ 7 = 8 (c) 45 ÷ 9 = 5
3. Compute.
   (a) $\frac{24}{4}$ = ____ (b) $\frac{45}{5}$ = ____ (c) $\frac{36}{9}$ = ____
4. Divide. You may verify your answers on a calculator.

(a) 9 ÷ 3	(e) 8 ÷ 4	(i) 25 ÷ 5	(m) 12 ÷ 4
(b) 8 ÷ 2	(f) 15 ÷ 5	(j) 24 ÷ 6	(n) 12 ÷ 6
(c) 6 ÷ 2	(g) 21 ÷ 3	(k) 16 ÷ 4	(o) 24 ÷ 8
(d) 9 ÷ 1	(h) 28 ÷ 7	(l) 12 ÷ 3	

5. Solve the following using division.
   (a) If there are 20 pennies on the table, how many times can you take 5 pennies away?
   (b) Forty-two cents were divided equally among 6 boys. How many cents did each boy receive?
   (c) There are 32 dimes on a table in 4 piles. How many are there in each pile?

Answer: **1.** 24 ÷ 3 and $\frac{24}{3}$ **2.** The first number is the dividend, the second number is the divisor, and the third number is the quotient. **3.** (a) $\frac{24}{4} = 6$ (b) $\frac{45}{5} = 9$ (c) $\frac{36}{9} = 4$ **4.** (a) 3 (b) 4 (c) 3 (d) 9 (e) 2 (f) 3 (g) 7 (h) 4 (i) 5 (j) 4 (k) 4 (l) 4 (m) 3 (n) 2 (o) 3 **5.** (a) 5 (b) 7 (c) 8

Lesson A5.3 Mental Division

Patterns in division give us simple formulas.

1. When a number is divided by itself, the quotient is always 1.

   (a) A number can be taken away from itself only once.

   8 ÷ 8 = 1

   (b) Therefore, for any number **N**,

   N ÷ N = 1

2. When a number is divided by 1, the quotient is always the number.

   (a) We can take 1 away from a number as many times as the number.

   8 ÷ 1 = 8

   (b) Therefore, for any number **N**,

   N ÷ 1 = N

3. When 0 is divided by a number, the quotient is always 0.

   (a) We cannot take away a number from zero (nothing) even once.

   0 ÷ 8 = 0

   (b) Therefore, for any number **N**,

   0 ÷ N = 0

4. When a number is divided by 0, the quotient is undefined.

   (a) When you take zero (nothing) out of a number, the number is still there. So, you cannot determine how many times zero can be taken out of a number.

   8 ÷ 0 = undefined

   (c) Therefore, for any number **N**,

   N ÷ 0 = undefined

☺ Exercise A5.3

1. Divide.

(a) 9 ÷ 9	(d 9 ÷ 0	(g) 0 ÷ 23	(j) 24 ÷ 1	(m) 12 ÷ 1
(b) 8 ÷ 1	(e) 8 ÷ 8	(h) 28 ÷ 0	(k) 0 ÷ 4	(n) 12 ÷ 12
(c) 0 ÷ 2	(f) 15 ÷ 1	(i) 25 ÷ 25	(l) 12 ÷ 0	(o) 24 ÷ 0

Answer: 1. (a) 1 (b) 8 (c) 0 (d) undefined (e) 1 (f) 15 (g) 0 (h) undefined (i) 1 (j) 24 (k) 0
(l) undefined (m) 12 (n) 1 (o) undefined

Lesson A5.4 Division is "Reverse Multiplication"

The quotient may be determined by "reverse multiplication."

1. We may question the quotient in division as follows.

   To determine **30 ÷ 6** **=** **what?**
   We may ask, **6 x what?** **=** **30**
   The answer is 5.

2. We use multiplication to determine the quotient. Remainder is what is left after division.

   Divide, **91 ÷ 13**

   Count by 13 → **13, 26, 39, 52, 65, 78, 91, ...**

   The seventh count is 91, hence
   13 x 7 = 91
   Therefore, **91 ÷ 13 = 7**

   Divide, **88 ÷ 11**

   We count by 11 to get, 11 x 8 = 88
   Therefore, **88 ÷ 11 = 8**

   Divide, **100 ÷ 12**

   We count by 12 to get, 12 x 8 = 96 and 12 x 9 =108
   Therefore, from 100, we can take 12 out, 8 times, as 96. The remainder is 4.
   Therefore, **100 ÷ 12 = 8 R4**

   *NOTE: We write the remainder with **R** in front of it,*

   Divide, **130 ÷ 15**

   We count by 15 to get, 15 x 8 = 120 and 15 x 9 = 135
   Therefore, from 130, we can take 15 out, 8 times, as 120. The remainder is 10.
   Therefore, **130 ÷ 15 = 8 R10**

   NOTE: The remainder will always be less than the divisor because we have taken the divisor out of the dividend as many times as possible.

3. We may verify a division by multiplying back.

   (a) When there is no remainder, we may verify the division by multiplying the quotient back to the divisor. The product should be equal to the dividend.

   If 20 ÷ 5 = 4 then **4 x 5 = 20** (The division is correct)
   If 42 ÷ 7 = 6 then **6 x 7 = 42** (The division is correct)

   In the following case we find that the division is incorrect.
   27 ÷ 3 = 8 but **8 x 3 = 24**

   (b) When there is a remainder, the product of the quotient and the divisor PLUS the remainder gives back the dividend.

   If 25 ÷ 3 = 8 R1 then **8 x 3 + 1 = 25** (The division is correct)
   If 35 ÷ 6 = 5 R5 then **5 x 6 + 5 = 35** (The division is correct)

   In the following case we find that the division is incorrect.
   37 ÷ 7 = 5 R3 but **5 x 7 + 3 = 38**

☺ Exercise A5.4

1. Fill in the blanks using the knowledge of multiplication

(a) 3 x ___ = 12	(d) 6 x ____ = 24	(g) 11 x ____ = 66
(b) 7 x ___ = 21	(e) 9 x ____ = 63	(h) 12 x ____ = 84
(c) 5 x ___ = 25	(f) 8 x ____ = 72	(i) 17 x ____ = 102

2. Convert division to "multiplication in reverse" and solve.
   - (a) 80 ÷ 8 = what? → 8 x what? = 80 → 8 x 10 = 80, Quotient is 10
   - (b) 35 ÷ 5 = what? → _____ → _____, Quotient is ___
   - (c) 54 ÷ 6 = what? → _____ → _____, Quotient is ___
   - (d) 72 ÷ 9 = what? → _____ → _____, Quotient is ___
   - (e) 96 ÷ 12 = what? → _____ → _____, Quotient is ___

3. In the following divisions state the quotient and the remainder.

(a) 8 ÷ 3	(d) 16 ÷ 5	(g) 20 ÷ 4
(b) 9 ÷ 4	(e) 25 ÷ 5	(h) 19 ÷ 6
(c) 8 ÷ 4	(f) 25 ÷ 6	(i) 24 ÷ 8

4. Divide the following.

(a) 72 ÷ 9	(d) 53 ÷ 7	(g) 45 ÷ 5
(b) 73 ÷ 9	(e) 55 ÷ 11	(h) 45 ÷ 3
(c) 49 ÷ 7	(f) 55 ÷ 12	(i) 45 ÷ 4

5. Verify the following divisions without remainder.

(a) 20 ÷ 4 = 6	(c) 36 ÷ 6 = 6	(e) 54 ÷ 6 = 9
(b) 35 ÷ 5 = 7	(d) 56 ÷ 7 = 7	(f) 24 ÷ 3 = 9

6. Verify the following divisions with remainder.

(a) 23 ÷ 4 = 5 R3	(c) 47 ÷ 5 = 8 R2	(e) 59 ÷ 6 = 9 R5
(b) 47 ÷ 7 = 6 R4	(d) 59 ÷ 7 = 8 R3	(f) 26 ÷ 3 = 8 R2

Answer: **1.** (a) 4 (b) 3 (c) 5 (d) 4 (e) 7 (f) 9 (g) 6 (h) 7 (i) 6 **2.** (a) 8 x what? =80, 10
(b) 5 x what? = 35, 7 (c) 6 x what? = 54, 9 (d) 9 x what? = 72, 8 (e) 12 x what? = 96, 8
3. (a) 2 R2 (b) 2 R1 (c) 2 (d) 3 R1 (e) 5 (f) 4 R1 (g) 5 (h) 3 R1 (i) 3 **4.** (a) 8 (b) 8 R1
(c) 7 (d) 7 R4 (e) 5 (f) 4 R7 (g) 9 (h) 15 (i) 11 R1 **5.** (a) 6x4=24, no (b) 7x5=35, yes
(c) 6 x6=36, yes (d) 7x7=49, no (e) 6x9=54, yes (f) 9x3=27, no **6.** (a) 4x5+3=23, yes
(b) 7x6+4=46, no (c) 5x8+2=42, no (d) 7x8+3=59, yes (e) 6x9+5=59, yes (f) 3x8+2=
26, yes

Lesson A5.5 Exact and Inexact Divisions

When the divisor can be taken out of the dividend an exact number of times the division is EXACT. If there is a remainder left then the division is inexact.

1. Here are some examples of EXACT division. There is no remainder.

   20 ÷ 5 = 4 (no remainder → exact division)
   30 ÷ 6 = 5 (no remainder → exact division)
   42 ÷ 7 = 6 (no remainder → exact division)

2. Here are some examples of INEXACT division. There is a remainder.

   20 ÷ 3 = 6 R2 (remainder of 2 → inexact division)
   35 ÷ 6 = 5 R5 (remainder of 5 → inexact division)
   44 ÷ 5 = 8 R4 (remainder of 4 → inexact division)

3. When the division is exact, the divisor is called the FACTOR of the dividend.

   20 ÷ 5 = 4 (exact division) → **5** is a factor of **20**
   42 ÷ 7 = 6 (exact division) → **7** is a factor of **42**

   In fact, in exact division, the quotient also happens to be a factor of the dividend,

   20 ÷ 5 = 4 → **4 x 5 = 20** → both **4** and **5** are factors of **20**
   42 ÷ 7 = 6 → **6 x 7 = 42** → both **6** and **7** are factors of **42**

   NOTE: The FACTORS are described in greater detail in a later MILESTONE.

4. When the division is inexact, the remainder is a fraction of the divisor.

   (a) When we divide 1 by 2, we get "half." This is a quantity LESS THAN ONE. We call such a quantity a FRACTION (a broken part). Whenever we divide a number by a larger number we get fractions.

   (b) In inexact division, the remainder is less than the divisor. When we divide the remainder also by the divisor, we get a "fraction" in the quotient.

   5 ÷ 2 → **2 and remainder 1** → **2 and 1 ÷ 2 = 2 and 1/2**
   25 ÷ 3 → **8 and remainder 1** → **8 and 1 ÷ 3 = 8 and 1/3**
   35 ÷ 6 → **5 and remainder 5** → **5 and 5 ÷ 6 = 5 and 5/6**

   NOTE: The FRACTIONS are described in greater detail in a later MILESTONE.

☺ Exercise A5.5

1. State if the following division is EXACT or INEXACT.

   (a) 9 ÷ 3 (d) 17 ÷ 5 (g) 20 ÷ 4
   (b) 9 ÷ 2 (e) 21 ÷ 4 (h) 20 ÷ 3
   (c) 6 ÷ 2 (f) 28 ÷ 7 (i) 12 ÷ 4

2. Answer the following

   (a) Is 4 a factor of 13? (d) Is 7 a factor of 49? (g) Is 6 a factor of 48?
   (b) Is 7 a factor of 35? (e) Is 6 a factor of 56? (h) Is 13 a factor of 91?
   (c) Is 5 a factor of 28? (f) Is 8 a factor of 56? (i) Is 15 a factor of 130?

3. In the following divisions divide the remainder also.

   (a) 8 ÷ 3 (d) 16 ÷ 5 (g) 20 ÷ 3
   (b) 9 ÷ 4 (e) 31 ÷ 7 (h) 19 ÷ 6
   (c) 7 ÷ 2 (f) 25 ÷ 6 (i) 24 ÷ 5

Answer: **1.** (a) exact (b) inexact (c) exact (d) inexact (e) inexact (f) exact (g) exact (h) inexact
(i) exact **2.** (a) no (b) yes (c) no (d) yes (e) no (f) yes (g) yes (h) yes (i) no
3. (a) 2 2/3 (b) 2 1/4 (c) 3 1/2 (d) 3 1/5 (e) 4 3/7 (f) 4 1/5 (g) 6 2/3 (h) 3 1/5 (i) 4 4/5

Lesson A5.6 Division by columns

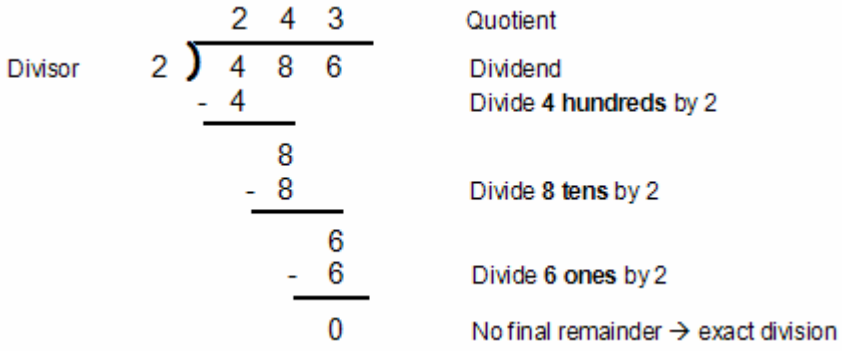
We divide the columns from left to right. When the division is not exact, the remainder is combined with the digit from the next column on the right.

1. We may divide large numbers by breaking them into ones, tens, hundreds, etc.

   Divide, **486 ÷ 2**

   486 ÷ 2 = (400 + 80 + 6) ÷ 2
   = (400 ÷ 2) + (80 ÷ 2) + (6 ÷ 2)
   = 200 + 40 + 3
   = 243

   We may follow the same procedure by using columns as follows.

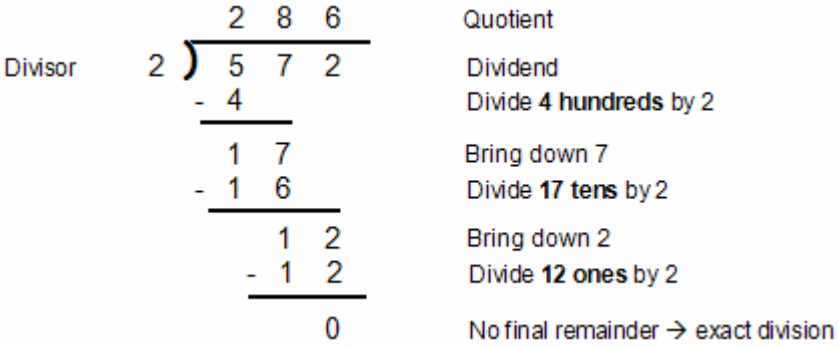


   The steps are:
   (a) For 100's: 4 divided by 2 is 2; place 2 hundreds in the quotient. Bring down the 8 tens.
   (b) For 10's: 8 divided by 2 is 4; place 4 tens in the quotient. Bring down the 6 ones.
   (c) For 1's: 6 divided by 2 is 3; place 3 ones.
   (d) The quotient is 2 hundreds, 4 tens and 3 ones or, 243.
   (e) There is no remainder meaning the division is exact.

   To verify if the division is correct, multiply the quotient by the divisor. The product should be the same as the dividend. **243 x 2 = 486**

2. Combine any remainder with the digit from the next column on the right.

   Divide, **572 ÷ 2**



The steps are:

(a) For 100's: 2 goes in 5, 2 times as 4; put down 2 hundreds in quotient. Subtract to get the remainder, 1 hundred.

(b) For 10's: Bring down 7 making it 17 tens. 2 goes in 17, 8 times as 16; put down 8 tens. Subtract to get the remainder, 1 ten.

(c) For 1's: Bring down 2 making it 12 ones. 2 goes in 12, 6 times as 12; put down 6 ones. There is no final remainder.

The quotient is **286.** There is no remainder meaning the division is exact. This quotient may be verified as, **286 x 2 = 572**

Divide, **617 ÷ 4**

```
    1 5 4      Quotient
  ---------
4 ) 6 1 7
  - 4
  -----
    2 1
  - 2 0
  -------
      1 7
    - 1 6
    -----
        1      Remainder
```

The division is inexact. This quotient may be verified as, **154 x 4 + 1 = 617**

3. The division may be done mentally using the short form.

Divide, **486 ÷ 2**

```
              2 4 3      Quotient
           ----------
Divisor  2 | 4 8 6      Dividend
```

The steps are (from left to right):

(a) 2 goes into 4, 2 times. Write 2 in the quotient. There is no remainder.

(b) 2 goes into 8, 4 times. Write 4 in the quotient. There is no remainder.

(c) 2 goes into 6, 3 times. Write 3 in the quotient. There is no remainder.

(d) The quotient is 243.

Divide, **572 ÷ 2**

```
              2 8 6      Quotient
           ----------
Divisor  2 | 5 1 7 1 2    Dividend
```

The steps are (from left to right):

(a) 2 goes into 5, 2 times. Write 2 in the quotient. Move remainder 1 in front of 7, making it 17.

(b) 2 goes into 17, 8 times. Write 8 in the quotient. Move remainder 1 in front of 2, making it 12.

(c) 2 goes into 12, 6 times. Write 6 in the quotient. There is no remainder.

(d) The quotient is 286.

4. Some special cases of division:

(a) When the first digit of the dividend cannot be divided, then start the division with the first two digits of the dividend. For example, compute **564 ÷ 6**

Long Form

```
      9  4
   ------
6 ) 5  6  4
    - 5  4
    ------
       2  4
     - 2  4
     ------
          0
```

Short Form

```
      9  4
   ------
6 | 5  6 24
```

The quotient is **94**. Verification: **94 x 6 = 564**.

(b) When a number in column cannot be divided, put a 0 in the quotient for that column. For example, computes 723 ÷ 7

Long Form

```
      1  0  3      Quotient
   ---------
7 ) 7  2  3
    - 7
    ------
       2
     - 0
     ------
       2  3
     - 2  1
     ------
          2       Remainder
```

Short Form

```
      1  0  3      R2
   ---------
7 | 7 02 23
```

The quotient is **103 R2**. Verification: **103 x 7 + 2 = 723**.

☺ Exercise A5.6

1. Compute the following using long form. Verify the quotient as above.

(a) 435 ÷ 5	(f) 826 ÷ 9	(k) 222 ÷ 4
(b) 324 ÷ 4	(g) 800 ÷ 5	(l) 876 ÷ 8
(c) 450 ÷ 6	(h) 315 ÷ 7	(m) 123 ÷ 3
(d) 325 ÷ 7	(i) 384 ÷ 6	(n) 735 ÷ 7
(e) 327 ÷ 3	(j) 612 ÷ 6	(o) 888 ÷ 9

2. Find the quotient using short form. Verify it as above.

(a) 363 ÷ 3	(g) 484 ÷ 4	(m) 309 ÷ 3	(s) 2244 ÷ 2
(b) 248 ÷ 2	(h) 936 ÷ 3	(n) 486 ÷ 2	(t) 5005 ÷ 5
(c) 444 ÷ 4	(i) 624 ÷ 2	(o) 844 ÷ 4	(u) 3996 ÷ 3
(d) 57 ÷ 3	(j) 345 ÷ 3	(p) 894 ÷ 6	(v) 8088 ÷ 8
(e) 75 ÷ 5	(k) 732 ÷ 6	(q) 372 ÷ 3	(w) 6072 ÷ 6
(f) 84 ÷ 7	(l) 847 ÷ 7	(r) 992 ÷ 8	(x) 9945 ÷ 5

3. A profit of $36,363 is to be divided equally among 6 business partners. How much does each partner receive?

4. If 7 computers cost $13,965 what is the cost per computer?

Answer: **1.** (a) 87 (b) 81 (c) 75 (d) 46 R3 (e) 109 (f) 91 R7 (g) 160 (h) 45 (i) 64 (j) 102
(k) 55 R2 (l) 109 R4 (m) 41 (n) 105 (o) 98 R6 **2.** (a) 121 (b) 124 (c) 111 (d) 19
(e) 15 (f) 12 (g) 121 (h) 312 (i) 312 (j) 115 (k) 122 (l) 121 (m) 103 (n) 243 (o) 211
(p) 149 (q) 124 (r) 124 (s) 1122 (t) 1001 (u) 1332 (v) 1011 (w) 1012 (x) 1989
3. 6060 and ⅔ **4.** $1995

Lesson A5.7 Division by large divisors

When the divisor is large, one may divide just the larger place values to narrow down to the quotient.

1. Write down the first ten multiples of the divisor using repeated addition.

Divide, **10853 ÷ 16**

Write down the multiplication table for 16 using repeated addition.

16 times →	1	2	3	4	5	6	7	8	9	10
	16	**32**	**48**	**64**	**80**	**96**	**112**	**128**	**144**	**160**

Then we can divide as before,

Long Form

```
          6  7  8
      ------------
16 ) 1  0  8  5  3
   -    9  6
      ---------
         1  2  5
      -  1  1  2
      ---------
            1  3  3
         -  1  2  8
         ---------
               5
```

Short Form

```
          6  7  8    R5
      ------------
16 ) 1  0  8  5  3
      12    13
```

The steps are:

(a) 16 cannot be taken out of the first two digits. Try taking it out the first 3 digits.

(b) 16 can be taken out of 108 six times (16 x 6 = 96). Place 6 in the column above 8. Subtract 96 from 108. The remainder is 12. Bring down the next digit 5. We get 125.

(c) 16 can be taken out of 125 seven times (16 x 7 = 112). Place 7 in the column above 5. Subtract 112 from 125. The remainder is 13. Bring down the next digit 3. We get 133.

(d) 16 can be taken out of 133 eight times (16 x 8 = 128). Place 8 in the column above 3. Subtract 128 from 133. The remainder is 5. There are no more digits.

(e) Therefore, the quotient is 678, and there is a remainder of 5.

10853 ÷ 16 = 678 R5

(f) Verify the quotient: **678 x 16 + 5 = 10853.**

2. When the divisor is very large, approximate the quotient as follows.

Divide, **563 ÷ 62**

Divide the TENS only as **56 ÷ 6**. This gives us 9 and remainder, so we check,
62 x 9 = 558, and **62 x 10 = 620**
We can take 558 out of 563 leaving a remainder of 5.
Therefore, **563 ÷ 62 = 9 R5**

Divide, **396 ÷ 44**

Divide the TENS, **39 ÷ 4 = 9 and remainder**
Check, **44 x 9 = 396**
Therefore, **396 ÷ 44 = 9**

Divide, **611 ÷ 87**

Divide the nearest TENS, **61 ÷ 9 = 6 and remainder**
Check, **87 x 6 = 522**, and **87 x 7 = 609**
Therefore, **611 ÷ 87 = 7 R2**

Divide, **630526 ÷ 37**

$$\begin{array}{r} 17041\ R9 \\ 37\overline{)\,6\,3\,0\,5\,2\,6} \\ \qquad\ \ {\scriptstyle 26}\ \ {\scriptstyle 1}\ {\scriptstyle 15}\ {\scriptstyle 4} \end{array}$$

The steps are:

(a) For 63 ÷ 37 check nearest TENS as 6 ÷ 4 = 1 and remainder. Check 37 x 1 = 37; 37 x 2 = 74. This gives us **63 ÷ 37 = 1 R 26**. Place 1 below 3, and 26 in front of 0.

(b) For 260 ÷ 37 check nearest TENS as 26 ÷ 4 = 6 and remainder. Check 37 x 6 = 222, 37 x 7= 259. This gives us **260 ÷ 37 = 7 R1**. Place 7 below 0, and 1 in front of 5.

(c) For 15 ÷ 37 you obviously get **0 R15**. Place 0 below 5, and 15 in front of 2.

(d) For 152 ÷ 37 check nearest TENS as 15 ÷ 4 = 3 and remainder. Check 37 x 3 = 111, 37 x 4 = 148. This gives us **152 ÷ 37 = 4 R4**. Place 4 below 2, and the remainder 4 in front of 6.

(e) We get **46 ÷ 37 = 1 R9**. Place 1 below 2. There are no more digits, so the remainder is 9.

(f) The quotient is **17041 R9.**

(g) Verify: **17041 x 37 + 9 = 630526**

☺ Exercise A5.7

1. Divide by first writing down the multiples of the divisor.

(a) 108 ÷ 12	(d) 3225 ÷ 12	(g) 976 ÷ 11
(b) 911 ÷ 11	(e) 1111 ÷ 11	(h) 322 ÷ 23
(c) 432 ÷ 14	(f) 4555 ÷ 25	(i) 3567 ÷ 18

2. Divide by approximating the quotient.

(a) 144 ÷ 16	(d) 217 ÷ 45	(g) 7488 ÷ 61
(b) 123 ÷ 23	(e) 318 ÷ 53	(h) 82593 ÷ 71
(c) 259 ÷ 37	(f) 419 ÷ 59	(i) 994720 ÷ 89

Answer: **1.** (a) 9 (b) 82 R9 (c) 30 R12 (d) 268 R9 (e) 101 (f) 182 R5 (g) 88 R8 (h) 14 (i) 198 R3
2. (a) 9 (b) 5 R8 (c) 7 (d) 4 R37 (e) 6 (f) 7 R6 (g) 122 R46 (h) 1163 R20 (i) 11176 R56

Lesson A5.8 Word Problems

Word problems give meaning to dividend, divisor, quotient and remainder.

1. Convert 26,623 seconds to hours, minutes, and seconds.

   We divide the seconds by 60 to convert them to minutes.

   26,623 ÷ 60 = 443 R43 → 443 minutes and 43 seconds

   We divide minutes by 60 to convert them to hours.

   443 ÷ 60 = 7 R23 → 7 hours and 23 minutes

   Therefore.

   26,623 seconds = 7 hours, 23 minutes, and 43 seconds

2. Convert 5,265 inches to yards, feet, and inches.

   We divide inches by 12 to convert them to feet.

   5,265 ÷ 12 = 438 R9 → 438 feet and 9 inches

   We divide feet by 3 to convert them to yards.

   438 ÷ 3 = 146 → 146 yards

   Therefore.

   5,265 inches = 146 yards, and 9 inches

3. If 6 apples cost 56 cents, how much would 9 apples cost?

   First find out how much one apple costs. Then you can find out how much 9 apples would cost.

   NOTE: You may translate word problem to mixed operations first before solving it.

   Cost of 6 apples = 56 cents
   Therefore,
   Cost of 1 apple = 56 ÷ 6
   Cost of 9 apples = (56 ÷ 6) x 9
   = 56 ÷ 6 x 9
   = $\frac{56 \times 9}{6}$
   = 84 cents

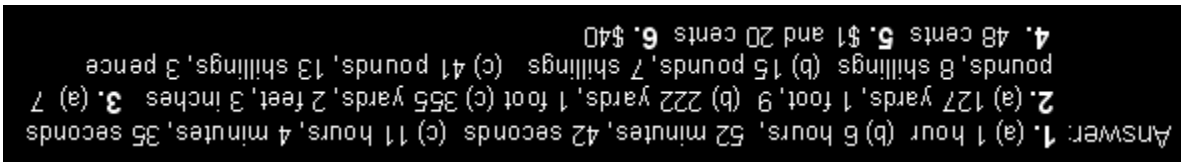
4. Joe, Bob and Lisa shared a prize of $108 equally among themselves. Lisa then shared her money equally with her two sisters. How much money did Lisa get?

   First find out how much Lisa got initially to share with her sisters. Then you can find out how much Lisa got after sharing the prize with her sisters.

   Lisa first got (dollars) = 108 ÷ 3
   After sharing with her sisters, Lisa got
   = (108 ÷ 3) ÷ 3
   = 108 ÷ 3 ÷ 3
   = (108) ÷ (3 x 3)
   = $\frac{108}{3 \times 3}$
   = 12 dollars

☺ Exercise A5.8

1. Convert the following to hours, minutes, and seconds.
   (a) 3,600 seconds (b) 24,762 seconds (c) 39,875 seconds
2. Convert the following to yards, feet, and inches.
   (a) 4,593 inches (b) 8004 inches (c) 12,807 inches
3. Convert the following to pounds, shillings and pence (Note that 12 pence make a shilling, and 20 shillings make a pound.)
   (a) 1,776 pence (b) 3,684 pence (c) 9,999 pence
4. You bought 5 bananas for 30 cents. How much will 8 bananas cost?
5. You bought 14 pencils for 80 cents. How much will 21 pencils cost?
6. You won a lottery of $600, which you shared with 4 of your friends. You then shared your winnings equally with your brother and sister. How much money did you end up with?



SUMMARY

Division is the opposite of multiplication. If you already know the various techniques of multiplication, you can learn to divide easily.

The operation of division computes how many times a quantity (**divisor**) can be taken out of another quantity (**dividend**). That computation provides the **quotient**.

When the division is exact, that is to say, the divisor can be taken out of the dividend an exact number of times with no remainder, the divisor is called a **factor** of the dividend.

When the division is not exact, a remainder is left after division. When the remainder is further divided by the divisor into portions less than a unit, then we get **fractions**.

Both factors and fractions are taken up in subsequent milestones.

A proper understanding of division helps one round up all the earlier concepts in math. By the time one completes the Elementary School, one should have developed the ability of divide mentally. This understanding then forms the basis of middle school math.

DIAGNOSTIC TEST

1. Mangoes cost $3 each. How many mangoes can you buy for $24?
2. How many times can you take 7 away from 56?
3. Divide the following mentally.
   (a) 12 ÷ 3 (b) 63 ÷ 7 (c) 72 ÷ 9 (d) 102 ÷ 17
4. Use the basic rules of division to answer the following.
   (a) 5 ÷ 5 (b) 5 ÷ 1 (c) 0 ÷ 5 (d) 5 ÷ 0
5. Fill in the blank using the knowledge of multiplication
   (a) 3 x __ = 12 (b) 7 x __ = 63 (c) 9 x __ = 72 (d) 17 x __ = 102
6. State the quotient and the remainder.
   (a) 8 ÷ 3 (b) 16 ÷ 5 (c) 18 ÷ 6 (d) 23 ÷ 7
7. State if the division is exact or inexact.
   (a) 9 ÷ 3 (b) 17 ÷ 5 (c) 20 ÷ 4 (d) 9 ÷ 2
8. State if the divisor is a factor of the dividend.
   (a) 17 ÷ 3 (b) 63 ÷ 7 (c) 72 ÷ 9 (d) 100 ÷ 17
9. In the following inexact divisions convert the remainder into a fraction in the quotient.
   (a) 9 ÷ 4 (b) 19 ÷ 6 (c) 24 ÷ 5 (d) 31 ÷ 7
10. Divide using the long form of division.
    (a) 725 ÷ 6 (b) 6243 ÷ 7 (c) 84425 ÷ 5 (d) 243618 ÷ 3
11. Divide using the short form of division.
    (a) 612 ÷ 6 (b) 3437 ÷ 7 (c) 54321 ÷ 5 (d) 700312 ÷ 3
12. Divide the following.
    (a) 563 ÷ 16 (b) 396 ÷ 23 (c) 7611 ÷ 62 (d) 630526 ÷ 87
13. If you cut a 56-inch long string into eight equal parts, what is the length of each part?
14. If 6 pens cost $16, how much will 9 pens cost?

Answer: **1.** 8 **2.** 8 **3.** (a) 4 (b) 9 (c) 8 (d) 6 **4.** (a) 1 (b) 5 (c) 0 (d) undefined **5.** (a) 4 (b) 9 (c) 8 (d) 6 **6.** (a) 2 R2 (b) 3 R1 (c) 3 R0 (d) 3 R2 **7.** (a) exact (b) inexact (c) exact (d) inexact **8.** (a) no (b) yes (c) yes (d) no **9.** (a) 2 and ¼ (b) 3 and 1/6 (c) 4 and 4/5 (d) 4 and 3/7 **10.** (a) 120 R5 (b) 891 R6 (c) 16885 (d) 81206 **11.** (a) 102 (b) 491 (c) 10864 R1 (d) 23437 R1 **12.** (a) 35 R3 (b) 17 R5 (c) 122 R47 (d) 7247 R37 **13.** 7 inches **14.** $24

GLOSSARY

[For additional words refer to the glossaries at the end of earlier Milestones]

Dividend — A **dividend** is the quantity that is being divided. In the expression "30 ÷ 6" the dividend is 30. The word DIVIDEND comes from the word *dvidere*, which means "to separate apart."

Division — **Division** means "to separate into portions and give out in shares." Division involves subtracting the divisor repeatedly from the dividend. It is the reverse of multiplication.

Division Sign — The sign **"÷"** when placed between two numbers, expresses that the first number is to be divided by the second number.

Divisor — A **divisor** denotes the number of shares the dividend is to be separated into. It is the quantity that is taken out of the divisor repeatedly. In the expression "30 ÷ 6" the divisor is 6.

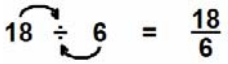
Exact Division — When the divisor can be taken out of the dividend an exact number of times the division is exact. In **exact division** there is no remainder.

Expression — An arithmetic **expression** describes operations on numbers. For example, "3 x 4 – 2 + 12 ÷ 6" is an arithmetic expression that reduces to a numeric value.

Factor — A **factor** is a divisor of a number in exact division. For example, 7 is a factor of 28 because 28 ÷ 7 = 4 (exact division). The other factor would be 4 because 7 x 4 = 28 and 28 ÷ 4 = 7. The word FACTOR comes from a Latin word meaning "doer, maker, performer." For more on fractions, see Milestone 7.

Fraction — The word FRACTION comes from a word, which means "a broken piece." A **fraction** shows how a smaller part compares to the whole in terms of size or quantity. For more on fractions, see Milestone 8.

"Fraction" Form — The "fraction" form shows the dividend above a line and the divisor below that line.



Inexact Division — When there is a remainder left after taking the divisor out of the dividend as many times as possible, the division is inexact. In **inexact division** the remainder is not zero but it is always less than the divisor.

Long Form — the **long form** is the form of division used in "Division by columns" (see Section 6.14).

Quotient — The **quotient** is the result of division. It shows how many times a divisor can be taken out of the dividend. The word QUOTIENT comes from a Latin word meaning "how many times."

Remainder	**Remainder** is the number, or quantity, that is left over after subtraction or division.
Short Form	the **short form** is the form of division used to assist "mental division" (see Section 6.16).
Term	A **term** is that part of an expression that consists entirely of multiplication and division. Multiple terms in an expression are separated by addition and subtraction.
Word Problem	A **word problem** is statement of a problem in English that may be solved numerically by translating it first into the language of mathematics.