MATH MILESTONE # A3

SUBTRACTION

The word, **milestone,** means "a point at which a significant change occurs." A Math Milestone refers to a significant point in the understanding of mathematics.

To reach this milestone one should be able to subtract single- and double-digit numbers mentally without effort.

A simple ABACUS, as described in Milestone #1, shall be required. Alternately, you may access the *VIRTUAL ABACUS* at *http://www.mathfundamentals.org/abacus.htm* for the purpose of this milestone. Also have some pennies, paper and pencil, and a calculator at hand.

Please consult the **Glossary** supplied with this Milestone for mathematical terms. Consult a regular dictionary at **www.dictionary.com** for general English words that one does not understand fully.

You may start with the Diagnostic Test on the next page to assess your proficiency on this milestone. Then continue with the lessons with special attention to those, which address the weak areas.

NOTE: The process of subtraction involves taking a quantity away from another. The quantity remaining after subtraction was earlier referred to as "remainder". This term **remainder** *is also used for the quantity left after division. This is because division is "repeated subtraction" (see Section A3.4, #4 on page 9).*

The quantity remaining after subtraction is also equal to the **difference** *between the two quantities involved (see below). Therefore, to prevent any confusion, this text uses the term* **difference** *for the quantity remaining after subtraction, and the term* **remainder** *for the quantity remaining after division.*

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DIAGNOSTIC TEST

1. Find the difference.
   (a) 8 – 7
   (b) 5 – 3
   (c) 7 – 4
   (d) 12 – 9
   (e) 9 – 5
   (f) 15 – 8
   (g) 15 – 13
   (h) 30 – 25
   (i) 71 – 67

2. Find the shortage.
   (a) 7 – 8
   (b) 3 – 5
   (c) 4 – 7
   (d) 9 – 12
   (e) 5 – 9
   (f) 8 – 15
   (g) 13 – 15
   (h) 25 – 30
   (i) 67 – 71

3. Find the difference by counting from the smaller to the larger number.
   (a) 7 and 8
   (b) 3 and 5
   (c) 4 and 7
   (d) 9 and 12
   (e) 5 and 9
   (f) 8 and 15
   (g) 13 and 15
   (h) 25 and 30
   (i) 67 and 71

4. Subtract the following on abacus.
   (a) 28 – 7
   (b) 37 – 6
   (c) 25 – 7
   (d) 44 – 8
   (e) 48 – 32
   (f) 55 – 23
   (g) 67 – 48
   (h) 72 – 56
   (i) 83 – 78

5. Subtract the following mentally.
   (a) 28 – 5
   (b) 35 – 4
   (c) 61 – 7
   (d) 51 – 47
   (e) 73 – 65
   (f) 85 – 79
   (g) 74 – 28
   (h) 57 – 25
   (i) 95 – 59

6. Subtract the following using columns.
   (a) 5,183 – 2,975
   (b) 44,731 – 17,842
   (c) 74,111 – 37,555
   (d) 600,000,000 – 523,987,999
   (e) 4,001,030,352 – 1,946,327,115
   (f) 5,082,359,777 – 3,193,461,857

Answer: **1.** (a) 1 (b) 2 (c) 3 (d) 3 (e) 4 (f) 7 (g) 2 (h) 5 (i) 4 **2.** (a) –1 (b) –2 (c) –3 (d) –3 (e) –4 (f) –7 (g) –2 (h) –5 (i) –4 **3.** (a) 1 (b) 2 (c) 3 (d) 3 (e) 4 (f) 7 (g) 2 (h) 5 (i) 4 **4.** (a) 21 (b) 31 (c) 18 (d) 36 (e) 16 (f) 32 (g) 19 (h) 16 (i) 5 **5.** (a) 23 (b) 31 (c) 54 (d) 4 (e) 8 (f) 6 (g) 46 (h) 32 (i) 36 **6.** (a) 2,208 (b) 26,889 (c) 36,556 (d) 76,012,001 (e) 2,054,703,237 (f) 1,888,897,920

LESSONS

Lesson A3.1   Subtraction and Difference

When we take a quantity away from another we have SUBTRACTION. The quantity remaining after subtraction is called the DIFFERENCE.

1. The word SUBTRACT comes from a word, *subtrahere*, which means, "drawn from under."

    (a) When we take some toys out from the storage box to play with, we are subtracting toys from the storage box. The quantity remaining is rest of the toys still in the box.

    (b) When we drink water, we are subtracting water from the glass. The quantity remaining is the water left in the glass.

    (c) When we spend money to purchase things, we are subtracting money from our account. The quantity remaining is what is left in the account.

2. We practice subtraction daily as follows.

    (a) When we take 2 pennies away from 5 pennies, we get 3 pennies as left over.

"The quantity remaining is 3 pennies."

This quantity remaining is equal to the **difference** between the two quantities.

"The DIFFERENCE is 3 pennies."

    (b) John had 8 apples; he gave 4 of them to his brother. He now has 4 apples left. This is the **difference** after subtraction.

    (c) Peter played at marbles. He had 23 when he began, but when he was done he had only 12. He lost 11 marbles. 12 is the **difference** after losing some marbles.

3. Please note that only the quantities of the same unit may be subtracted. To subtract quantities of different units, we must first express them in a common unit.

    (a) We may subtract 2 apples from 5 apples. The difference is 3 apples.

(b) But we cannot subtract 2 peaches from 5 apples. To do so, we must express both peaches and apples as fruits. We may then subtract 2 fruits from 5 fruits.

4. In general, when we subtract 2 counts from 5 counts of anything we are left with 3 counts of that thing. This makes it possible to carry out subtraction using fingers, abacus, or just mentally.

☺ Exercise A3.1

1. Subtract to find the difference.
   (a) 2 marbles from 9 marbles   (b) 3 quarters from 8 quarters   (c) 4 feet from 8 feet

2. A boy, being asked how old be was, said that he was 25 years younger then his father, whose age was 33 years; how old was the boy?

3. If you don't have cats how you would you know the number of cars left when 5 cats out of 9 cats have ran away?

Lesson A3.2   Terms and Definitions

Subtraction is counting less from a number.

1. To subtract 3 from 12, start from 12 and count 3 less (11, 10, 9). The difference is 9.

(3 less counts from 12)

2. We write subtraction with a minus sign (–) in front of the number being subtracted. For example, if we are subtracting 3 counts from 12, we may express it as follows.

$$12 \ - \ 3 \ = \ 9$$

   (a) "**–**" is the **minus sign** for subtraction. "**=**" is the **equals sign**.
   (b) We read the above expression as, "**12 minus 3 equals 9.**" The word "minus" means *less*.
   (c) The **units** of all numbers above are assumed to be the same.
   (d) "**12**" is the first number called the **minuend** (the number to be made smaller).
   (e) "**3**" is the second number called the **subtrahend** (the number to be taken away).
   (f) "**9**" is the result of subtraction called the **difference**, which remains.

3. We can subtract smaller number from a larger number. But when we try to subtract a larger number from a smaller number, we run into a **shortage** to be compensated from elsewhere.

4. Therefore, to subtract we may start with the minuend and count as many less as the subtrahend.

   (a) To subtract 5 from 9, we start from 9 and count 5 less (8, 7, 6, 5, 4). The difference is what remains as 4.
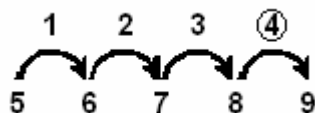

(5 less counts from 9)
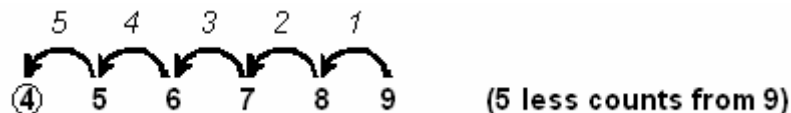
(b) This method is helpful when the number to be subtracted is small, as when we subtract 5 from 72. The difference is what remains as 67.
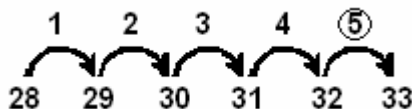

67　68　69　70　71　72

5. Alternatively, we may count the difference directly from the subtrahend to the minuend.

(a) To subtract 5 from 9, we start from 5 and count up to 9 (6, 7, 8, 9). There are 4 counts. This is the difference.

1　2　3　④
5　6　7　8　9

(b) This method is helpful when the difference is small, as when we subtract 28 from 33. The difference is counted directly as 5.

1　2　3　4　⑤
28　29　30　31　32　33

6. Some simple word problems require subtraction only:

(a) *John bought 17 books, and read 5 of them; how many books are still unread?*
This is clearly a subtraction problem. We start from 17 and count 5 less. There are 12 books remaining to be read.

(b) *Charles is 19 years old, and Andrew is 13; what is the difference in their ages?*
This is clearly a subtraction problem. We start from 13 and count up to 19. There are 6 counts. Therefore, the difference in ages is 6 years.

☺ Exercise A3.2

1. Write down "8 minus 3 equal 5" using mathematical symbols.

2. Identify the minuend, subtrahend and the difference in the following subtractions.
   (a) 12 – 3 = 9 　　　(b) 28 – 23 = 5 　　　(c) 45 – 37 = 8

3. Which of the numbers 3 and 16 would you subtract from the other mentally? Why?

4. Subtract the following by "counting less" or by "counting the difference."
   (a) 78 – 3 　　(e) 49 – 4 　　(i) 67 – 8 　　(m) 87 – 5 　　(q) 48 – 8
   (b) 74 – 69 　(f) 38 – 32 　(j) 44 – 38 　(n) 75 – 69 　(r) 48 – 40
   (c) 54 – 5 　　(g) 95 – 5 　　(k) 43 – 5 　　(o) 17 – 3
   (d) 65 – 60 　(h) 87 – 79 　(l) 72 – 65 　(p) 17 – 13

5. Solve the following using subtraction.
   (a) John has 9 hamsters. 5 of them ran away. How many hamsters are left?
   (b) Billy caught 13 butterflies during the week. 4 of these flew away. How many butterflies were left?

　　(01/01/2008)

Lesson A3.3   Subtracting on Abacus

Subtraction is moving beads to the left on the wire of abacus.

NOTE: Obtain a simple ABACUS, as described in Milestone #1. Alternately, you may access the VIRTUAL ABACUS at http://www.mathfundamentals.org/abacus.htm for the purpose of this milestone.

1. To subtract **3** from **8**, start with 8 beads on the right on the first wire. Then count 3 beads to the left. The difference is the 5 beads remaining on the right.
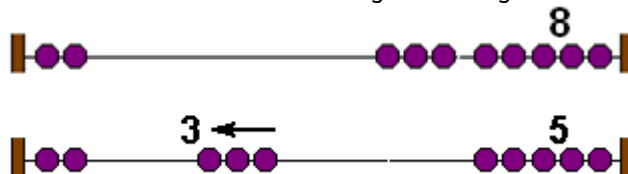
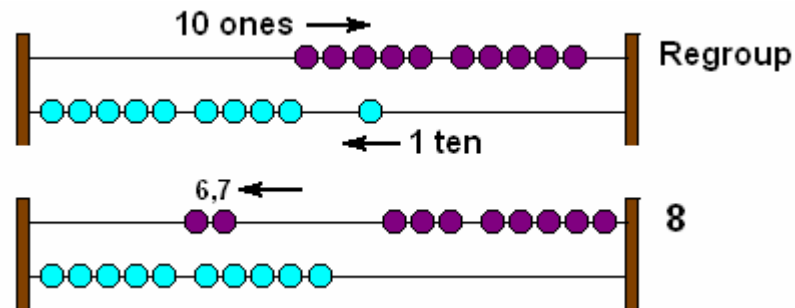2. To subtract **23** from **38**, start with the count of 38 as 3 TENS and 8 ONES to the right. Then count back 23 as 2 TENS and 3 ONES to the left. The difference is 15.
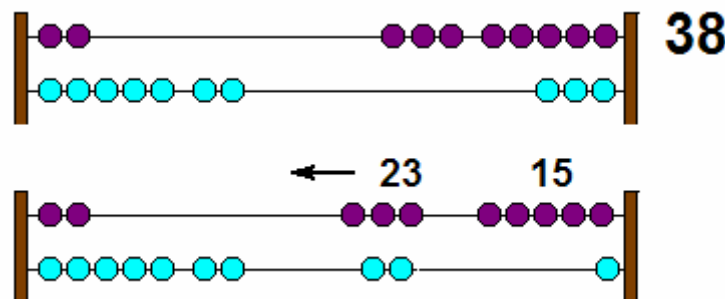
3. When more beads need to be taken away than available on the first wire (ONES), then exchange 1 TEN on the second wire for 10 ONES on the first wire. This is called REGROUPING. To subtract **7 from 15** we use regrouping as follows.
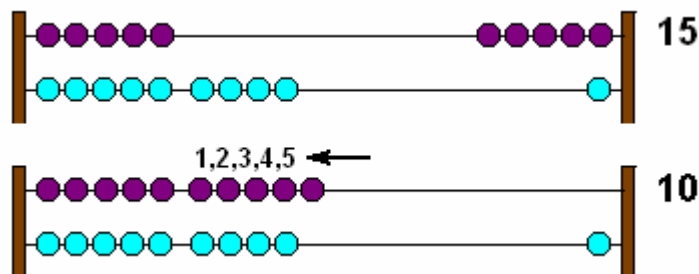
The difference may be read from the beads on the right as **8**.

$$15 - 7 \quad = \quad 15 - 5 - 2 \quad = \quad 8$$

4. On abacus, we subtract TENS first, and then the ONES. To subtract 59 from 84, first take away 50, and then take away 9.
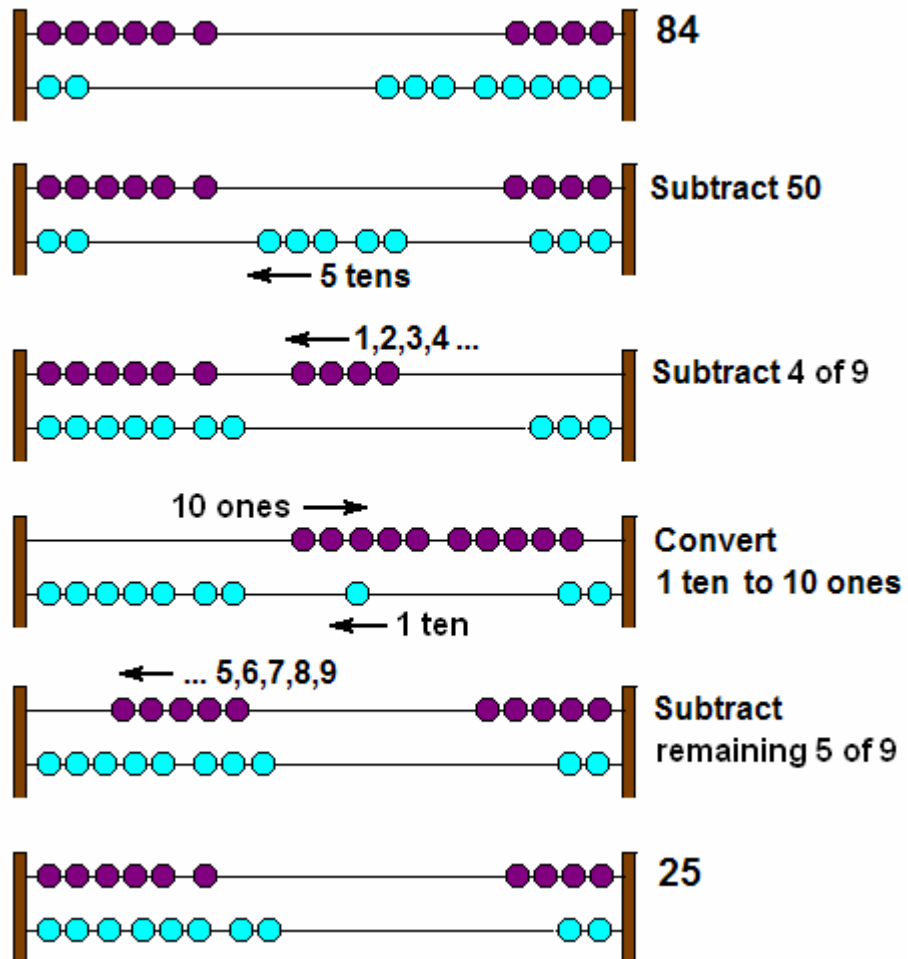
The difference may be read from the beads on the right as **25**.

$$84 - 59 \quad = \quad 84 - 50 - 4 - 5 \quad = \quad 25$$

☺ Exercise A3.3

1. Subtract on abacus.

(a) 3 from 9	(g) 35 from 65	(m) 47 from 74
(b) 5 from 8	(h) 42 from 85	(n) 33 from 51
(c) 4 from 9	(i) 71 from 96	(o) 35 from 64
(d) 21 from 31	(j) 9 from 14	(p) 57 from 91
(e) 44 from 56	(k) 5 from 13	(q) 49 from 85
(f) 35 from 77	(l) 7 from 11	(r) 77 from 96

2. Subtract on abacus. Check the result on a calculator.

(a) 7 – 4	(g) 47 – 24	(m) 44 – 8
(b) 8 – 5	(h) 77 – 38	(n) 48 – 32
(c) 19 – 6	(i) 63 – 47	(o) 55 – 23
(d) 13 – 7	(j) 28 – 7	(p) 67 – 48
(e) 33 – 8	(k) 37 – 6	(q) 72 – 56
(f) 39 – 15	(l) 25 – 7	(r) 83 – 78

Lesson A3.4   Mental Subtraction

We may use visualization of Number Line to subtract mentally.

1. (a) We may visualize counting back on a number line as follows.

   <u>Subtract 3 from 8</u>: Start from 8 and count back 3,   8 – 3   =   5
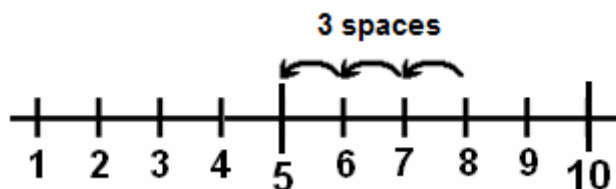
   <u>Subtract 3 from 38</u>: Start from 38 and count back 3:   38 – 3   =   35

   (b) When counting across TEN, we count back in two moves stopping at TEN in between as we did on abacus.

   <u>Subtract 8 from 15</u>:
   Start from 15 and count back 5 to 10, then count the remaining 3 back to 7:
      15 – 8   =   15 – 5 – 3   =   10 – 3   =   7

Subtract 8 from 45

Start from 15 and count back 5 to 10, then count the remaining 3 back to 7:

  45 – 8   =   45 – 5 – 3   =   40 – 3   =   37

(c) We may count back across many TENS as follows.

Subtract 43 – 26.

  43 – 26   =   43 – 3 – 20 – 3   =   17

2. From the above we get the following rules for subtracting single-digit numbers.

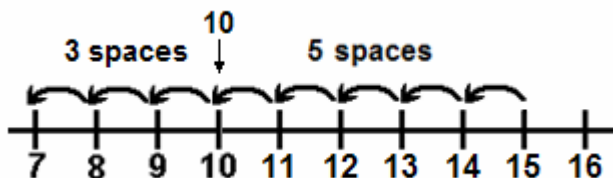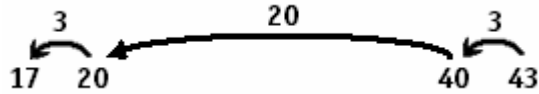(a) When ONES in subtrahend are less than ONES in minuend, we may add to TENS the difference of ONES.

Subtract 6 from 39

  39 – 6   =   30 + (9 – 6)   =   30 + 3   =   33

Similarly,

  44 – 3   =   40 + (4 – 3)   =   40 + 1   =   41
  87 – 5   =   80 + (7 – 5)   =   80 + 2   =   82

(b) When the ONES in subtrahend are greater than ONES in minuend, we may subtract from TENS the difference of ONES.

Subtract 6 from 33

  36 – 9   =   30 – (9 – 6)   =   30 – 3   =   27

Similarly,

  44 – 7   =   40 – (7 – 4)   =   40 – 3   =   37
  82 – 8   =   80 – (8 – 2)   =   80 – 6   =   74

3. To subtract a double-digit number, first subtract TENS, and then add or subtract the difference of ONES as above.

  34 – 16 =   (30 – 10) – (6 – 4)   =   20 – 2   =   18
  49 – 26 =   (40 – 20) + (9 – 6)   =   20 + 3   =   23
  33 – 24 =   (30 – 20) – (4 – 3)   =   10 – 1   =   9
  77 – 32 =   (70 – 30) + (7 – 2)   =   40 + 5   =   45

4. One may subtract a subtrahend repeatedly from the minuend until the difference is smaller than the subtrahend. This process is called division, and the difference remaining at the end is called the **remainder**.

(a) Subtract 2 from 10 repeatedly to see how many times it can be done.

  10 – 2   =   8
   8 – 2   =   6
   6 – 2   =   4
   4 – 2   =   2
   2 – 2   =   0

Therefore,   10 – 2 – 2 – 2 – 2 – 2   =   0

Or, 2 may be subtracted <u>five</u> times from 10, until the remainder is 0.

(b) Subtract 3 from 20 repeatedly. How many times can it be done?

     20 − 3 − 3 − 3 − 3 − 3 − 3   =   2

Or, 3 may be subtracted <u>six</u> times from 20, until the remainder is 2 (less than 3).

Note that the remainder is always less than what is being subtracted repeatedly.

☺ Exercise A3.4

1. Find the difference by mental subtraction.
    (a) 15 − 8   (d) 35 − 6   (g) 87 − 8
    (b) 27 − 9   (e) 44 − 8   (h) 73 − 7
    (c) 23 − 6   (f) 65 − 7   (i) 66 − 9

2. Find the difference by mental subtraction.
    (a) 42 − 3   (g) 32 − 8   (m) 84 − 5
    (b) 37 − 4   (h) 59 − 3   (n) 63 − 8
    (c) 89 − 8   (i) 71 − 3   (o) 82 − 2
    (d) 46 − 9   (j) 23 − 9   (p) 75 − 8
    (e) 73 − 9   (k) 66 − 6   (q) 44 − 8
    (f) 57 − 4   (l) 51 − 5   (r) 93 − 4

3. Find the difference by mental subtraction.
    (a) 45 − 18   (d) 35 − 16   (g) 87 − 58
    (b) 37 − 59   (e) 44 − 68   (h) 73 − 47
    (c) 53 − 26   (f) 65 − 47   (i) 66 − 89

4. Find the difference by mental subtraction.
    (a) 42 − 23   (g) 32 − 28   (m) 84 − 45
    (b) 37 − 14   (h) 59 − 53   (n) 63 − 38
    (c) 89 − 48   (i) 71 − 63   (o) 82 − 28
    (d) 46 − 69   (j) 23 − 39   (p) 75 − 87
    (e) 73 − 49   (k) 66 − 66   (q) 44 − 83
    (f) 57 − 24   (l) 51 − 15   (r) 93 − 47

5. How many times can you repeatedly subtract
    (a) 5 from 30?   Number of times = _____;   Remainder = _____
    (b) 7 from 50?   Number of times = _____;   Remainder = _____
    (c) 6 from 53?   Number of times = _____;   Remainder = _____

6. Have another person call out a double-digit and a single-digit number. Subtract the smaller from the larger number mentally. Repeat this exercise until you can do so effortlessly. *Do this exercise on a turn about basis, if possible.*

7. Have another person call out a double-digit and a single-digit number. Subtract the smaller from the larger number as many times as you can and tell the remainder. Repeat this exercise until you can do so effortlessly. *Do this exercise on a turn about basis, if possible.*

8. Have another person call out two double-digit numbers. Subtract the smaller from the larger number mentally. Repeat this exercise until you can do so effortlessly. *Do this exercise on a turn about basis, if possible.*

Lesson A3.5   Subtraction by column

We subtract by arranging the digits in columns by their place values, and subtracting the columns from right to left. Use the Rule of Abacus in reverse in regrouping.

1. When subtracting by columns on paper, start from the right-most column, and work your way to the left. For example, subtract 62 from 76 by columns.

   The mental steps are:
   (a) For 1's: 6 minus 2 equals 4, put down 4 **ones.**
   (b) For 10's: 7 minus 6 equals 1, put down 1 **ten.**
   (c) The <u>difference</u> is 14.

	tens	ones	
	7	6	minuend
−	6	2	subtrahend
	1	4	difference

2. When the subtrahend at the bottom is larger, then regroup a TEN from left to increase the minuend at the top.

   <u>Subtract 49 from 82</u>:

   (a) For 1's: We cannot take away 9 from 2. Regroup 1 ten as 10 ones. You now have 12 ones. 12 minus 9 equal 3; put down 3.
   (b) For 10's: 7 tens are left; 7 minus 4 equal 3; put down 3.
   (c) The <u>difference</u> is 33.

	7	12	regroup
	8̶	2̶	minuend
−	4	9	subtrahend
	3	3	difference

3. We regroup from the column on the left as necessary.

   <u>Subtract 387 from 623</u>:

   ```
      1  13                5  11
     6  2̶  3̶               4̶  13
                    →    6̶  2̶  3̶
   − 3  8  7            − 3  8  7
   ─────────            ─────────
          6              2  3  6
   ```

   (a) For 1's: 7 is more than 3; regroup 1 from 10's column to have 13 ones; 13 minus 7 equal 6, put down 6.
   (b) For 10's: 1 is left in 10's column; regroup 1 from 100's column to have 11 tens; 11 minus 8 equal 3, put down 3.
   (c) For 100's: 5 is left in 100's column; 5 minus 3 equal 2; put down 2.
   (d) The <u>difference</u> is 236.

4. To check for the correctness of the answer, add the difference to the subtrahend. The result should be the minuend.

Subtract 369 from 657.

```
    6 5 7                   2 8 8
  - 3 6 9     Verify →    + 3 6 9
  -------                  -------
    2 8 8                   6 5 7
```

5. If there is 0 in a column we may have to regroup from farther left. For example, to subtract 509 from 2005, the mental steps are:

    (a) For 1's: 9 is more than 5; regroup 1 from 10's column. To do so, regroup 1 from 1000's column to have 10 in 100's column; then regroup 1 from 100's column to have 10 in 10's column; then regroup 1 from 10's column to have 15 in 1's column. 15 minus 9 equal 6; put down 6.

```
            9   9
      1    10  10  15
      2     0   0   5
    -       5   0   9
    ---------------
      1     4   9   6
```

    (b) For 10's: 9 is left in 10's column; 9 minus 0 equal 9, put down 9.
    (c) For 100's: 9 is left in 100's column; 9 minus 5 equal 4; put down 4.
    (d) For 1000's: 1 is left in 100's column; put down 1.
    (e) The difference is 1496.

6. We may subtract large numbers the same way.

    (a) Subtract **4,001,030,352 – 1,946,327,115**.

    We regroup as shown on the right.

    Note that when there are several zeros following each other in the minuend, regrouping takes place far from the left.

```
             9  9 10
    3   10 10  0 10  2 10    4 12
    4,  0  0  1, 0  3  0,  3  5  2
    1,  9  4  6, 3  2  7,  1  1  5
    -----------------------------
    2,  0  5  4, 7  0  3,  2  3  7
```

    (b) Subtract **5,082,359,777 – 3,193,461,857**.

    We regroup as shown on the right.

```
              9  17 11 12
    4   10  7  1  2 15  8 17
    5,  0  8  2, 3  5  9,  7  7  7
    3,  1  9  3, 4  6  1,  8  5  7
    -----------------------------
    1,  8  8  8, 8  9  7,  9  2  0
```

☺ Exercise A3.5

1. Subtract by columns.

(a) 19 – 8	(e) 67 – 33	(i) 66 – 42	(m) 703 – 487	(q) 7030 – 3657
(b) 27 – 15	(f) 75 – 44	(j) 22 – 8	(n) 6452 – 4666	(r) 2003 – 1956
(c) 35 – 12	(g) 87 – 24	(k) 43 – 15	(o) 1001 – 999	
(d) 58 – 35	(h) 73 – 32	(l) 531 – 362	(p) 8500 – 5555	

2. Subtract and verify your answer by adding the difference to the subtrahend to see if you get back the minuend.

(a) 734 – 569 = _____	(c) 583 – 275 = _____	(e) 727 – 458 = _____
(b) 306 –187 = _____	(d) 666 – 337 = _____	(f) 600 – 387 = _____

3. Subtract.

(a) 36,251– 14, 532	(d) 400,315 – 291,827	(g) 5,423,234,015 – 3,567,888,146
(b) 30,000 – 24,762	(e) 549,321 – 485,789	(h) 9,876,543,210 – 6,012,345,678
(c) 77,004 – 45, 675	(f) 707,303 – 596,276	(i) 5,937,123,472 – 4,999,847,503

Lesson A3.6   Subtraction is "Reverse Addition"

The difference may be determined by "reverse addition."

1. We may question the difference between subtrahend and minuend as follows.

To determine	11 – 6	=	what?
We may ask,	6 + what?	=	11
The answer is 5.			

2. We may use columns to count the difference.

   <u>Subtract 62 from 76</u>:

   (a) For 1's: 2 plus **what** is "6"? The answer is 4. Put down **4 ones.**

   (b) For 10's: 6 plus **what** is "7"? The answer is 1. Put down **1 ten.**

   (c) The <u>difference</u> is 14.

	tens	ones	
	7	6	minuend
–	6	2	subtrahend
	1	4	difference

3. When the minuend in a column is smaller than the subtrahend, we increase the minuend in that column by "ten," and then count the difference. We compensate by regrouping this increase in the next column on left of the subtrahend.

   <u>Subtract 49 from 82</u>:

   (a) For 1's: 2 is smaller than 9, so increase 2 to "12." Ask 9 plus what is "12"? The answer is 3; put down 3.

   (b) For 10's: Regroup 1 from "12" to the subtrahend 4, making it 5. Ask 5 plus what is "8"? Put down the answer 3.

   (c) The <u>difference</u> is 33.

	8	¹2	minuend
–	4	9	subtrahend
	1		regroup
	3	3	difference

4. The "regrouping to subtrahend" is used in subsequent columns as necessary.

   <u>Subtract 387 from 623</u>: (Compare to Lesson A3.5, #3)

   (a) For 1's: 3 is smaller than 7, so increase 3 to "13." Ask 7 plus what is "13." Put down the answer 6.

   (b) For 10's: Regroup 1 from "13" to the subtrahend 8, making it 9. 2 is less than 9, so increase 2 to "12." Ask 9 plus what is "12"? Put down the answer 3.

   (c) For 100's: Regroup 1 from "12" to the subtrahend 3, making it 4. Ask 4 plus what is "6"? Put down the answer 2.

   (d) The difference is 236.

	6	¹2	¹3	
–	3	8	7	
	1	1		regroup
	2	3	6	

5. Here we use our earlier skill of addition to accomplish subtraction. Regrouping is much simpler.

Subtract 509 from 2005: (Compare to Lesson A3.5, #5)

```
  2 ¹0 ¹0 ¹5
 −    5  0  9
    1  1  1
 ──────────────
    1  4  9  6
```

(a) For 1's: 5 is smaller than 9, so increase 5 to "15." Ask 9 plus what is "15." Put down the answer 6.

(b) For 10's: Regroup from "15" to the subtrahend 0, making it 1. 0 at top is less than 1, so increase 0 to "10." Ask 1 plus what is "10"? Put down the answer 9.

(c) For 100's: Regroup 1 from "10" to the subtrahend 5, making it 6. 0 at top is less than 6, so increase 0 to "10." Ask 6 plus what is "10"? Put down the answer 4.

(d) For 1000's: Regroup 1 from "10" making the subtrahend 1. Ask 1 plus what is "2"? Put down the answer 1.

(e) The difference is 1496.

6. We may subtract large numbers the same way (compare to Lesson A3.5, #6).

(a) Subtract

4,001,030,352 – 1,946,327,115.

We "regroup to subtrahend" to get the following. Note that we do not get into complicated regrouping as in Lesson A3.4, #6.

```
   4, 0 0 1, 0 3 0, 3 5 2
 − 1, 9 4 6, 3 2 7, 1 1 5
   1  1 1 1     1       1
 ────────────────────────
   2, 0 5 4, 7 0 3, 2 3 7
```

(b) Subtract

5,082,359,777 – 3,193,461,857.

We "regroup to subtrahend" to get the following.

```
   5, 0 8 2, 3 5 9, 7 7 7
 − 3, 1 9 3, 4 6 1, 8 5 7
   1  1 1 1 1   1
 ────────────────────────
   1, 8 8 8, 8 9 7, 9 2 0
```

☺ Exercise A3.6

1. Subtract by "reverse addition" as shown above.
   (a) 10 – 8 → 8 + what? = 10   (d) 14 – 8 → 8 + what?   = 14
   (b) 12 – 5 → 5 + what? = 12   (e) 15 – 12 → 12 + what? = 15
   (c) 16 – 9 → 9 + what? = 16   (f) 13 – 5 → 5 + what?   = 13

2. Subtract by "reverse addition" and verify your answer on a calculator.
   (a) 19 – 8   (g) 87 – 24   (m) 703 – 487
   (b) 27 – 15   (h) 73 – 32   (n) 6452 – 4666
   (c) 35 – 12   (i) 66 – 42   (o) 1001 – 999
   (d) 58 – 35   (j) 22 – 8   (p) 8500 – 5555
   (e) 67 – 33   (k) 43 – 15   (q) 7030 – 3657
   (f) 75 – 44   (l) 531 – 362   (r) 2003 – 1956

3. Subtract by "reverse addition" and verify your answer on a calculator.
   (d) 36,251– 14, 532   (d) 400,315 – 291,827   (g) 5,423,234,015 – 3,567,888,146
   (e) 30,000 – 24,762   (e) 549,321 – 485,789   (h) 9,876,543,210 – 6,012,345,678
   (f) 77,004 – 45, 675   (f) 707,303 – 596,276   (i) 5,937,123,472 – 4,999,847,503

☺ Practice #1

1. Subtract the following. Verify your answer on a calculator.

(a)	(b)	(c)	(d)	(e)	(f)
3 5	7 6	5 5	8 7	4 9	5 6
− 2 4	− 1 2	− 3 3	− 1 0	− 3 4	− 3 1

(g)	(h)	(i)	(j)	(k)	(l)
6 3	5 4	6 5	3 7	6 8	9 9
− 2 2	− 3 1	− 2 4	− 5	− 4 3	− 2 7

2. Subtract the following by columns. Verify your answer mentally or on a calculator.

(a)	(b)	(c)	(d)	(e)	(f)
9 3	7 5	9 4	8 7	8 6	3 6
− 8 9	− 6 6	− 5 9	− 4 7	− 4 8	− 2 3

(g)	(h)	(i)	(j)
2 6 4	2 9 6	1 0 0 0	8 0 3 1
− 5 9	− 1 9 8	− 8 7 5	− 5 6 7 3

3. Subtract the following by columns. Verify your answer mentally or on a calculator.

(a)	(b)	(c)	(d)	(e)	(f)
9 3	7 5	9 4	8 7	8 6	3 6
− 8 9	− 6 6	− 5 9	− 4 7	− 4 8	− 2 3

(g)	(h)	(i)	(j)
2 6 4	2 9 6	1 0 0 0	8 0 3 1
− 5 9	− 1 9 8	− 8 7 5	− 5 6 7 3

4. Subtract the following mentally or by column. Verify your answers on a calculator.

(a) 33 – 13 = _____   (f) 32 – 15 = _____   (k) 163 – 57 = _____
(b) 55 – 22 = _____   (g) 46 – 29 = _____   (l) 177 – 88 = _____
(c) 87 – 65 = _____   (h) 81 – 37 = _____   (m) 214 – 59 = _____
(d) 48 – 31 = _____   (i) 63 – 26 = _____   (n) 537 – 248 = _____
(e) 74 – 32 = _____   (j) 64 – 27 = _____   (o) 792 – 499 = _____

5. Subtract the following using the most convenient method. Verify your answers on a calculator.

(a)	(b)	(c)	(d)	(e)
4 5 6	5 0 6	7 2	2 6 3 5	7 0 0
– 2 8 9	– 2 4 7	– 4 6	– 4 7 3	– 4 6 7

(f)	(g)	(h)	(i)	(j)
7 6 5	6 6 3	8 9	3 2 2 1	3 3 3
– 3 7 8	– 3 5 5	– 2 4	– 6 5 3	– 2 2 4

6. Have another person call out two numbers randomly. Subtract the numbers mentally or by column using any method. Verify the answer on a calculator. Repeat this exercise until you can do so effortlessly. Do this exercise on a turn about basis, if possible.

Answer: 1. (a) 11 (b) 64 (c) 22 (d) 77 (e) 15 (f) 25 (g) 41 (h) 23 (i) 41 (j) 32 (k) 25 (l) 72
2. (a) 4 (b) 4 (c) 9 (d) 40 (e) 38 (f) 13 (g) 205 (h) 98 (i) 125 (j) 2358 3. (a) 4 (b) 9
(c) 35 (d) 40 (e) 38 (f) 13 (g) 205 (h) 98 (i) 125 (j) 2358 4. (a) 20 (b) 33 (c) 22 (d) 17
(e) 42 (f) 17 (g) 17 (h) 44 (i) 37 (j) 37 (k) 106 (l) 89 (m) 155 (n) 289 (o) 293
5. (a) 167 (b) 259 (c) 26 (d) 2162 (e) 233 (f) 387 (g) 308 (h) 65 (i) 55 (j) 2558 (l) 109

Lesson A3.7   Shortage

When we need to take away more quantity than what is there, we have a shortage to be compensated from elsewhere.

1. A shortage is the amount one needs to compensate from elsewhere.

(a) After drinking all of the water in the glass, if you are still thirsty, then there is a shortage of water compared to what you need

(b) When you need to take away 7 pennies but only 5 are available, you are 2 pennies short. This we write as,

$$5 - 7 = -2$$
(A **shortage** of 2 pennies)

We represent the shortage of two pennies by leaving the minus (–) sign in front of the number. This means that this amount is yet to be taken away from any additional quantity.

(c) There are 7 candies. But we need to take away 10 candies. We have a shortage of 3 candies.

$$7 - 10 = -3$$
(A **shortage** of 3 candies)

(d) Subtract 70 from 63. We have a shortage of 7.

$$63 - 70 = -7$$
(A **shortage** of 7)

2. The shortage is opposite of the difference.

(a) When we take 2 pennies away from 5 pennies we have a difference of 3 pennies.

$$5 - 2 = 3 \qquad \text{(difference)}$$

When we try to take 5 pennies away from 2 pennies we have a shortage of 3 pennies.

$$2 - 5 = -3 \qquad \text{(shortage)}$$

(b) A shortage is equal in amount but opposite of the difference.

$$2 - 5 = -(5 - 2) = -3$$
$$5 - 7 = -(7 - 5) = -2$$
$$5 - 63 = -(63 - 5) = -58$$

☺ Exercise A3.7

1. Compute the shortage.

(a) $8 - 5 = 3$, $5 - 8 = $ _____
(b) $14 - 6 = 8$, $6 - 14 = $ _____
(c) $25 - 20 = 5$, $20 - 25 = $ _____
(d) $38 - 35 = 3$, $35 - 38 = $ _____
(e) $53 - 48 = 5$, $48 - 53 = $ _____

2. Show that shortage is opposite of the difference.

(a) $7 - 12 = -($ ____ $) = $ _____
(b) $13 - 23 = -($ ____ $) = $ _____
(c) $27 - 30 = -($ ____ $) = $ _____
(d) $35 - 45 = -($ ____ $) = $ _____
(e) $53 - 56 = -($ ____ $) = $ _____

3. Subtract and present the result as a difference or a shortage.

(a) $9 - 1$ (g) $9 - 73$ (m) $13 - 7$
(b) $1 - 9$ (h) $42 - 4$ (n) $23 - 8$
(c) $7 - 3$ (i) $5 - 33$ (o) $34 - 6$
(d) $3 - 7$ (j) $15 - 8$ (p) $4 - 41$
(e) $17 - 7$ (k) $8 - 15$ (q) $62 - 5$
(f) $28 - 6$ (l) $7 - 13$ (r) $7 - 66$

☺ Practice #2

1. Subtract the following mentally.

(a) $9 - 3 = $ _____ (f) $13 - 9 = $ _____ (k) $54 - 23 = $ _____
(b) $6 - 2 = $ _____ (g) $42 - 38 = $ _____ (l) $41 - 17 = $ _____
(c) $17 - 4 = $ _____ (h) $75 - 69 = $ _____ (m) $63 - 38 = $ _____
(d) $28 - 5 = $ _____ (i) $51 - 47 = $ _____ (n) $74 - 28 = $ _____
(e) $35 - 4 = $ _____ (j) $85 - 85 = $ _____ (o) $95 - 59 = $ _____

2. Subtract the following by columns.

(a) 734 – 569	= _____		(f) 6,060,066 – 387,737	= _____
(b) 3,006 – 987	= _____		(g) 73,503,627 – 45,732,358	= _____
(c) 5,183 – 2,975	= _____		(h) 600,000,000 – 523,987,999	= _____
(d) 74,111 – 37,555	= _____		(i) 5,082,359,777 – 3,193,461,857	= _____
(e) 835,254 – 657,578	= _____		(j) 73,321,654,987 – 37,789,123,456	= _____

Lesson A3.8   Word Problems

To solve a word problem one must translate it from English into a mathematical expression.

1. Some of the verbal expressions that may be translated as subtraction are as follows. *NOTE: The letter 'n' means a number.*

A number diminished by **20** .... n – 20
15 minus a number .... 15 – n
The difference of **20** and a number.. 20 – n
25 less than a number... n – 25
25 less a number .... 25 – n
A number subtracted from **50** .... 50 – n
A number reduced by **75** .... n – 75
30 decreased by a number .... 30 – n
A number dropped by **35** .... n – 35
46 lower than a number .... n – 46
32 fewer than a number .... n – 32
A number shortened by **60** .... n – 60
83 depreciated by a number .... 83 – n
11 smaller than a number .... n – 11

2. To solve a word problem one must be able to think logically with all the words used in describing that problem.

    NOTE: Notice that each word problem solution is ended by answer to the task in words including units, such as, "Therefore, it is 92,650,000 miles farther to the sun than to the moon."

    (a) The average distance to the sun is about 92,889,000 miles. The average distance to the moon is about 239,000 miles. How much farther is it to the sun than to the moon?

    SOLUTION:
    The average distance to the sun   =   92,889,000 miles
    The average distance to the moon   =   239,000 miles
    The difference between the distances   =   92,889,000 – 239,000
    *   =   92,650,000 miles*
    Therefore, it is 92,650,000 miles farther to the sun than to the moon.

(b) The height of Mt. Everest is twenty-nine thousand, twenty-eight feet. The height of Mt. Whitney is fourteen thousand, four hundred ninety-five feet. How much higher is Mt. Everest than Mt. Whitney?

SOLUTION:
The height of Mt. Everest   =   *29,028 feet*
The height of Mt. Whitney   =   *14,495 feet*
The difference between the heights   =   *29,028 – 14,495*
                                           =   *14,533 feet*
Therefore, Mt. Everest is 14,533 feet higher than to Mt. Whitney.

(c) At the beginning of the month Mr. Hanson's checking account balance was $356. He made deposits of $225, $57, and $375. He wrote checks for $56, $135, $157, $38, and $417. What was his balance at the end of the month?

SOLUTION:
Initial checking account balance   =   *$356*
Total deposits   =   *$(225 + 57 + 375)* =   *$657*
Total checks written   =   *$(56 + 135 + 157 + 38 + 417)* =   *$803*
Balance at the end of the month   =   *$(356 + 657 – 803)* =   *$210*
Mr. Hanson's balance at the end of the month is $210.

(d) A man gave his son the following riddle. "Take a 5-gallon and a 3-gallon bucket down to the lake, and bring back exactly 4 gallons of water.' How would you solve this riddle?

SOLUTION:
Fill the 3-gallon bucket and empty it in the 5-gallon bucket.
Fill the 3-gallon bucket again and pour it till the 5-gallon bucket is full. You will have 1 gallon of water left in the 3-gallon bucket.
Empty the 5-gallon bucket back into the lake, and pour 1 gallon of water into it from the 3-gallon bucket.
Fill the 3-gallon bucket and empty it in the 5-gallon bucket.
You now have 4 gallons of water in the 5-gallon bucket.
      3 + 3 – 5 + 3   =   *4 gallons*

☺ Exercise A3.8

1. Express the following mathematically.
    (a) The number decreased by **10**.   (c) The number minus **45**.
    (b) **13** depreciated by the number.   (d) **8** fewer than the number.

2. Answer the following questions.
    (a) Joey has 12 pennies. He lost 3 pennies. How many pennies does he have now?
    (b) Bill is 60 inches tall. John is only 45 inches. How much taller than John is Bill?
    (c) There are 27 students in the class. 4 new students left the class last week. How many students are in the class now?
    (d) John drove 90 miles one day. Bill drove 135 miles the same day. How much more distance did Bill travel?
    (e) Lisa weighed 125 lbs. After changing her diet and exercising she lost 15 lbs last month. How much does she weigh now?

3. On a particular day the sun is about 92,786,000 miles from earth, while the moon is only about 245,000 miles. How much farther is it to the sun than to the moon?

4. If the population of United States is 303,321,825 and the population of Mexico is 106,535,000, how many more people are there in Asia than in North America?

5. If at the beginning of a trip your odometer reading was 67,856 miles and at the end of the trip it read 71,304 miles, how many miles did you drive?

6. A new car had a list price of $24,120. Mr. Jones bought the car and was given a discount of $2455. What did he pay for the car?

7. Jim has $75 and Joe has $57. Jack has $17 more than Jim, and Mike has $13 less than Jim and Joe together. How much money do all four boys have together?

8. If the population of China is 1,321,847,351 and the population of the United States is 303,321,825, how many more people are there in China than in the United States?

9. The highest point in North America, Mt. McKinley, is twenty thousand, three hundred twenty feet. The highest point in South America, Mt. Aconcagua, is twenty-two thousand, eight hundred thirty-four feet. How much higher is Mt. Aconcagua than Mt. McKinley?

10. At the beginning of the month Mr. Houmis' checking account balance was $417. He made deposits of $179, $83, and $216. He wrote checks for $68, $24, $162, $46, and $185. What was his balance at the end of the month?

11. Riddle: Two coins have a value of 60¢. One is not a dime. What are the coins?

12. Riddle: Four sheep ahead of a sheep, four sheep behind a sheep, and a sheep in the middle. How many sheep?

SUMMARY

When we take a quantity away from another quantity we have SUBTRACTION. The word SUBTRACT comes from a word, *subtrahere*, which means, *"drawn from under."*

When we take away some amount from what we have then we are left with the **difference**. When we take away all that is there, we have **nothing**. When we need to take away more than what we have then we have a **shortage**. Shortage is the difference with a negative sign.

At first one learns to subtract by counting less from a number using one's fingers. Then one learns the concept of regrouping across the place values on abacus. From this comes about various methods to subtract numbers mentally. Larger numbers may be subtracted rapidly with some help from paper and pencil while using this **mental math**.

More methods of subtraction may be derived from turning subtraction into "reverse addition." Here one may use already learned skills of addition to carry out subtraction.

Calculators may be used as an aid; but still mental awareness is necessary to know if the calculated sum is correct. This awareness comes from the practice with **mental math** as outlined above.

DIAGNOSTIC TEST

1. Find the difference.
   (a) 8 – 7   (d) 12 – 9   (g) 15 – 13
   (b) 5 – 3   (e) 9 – 5   (h) 30 – 25
   (c) 7 – 4   (f) 15 – 8   (i) 71 – 67

2. Find the shortage.
   (a) 7 – 8   (d) 9 – 12   (g) 13 – 15
   (b) 3 – 5   (e) 5 – 9   (h) 25 – 30
   (c) 4 – 7   (f) 8 – 15   (i) 67 – 71

3. Find the difference by counting from the smaller to the larger number.
   (a) 7 and 8   (d) 9 and 12   (g) 13 and 15
   (b) 3 and 5   (e) 5 and 9   (h) 25 and 30
   (c) 4 and 7   (f) 8 and 15   (i) 67 and 71

4. Subtract the following on abacus.
   (a) 28 – 7   (d) 44 – 8   (g) 67 – 48
   (b) 37 – 6   (e) 48 – 32   (h) 72 – 56
   (c) 25 – 7   (f) 55 – 23   (i) 83 – 78

5. Subtract the following mentally.
   (a) 28 – 5   (d) 51 – 47   (g) 74 – 28
   (b) 35 – 4   (e) 73 – 65   (h) 57 – 25
   (c) 61 – 7   (f) 85 – 79   (i) 95 – 59

6. Subtract the following using columns.
   (d) 5,183 – 2,975   (d) 600,000,000 – 523,987,999
   (e) 44,731 – 17,842   (e) 4,001,030,352 – 1,946,327,115
   (f) 74,111 – 37,555   (f) 5,082,359,777 – 3,193,461,857

GLOSSARY

[For additional words refer to the glossaries at the end of earlier Milestones]

Difference   **Difference** is what is left after subtraction.

Minuend   **Minuend** is the number, or quantity, that is lessened, or made smaller, through subtraction. The word MINUEND comes from a word *minuere*, which means "to lessen."

Minus Sign	A **minus (–),** when placed between two numbers, expresses that the second number is to be subtracted from the first number.
Remainder	The difference remaining after repeated subtraction until it is less than the subtrahend.
Regrouping	Rearranging (a) "Ten ones" as "one ten," or vice versa (b) "Ten tens" as "one hundred" or vice versa (c) "Ten hundreds" as "one thousand" or vice versa, and so on.
Subtraction	When we take away a smaller number from a larger number, we have **subtraction**. The word SUBTRACT comes from a word *subtrahere*, which means "drawn from under." To subtract, we start from the larger number, and count as many less as the second number. Only the quantities of the same unit may be subtracted.
Subtrahend	**Subtrahend** is the number, or quantity, that is taken away during subtraction. See SUBTRACTION for root meaning.
Turn About	Doing an exercise on a **turn about** basis means doing that exercise again with the roles of the student and the coach switched.