NUMBERS & PLACE VALUES

Numbers and Place Values forms the foundation of the subject of ARITHMETIC. It introduces a system of whole numbers to represent quantities in a simple manner.

This is the second of the seven levels of the troubleshooting guide for KINDERGARTEN MATH. See *Summary* for details on all seven levels.

These lessons are designed for kindergarten, but they may be applied to anybody to fill earlier blanks in understanding.

Start with the Diagnostic. If the diagnostic fails, then do the Lesson & Exercise.

Follow these guidelines.

(a) When helping, make sure you have the attention of the student.

(b) If you lose the attention, then go back to the point in the lesson where the student was attentive. Then come forward checking student's understanding.

(c) Always approach any situation in an affectionate and relaxed manner.

(d) Always encourage the student to ask questions.

(e) Carefully listen to what the student has to say, and let the student know that you have heard him (or her).

(f) Answer all questions matching the interest and understanding of the student.

(g) Always talk to the student at his (or her) level. Use only those terms and words that the student can easily understand.

(h) When teaching a new concept, ask the student to think examples of his own. Allow enough time even days to let that happen.

(i) Get the student involved and thinking with mathematical principles.

(j) In the final analysis, make sure that the student can apply mathematics with confidence.

Researched and written by Vinay Agarwala
Edited by Ivan Doskocil

(Issued 01/30/08)

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DIAGNOSTICS & LESSONS

☺ Diagnostic K2.1   Count up to ten or more objects

To pass, the student should be able to count up to 10 or more objects using verbal names and one-to-one correspondence, correctly and with confidence.

"Count pennies on the table."
"Count chairs in the room."
"Count the corners in the room."

If the diagnostic fails, then do the Lesson & Exercise.

Lesson & Exercise

In this lesson the student learns to count up to 10 or more objects using verbal names and one-to-one correspondence.
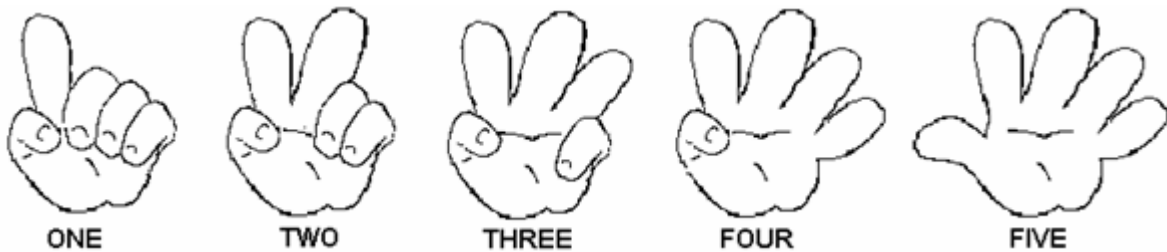
(a) Explain to the student,

"I am going to count from ONE to FIVE."

(b) Count the fingers by raising them as shown below. It is important that the student learns the following finger patterns for the numbers.
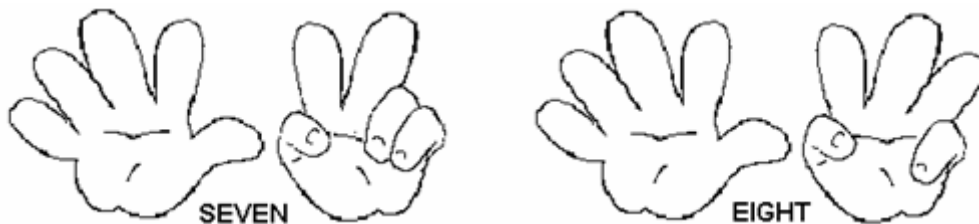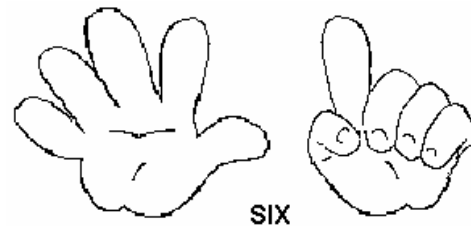
"ONE... TWO... THREE... FOUR... FIVE."

Have the student mimic your actions.

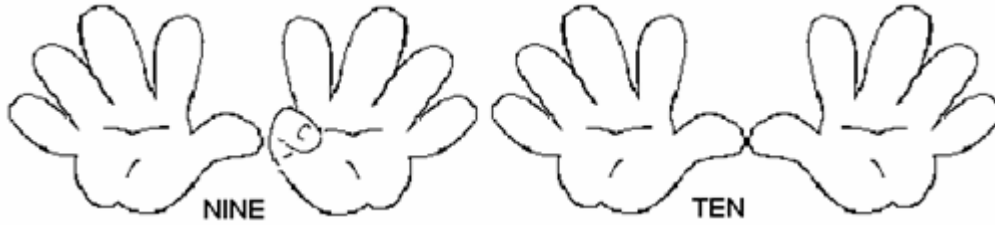ONE   TWO   THREE   FOUR   FIVE

(c) Count the fingers up to five on one hand as before. Continue counting by raising the fingers on the second hand as shown here. Have the student mimic your actions.
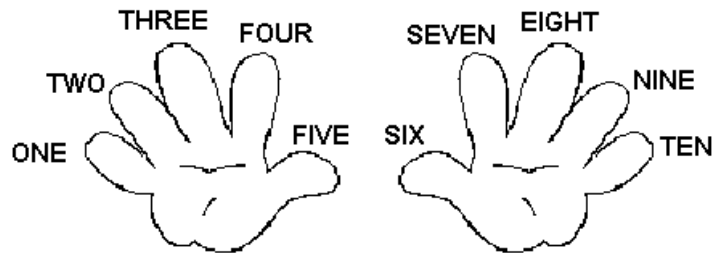
"ONE... TWO... THREE... FOUR... FIVE."

"SIX... SEVEN... EIGHT... NINE... TEN."

SIX

SEVEN   EIGHT

NINE   TEN

(d) Count the fingers on both hands by touching. Have the student mimic your actions.

(e) Count some pennies by placing one penny at a time on the table in rows of FIVE.

"We are going to count some pennies now."

"This is ONE penny... TWO, THREE, FOUR, FIVE, SIX, SEVEN, EIGHT, NINE, TEN."

Have the student count pennies from one to ten.

(f) Count pennies beyond ten up to twenty.

"We are going to count numbers beyond ten."

"ELEVEN, TWELVE, THIRTEEN, FOURTEEN, FIFTEEN, SIXTEEN, SEVENTEEN, EIGHTEEN, NINETEEN, TWENTY."

Have the student count pennies beyond ten up to twenty.

(g) Have the student count objects in the surroundings, such as, walls, windows, chairs, desks, corners, books, etc.

(h) Have the student ask you to count objects in the surroundings.

(i) Continue with this lesson until the student can comfortably count up to 10 or more objects.

(j) Repeat the diagnostic test.

☺ <u>Diagnostic K2.2</u>   <u>Read and write the numbers 0 to 9</u>

To pass, the student should be able to read and write the numbers up to 9, correctly and with confidence.

"Read the following numbers: 6, 9, 7, and 3."
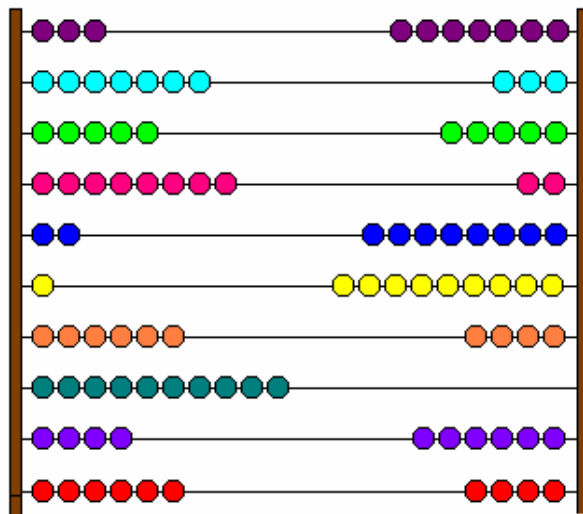"Write these numbers: FOUR, SEVEN, EIGHT, and ZERO."

If the diagnostic fails, then do the Lesson & Exercise.

<u>Lesson & Exercise</u>

In this lesson the student learns to read and write numbers up to 9.

(a) Introduce the student to a child's version of the abacus.

"This is a COUNTING BOARD called Abacus."

(b) Point to the various parts of the abacus.

"This is the frame of the Abacus."
"There are ten wires in the frame."
"There are ten beads on each wire."

(c) Slide a bead from left to right on the top (first) wire.

"You count a bead by sliding it from left to right."
"Beads on the right are counted."
"Beads on the left are in storage."

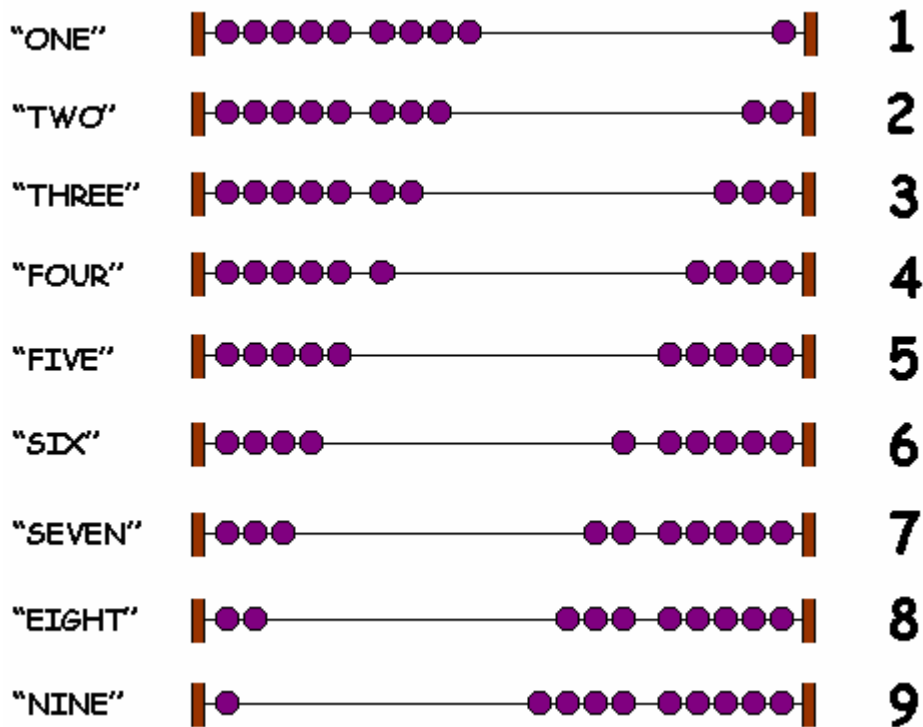(d) Move all the beads to the left. Point to the empty part of the wire on the right.

"There are no beads counted. There is nothing on the right."
"We write NOTHING with the digit 0 (ZERO)."

(e) Move beads to the right one at a time. Introduce the corresponding numbers. *NOTE: The extra space between the fifth and the sixth beads is to show "groupings by five."*

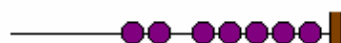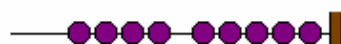"We are now going to show how numbers are written with digits."

"ONE"	●●●●● ●●●● ●	1
"TWO"	●●●●● ●●● ●●	2
"THREE"	●●●●● ●● ●●●	3
"FOUR"	●●●●● ● ●●●●	4
"FIVE"	●●●●● ●●●●●	5
"SIX"	●●●● ● ●●●●●	6
"SEVEN"	●●● ●● ●●●●●	7
"EIGHT"	●● ●●● ●●●●●	8
"NINE"	● ●●●● ●●●●●	9
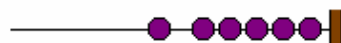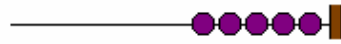
(f) Practice writing the digits 0 to 9 as necessary.

"Now you show the numbers on abacus and write them with digits."
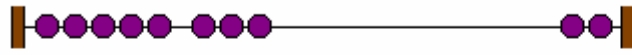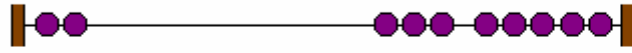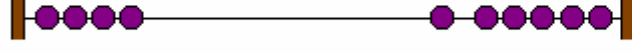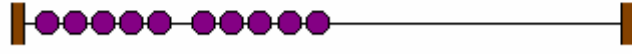
Zero	0	0	0	__	__	__	__
One	1	1	1	__	__	__	__
Two	2	2	2	__	__	__	__
Three	3	3	3	__	__	__	__
Four	4	4	4	__	__	__	__
Five	5	5	5	__	__	__	__
Six	6	6	6	__	__	__	__
Seven	7	7	7	__	__	__	__

(g) Match the numbers on abacus to the digits:

5

4

2

3

7

8

9

6

1

0

(h) Write the digit for the number on abacus.

____

____

____

____

____

____
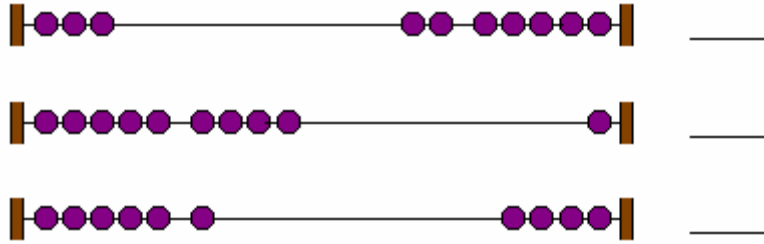
(i) Have the student ask you to write the numbers zero to nine using digits.

(j) Continue with this lesson until the student can comfortably read and write the numbers 0 to 9 using the single digits.

(k) Repeat the diagnostic test.

☺ Diagnostic K2.3　　　Read and write two-digit numbers

To pass, the student should be able to read and write two-digit numbers up to twenty, correctly and with confidence.

"Explain why the number for TEN is written with two digits 1 and 0."
"Read the following numbers:　　16, 19, 14, and 18."
"Write these numbers:　　　　FIFTEEN, ELEVEN, NINETEEN, and SEVENTEEN."
"What is a single-digit number? What is a double-digit number?"

If the diagnostic fails, then do the Lesson & Exercise.

Lesson & Exercise

In this lesson the student learns to read and write two-digit numbers up to twenty.

(a) Introduce the student to a child's version of the abacus using the previous lesson.

"This is a COUNTING BOARD called Abacus."

(b) Count all the ten beads to the right on the first wire.
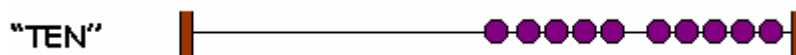
(c) Explain the reason for second wire.

"We count by ONES on the first wire only; but we run out beads after counting to ten. We must move the beads back to be able to count more."

"This is exactly what we do; but we move one bead on the second wire to account for the ten counted already.

"Therefore, one bead by itself on the second wire also means a count of TEN.

(d) Explain why the number for TEN is written with digits 1 and 0.

"You write the number TEN just the way beads appear on the abacus, as 1 and 0."
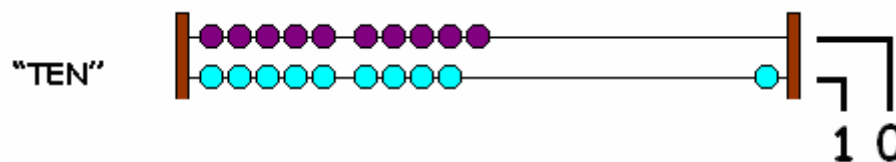
(e) Explain this in terms of regrouping.

"Ten beads on the first wire are like <u>ten pennies</u>. One bead on the second wire is like <u>one dime</u>."

"We regroup <u>ten pennies</u> as <u>one dime</u>. Similarly, at count TEN, we regroup <u>ten ONES</u> as <u>one TEN</u> on abacus."

(f) Explain how the numbers for counts beyond 10 are written.

"You count beyond 10 by moving beads on the first wire again. You write the corresponding number the way beads appear on the wires."

"Ten and one are ELEVEN."

1 1

"Ten and two are TWELVE."

1 2

"Ten and three are THIRTEEN."

1 3

"Ten and four are FOURTEEN."

1 4

"Ten and five are FIFTEEN."

1 5

"Ten and six are SIXTEEN."

1 6

"Ten and seven are SEVENTEEN."

1 7

"Ten and eight
are EIGHTEEN."

1 8
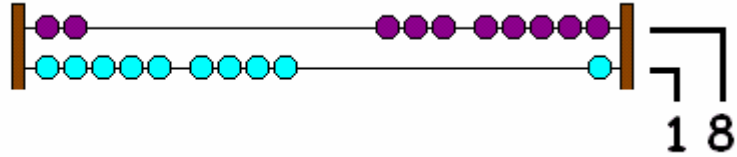
"Ten and nine
are NINETEEN."

1 9
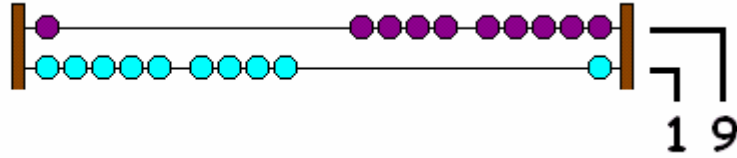
"At count TWENTY, we once again regroup the ten beads on the first wire as one bead on the second wire."

"TWENTY"

"TWENTY"

2 0
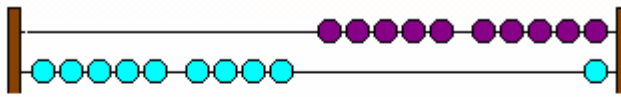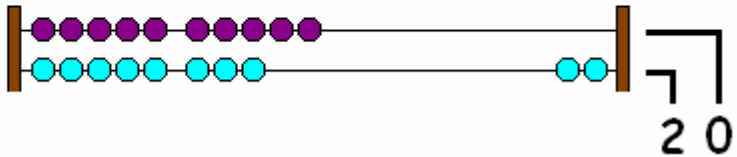
"THE RULE OF ABACUS IS WHENEVER ALL THE TEN BEADS ARE TO THE RIGHT ON A WIRE; WE REGROUP THEM AS ONE BEAD ON THE NEXT WIRE."

(g) Write the following numbers using digits.
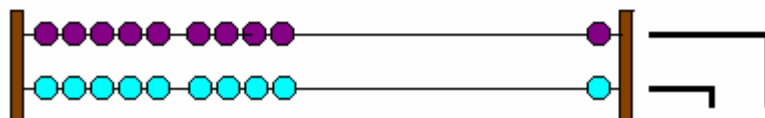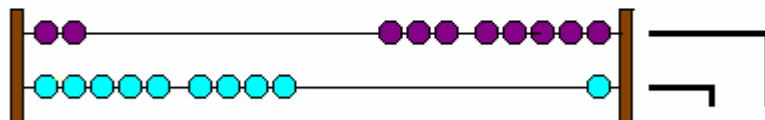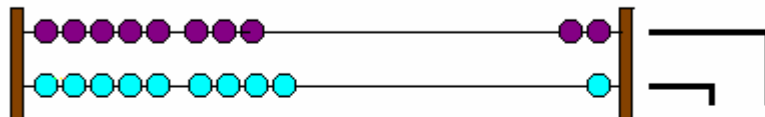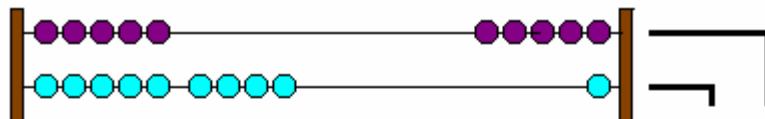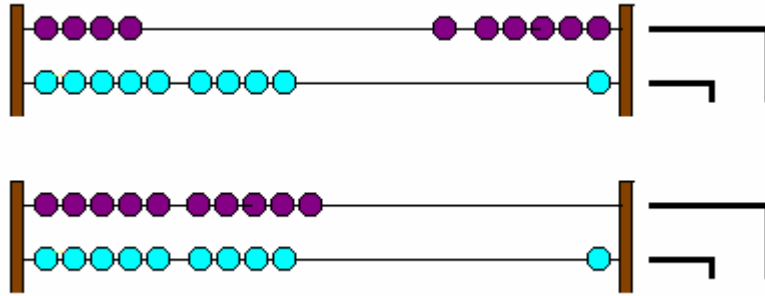
(h) Explain what digits are.

"The digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 are used to write numbers just like letters are used to write words."

(i) Explain how digits are used to write numbers.

"5 is a single-digit number; just like the word 'I' is a single-letter word."

"15 is a double-digit number, just like the words 'we' is a double-letter word."

"The numbers 0 to 9 are called single-digit numbers because they are written using a single digit only. The numbers from ten to twenty are double-digit numbers because they are written with two digits."

(j) Have the student ask you to write the numbers ten to twenty using digits.

(k) Continue with this lesson until the student can comfortably read and write the numbers 10 to 20 using digits.

(l) Repeat the diagnostic test.

☺ Diagnostic K2.4   Visualize numbers as sets of objects

To pass, the student should able to visualize a number not only as it is written, but also as a set of objects.

"Show the number FIVE.
 (a) As it is written.
 (b) As a set of objects."

If the diagnostic fails, then do the Lesson & Exercise.

Lesson & Exercise

In this lesson the student learns to visualize numbers as a set of objects.

(a) Write a word and draw what it represents.

"This is how the word CAT is written; and this is how a cat looks."
"There can be many different looking cats."

(b) Write a number and draw what it represents.

"This is how the number 5 is written; and this is how a five appears."
"There can be many different looking fives."

(c) The student should understand that a number represents a set of objects.

"Just as the word CAT can represent many different looking cats; similarly, the number 5 represents a SET of any five things, such as, five fingers, five beads, or five marks."

(d) Explain what a SET is.

"A SET is simply a collection of things."

(e) Have the student describe the following sets.

"What numbers do you see here as sets?"
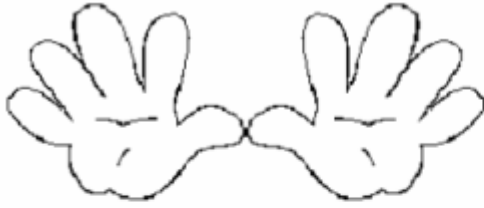
Answer: 10 fingers, or 2 hands

Answer: 5 chairs

(f) Explain to the student.

"To be good in mental math one must be able to visualize numbers as a set of things."

(g) Have the student locate sets in the surroundings for the following numbers.

FOUR, TWELVE, SEVEN, SIXTEEN

(h) Write the number SEVEN. Show a set for number SEVEN.

(i) Have the student ask you to write and visualize numbers.

(j) Repeat the diagnostic test.

☺ **Diagnostic K2.5   Show the concept of numbers**

To pass, the student should be able to understand that a number may be applied to a set of any sort of objects.

"Show me the number FOUR in at least three different ways."

If the diagnostic fails, then do the Lesson & Exercise.

Lesson & Exercise

In this lesson, the student uses sets of things to show the concept of numbers.

NOTE: A child can understand three spoons, three cups, and three plates much before he can grasp what THREE means. Understanding always starts from the concrete and then proceeds toward the abstract.
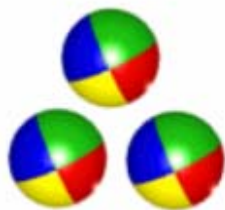
Three Clocks	Three Balls	Three Chairs

(a) Demonstrate the number THREE.

"I am going to show you the number THREE as a set of different things."
"These are THREE Fingers"   (Extend your hand showing THREE fingers)
"These are THREE Beads"   (Move THREE beads to the right on the abacus)
"These are THREE Pennies"   (Place THREE pennies on the table)

 "The number THREE is the same in all these sets."

(b) Invite the student to show the number THREE using different objects.

(c) Demonstrate the number FIVE.

"I am going to show you the number FIVE."
"These are FIVE Fingers"   (Extend your hand showing FIVE fingers)
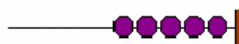
Five Fingers	Five Beads
	Five Pennies

| "These are FIVE Beads" | (Move FIVE beads to the right on the abacus) |
| "These are FIVE Pennies" | (Place FIVE pennies on the table) |

"The number FIVE is common to all these sets."

(d) Have the student to demonstrate the number FIVE using different objects.

(e) Demonstrate the number SEVEN.

"I am going to show you the number SEVEN."

"These are SEVEN Fingers"	(Extend your hands showing SEVEN fingers)
"These are SEVEN Beads"	(Move SEVEN beads to the right on the abacus)
"These are SEVEN Pennies"	(Place SEVEN pennies on the table)

"The number SEVEN is common to all these sets."

(f) Have the student to demonstrate the number SEVEN using different objects.

(g) Point to objects in the room.

"Numbers tell you how many things there are in a set."

(h) Have the student ask you to demonstrate a number using different sets.

(i) Continue with this lesson until the student can show numbers up to 10 or more with different sets of objects comfortably.

(j) Repeat the diagnostic test.

To pass, the student should be able to count backward from ten correctly, and with confidence.

"Count backwards from ten to one."

If the diagnostic fails, then do the Lesson & Exercise.

Lesson & Exercise

In this lesson the student learns to count backwards from ten to one.

(a) Start the lesson with the count of TEN on the first wire of abacus.

"All the beads are to the right. The count is TEN."

"TEN"

(b) Demonstrate how to count backwards by moving beads back one at a time.

"You count backwards like this."
"The count is NINE."

"NINE"

(c) Call out the subsequent counts as follows.

"EIGHT"

"SEVEN"

"SIX"

"FIVE"

"FOUR"

"THREE"

"TWO"

"ONE"
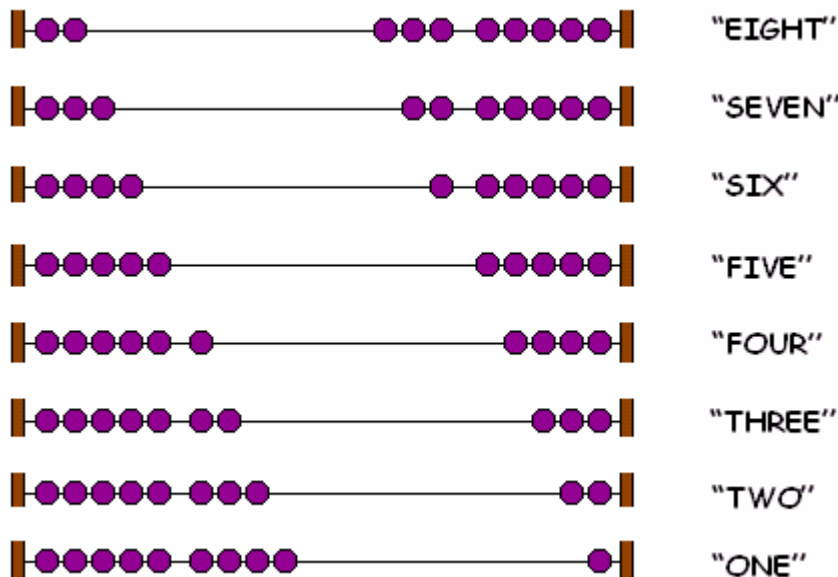
(d) Move the last bead to the left.

"When the last bead is moved to the left the count is ZERO."

(e) Move all the beads to the right to show the count of TEN. Have the student count backward from ten to one.

"Count backwards from ten to one."

(f) Have the student ask you to count backward from ten to one.

(g) Repeat this lesson until the student can recite the first ten counts forward and backward rapidly.

(h) Repeat the diagnostic test.

SUMMARY

This is the second of the seven levels of the Troubleshooting Guide for KINDERGARTEN MATH. The Troubleshooting Guide for Kindergarten introduces the concept of UNIT, and explores ways to measure length, weight, capacity and time. It further develops the concept of counting into the concepts of addition and subtraction.

The Kindergarten troubleshooting guide is divided into the following levels:

(1) ORIENTATION & SPATIAL SENSE

Orientation and Spatial Sense forms the foundation of the subject of GEOMETRY. It introduces the elements of space and how they relate to us.

(2) NUMBERS & PLACE VALUES

Numbers and Place Values form the foundation of the subject of ARITHMETIC. It introduces a system of whole numbers to represent quantities in a simple manner.

(3) UNITS & FRACTIONS

Units & Fractions addresses ways to represent quantities, which cannot be represented by whole numbers.

(4) COUNTING & MEASUREMENTS

Counting & Measurements provides ways to determine the various magnitudes. It helps to bring familiarization with the use of numbers.

(5) NUMBERS & OPERATIONS

Numbers & Operations introduces the basic operations with numbers and how such operations may be executed with skill.

(6) PATTERNS & RELATIONAL SENSE

Patterns and Relational Sense forms the foundation of the subject of ALGEBRA. It is a study of patterns underlying numbers, and quantitative relationships.

(7) DATA ANALYSIS & PROBABILITY

Data Analysis & Probability shows how to display quantitative comparisons graphically. It introduces the estimation of likelihood.

Though these lessons are designed for the kindergarten level, these diagnostic actions may be used for students at higher grades to help discover and resolve missing basics.

GLOSSARY

[For additional words refer to the glossary at the end of Level K1.]

Abacus	An **abacus** is a counting board on which one can count to very large numbers. The RULE OF ABACUS is, "When all beads are counted to the right on a wire, they are replaced by counting one bead to the right on the next wire.
Arithmetic	**Arithmetic** means literally, "skill with numbers." It helps one find the answers to problems involving numbers and quantities.
Counting	The purpose of **counting** is to find out how many things there are. One counts by sequentially calling out one, two, three, four, and so on, for each item.
Digits	**Digits** are used to write numbers, much like letters are used to write words. The number FIVE is written with only one digit 5. The number FIFTEEN is written with two digits 1 and 5 as 15. There are ten different digits – 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. All numbers are written just with these ten digits.
Number	In counting, each count is given a different name called a **number**, such as, one, two, and three. A number answers the question, "How many?"
Place value	**Place value** is the value a digit gets from its place in a number.
Quantity	A **Quantity** refers to "how many" or "how much" of something, as opposed to quality, or the description of that thing.
Regroup	We **regroup** ten pennies as one dime. Similarly, at count TEN, we regroup <u>ten ONES</u> as <u>one TEN</u> on abacus."
Set	A **Set** is a collection of things, such as, a set of chairs, a set of books, etc.