## KINDERGARTEN MATH \#6

## PATTERNS \& RELATIONAL SENSE

Patterns and Relational Sense forms the foundation of the subject of ALGEBRA. It is a study of patterns underlying numbers, and quantitative relationships.
Index ..... 3K6.1 Identify Simple Patterns
K6.2 Identify Objects that do not Match ..... 5
K6.3 Sort and Classify Objects ..... 7
K6.4 Predict and Extend Existing Patterns ..... 9
K6.5 Create Patterns ..... 11
K6.6 Transfer Patterns between Mediums ..... 13
K6.7 Locate Known and Unknown Numbers ..... 15
K6.8 Represent Missing Quantities ..... 17
K6.9 Determine if Two Groups are Equal ..... 19
Summary ..... 21
Glossary ..... 22

This is the sixth of the seven levels of the troubleshooting guide for KINDERGARTEN MATH. See Summary for details on all seven levels.

These lessons are designed for kindergarten, but they may be applied to anybody to fill earlier blanks in understanding.

## Start with the Diagnostic. If the diagnostic fails, then do the Lesson \& Exercise.

Follow these guidelines.
(a) When helping, make sure you have the attention of the student.
(b) If you lose the attention, then go back to the point in the lesson where the student was attentive. Then come forward checking student's understanding.
(c) Always approach any situation in an affectionate and relaxed manner.
(d) Always encourage the student to ask questions.
(e) Carefully listen to what the student has to say, and let the student know that you have heard him (or her).
(f) Answer all questions matching the interest and understanding of the student.
(g) Always talk to the student at his (or her) level. Use only those terms and words that the student can easily understand.
(h) When teaching a new concept, ask the student to think examples of his own. Allow enough time even days to let that happen.
(i) Get the student involved and thinking with mathematical principles.
(j) In the final analysis, make sure that the student can apply mathematics with confidence.

## PLEASE PRINT DOUBLE-SIDED

## DIAGNOSTICS \& LESSONS

## Diagnostic K6.1 Identify Simple Patterns

To pass, the student should be able to identify simple patterns of sounds, physical movements, and concrete objects with confidence.
"Make a simple pattern of sound."
"Show a simple pattern of physical movement."
"Arrange a simple pattern of objects."
"Show something with no pattern."
If the diagnostic fails, then do the Lesson \& Exercise.

## Lesson \& Exercise

In this lesson the student identifies simple patterns of sounds, physical movements, and concrete objects.
(a) Place a small drum, glass and spoon, two sticks, etc.
"We are going to identify some simple patterns of sound."
(b) Make a pattern on the drum with two sticks,
"Here is a simple patterns of sound: DRUM - drum - DRUM - drum

- DRUM - drum - DRUM - drum."

"Repetition of sounds gives you a pattern."
(c) Make another pattern of sounds on the drums.
"This is also a pattern: DRUM - drum, drum - DRUM - drum, drum - DRUM - drum, drum DRUM - drum, drum."
(d) Make some sounds with pattern and with no pattern to show the contrast. "This is a pattern... but this is not a pattern."
(e) Have the student make sounds with a simple pattern and with no pattern.
(f) Stand up in front of the student.
"We are going to identify some simple patterns of physical movement."
(g) Pick up a book and raise it up and down.
"Here is a simple pattern of physical movement."
"Repetition of physical movements gives you a pattern."
(h) Do some jumping jacks,
"This is a pattern with both hands and feet."

(i) Make some physical movements with pattern and with
no pattern to show the contrast.
"This is a pattern. But this is not a pattern."
(j) Have the student make physical movements with a simple pattern and with no pattern.
(k) Place some paper clips and erasers in front of the student.
"We are going to identify some simple patterns of concrete objects."
(l) Place the paper clips and erasers alternately.
"Here is a simple pattern with paper clips and erasers."

"Repetition of objects gives you a pattern."
(m) Place crayons, paper clips and erasers in a repeating array. "Here is another pattern with objects."
(n) Pile up all the objects with no order among them.
"But this is not a pattern."
(o) Have the student arrange objects in simple pattern and with no pattern.
(p) Have the student make a simple pattern of sound, and then sound with no pattern at all.
(q) Have the student make show a simple pattern of physical movement, and then physical movement with no pattern at all.
(r) Have the student make arrange a simple pattern of concrete objects, and then concrete objects in no pattern at all.
(s) Continue with this lesson until the student can comfortably identify simple patterns.
( t$)$ Repeat the diagnostic test.


## © Diagnostic K6.2 Identify Objects that do not Match

To pass, the student should be able to identify objects that do not belong to a particular group, correctly and with confidence.
"Pick out from a mixture of beads (of same color, size, or shape)."
(a) The only bead of different COLOR than the rest
(b) The only bead of different SIZE than the rest
(c) The only bead of different SHAPE than the rest

If the diagnostic fails, then do the Lesson \& Exercise.

## Lesson \& Exercise

In this lesson the student identifies objects that do not belong to a particular group (for example, blue lid in set of red lids).

(a) Place a pile of beads of the same color on the table. Mix a bead of a different color in that pile. Then ask the student
"Find the bead of a different color than the rest."
(b) Help the student find the bead of different color.

(c) Place a pile of beads of the same size on the table. Mix a bead of a different size in that pile. Then ask the student
"Find the bead of a different size than the rest."
(d) Help the student find the bead of different size.

(e) Place pieces of paper of the same shape on the table. Place a piece of paper of a different shape among them. Then ask the student
"Find the piece of paper of a different shape than the rest."
(f) Help the student find the piece of different shape.
(g) Mix a dime with lots of pennies. Have the student find the dime.
(h) Mix a dime with lots of nickels. Have the student find the dime.
(i) Get a tab from a can of some drink. Mix it with pennies, nickels, and dimes. Have the student find the tab.
(j) Continue with this lesson until the student can comfortably identify objects that do not belong to a particular group.
(k) Repeat the diagnostic test.

## © Diagnostic K6.3 Sort and Classify Objects

To pass, the student should be able to sort and classify objects by color, shape, size, or kind, with confidence.
"Mix beads of three different COLORS and sort them."
"Mix beads of three different SIZES and sort them."
"Mix beads of three different SHAPES and sort them."
"Mix objects of three different KINDS and classify them."
If the diagnostic fails, then do the Lesson \& Exercise.

## Lesson \& Exercise

In this lesson the student sorts and classifies objects by color, shape, size, or kind.
(a) Mix beads of two different colors (for example, RED and BLUE) in one pile.
"I am going to separate these beads by color."
(b) Pick out a RED bead and put it in a separate location. "This is a pile for RED beads."
(c) Pick out a BLUE bead and put it in a separate location. "This is a pile for BLUE beads."

(d) Pick out subsequent beads.
"This RED bead goes to the RED pile."
"This BLUE bead goes to the BLUE pile."
(e) Continue till all the beads are sorted out.

"The beads are now sorted by color."
(f) Mix the beads in a pile again, and have the student sort them out by colors.
(g) Mix beads of the same color but of two different sizes in one pile.
"I am going to separate these beads by size."
(h) Pick out a LARGE bead and put it in a separate location. "This is a pile for LARGE beads."

(i) Pick out a SMALL bead and put it in a separate location.
"This is a pile for SMALL beads."
(j) Pick out subsequent beads.
"This LARGE bead goes to the LARGE beads pile."
"This SMALL bead goes to the SMALL beads pile."

(k) Continue till all the beads are sorted out.
"The beads are now sorted by size."
(l) Mix the beads in a pile again, and have the student sort them out by sizes.
(m) Mix beads of two different shapes (for example, CIRCULAR and TRIANGULAR) in one pile.
"I am going to separate these beads by shape."
(n) Pick out a CIRCULAR bead and put it in a separate location.
"This is a pile for CIRCULAR beads."
(o) Pick out a TRIANGULAR bead and put it in a separate location.
"This is a pile for TRIANGULAR beads."

(p) Pick out subsequent beads.
"This CIRCULAR bead goes to the CIRCULAR beads pile."
"This TRIANGULAR bead goes to the TRIANGULAR beads pile."

(q) Continue till all the beads are sorted out.
"The beads are now sorted by shape."
(r) Mix the beads in a pile again, and have the student sort them out by shapes.
(s) Mix two different kinds of objects (for example, PAPER CLIPS and ERASERS) of mixed colors, shapes and sizes in one pile. "I am going to separate these objects by kind."
( t$)$ Pick out an ERASER and put it in a separate location. "This is a pile for ERASERS."
(u) Pick out a PAPER CLIP and put it in a separate location. "This is a pile for PAPER CLIPS."
(v) Pick out the subsequent objects.
"This ERASER goes to the ERASERS pile."
"This PAPER CLIP goes to the PAPER CLIPS pile."
(w) Continue till all the objects are sorted out.
"The objects are now sorted by kind."
(x) Mix the beads in a pile again, and have the student sort them out by shapes.
(y) Mix some pennies and dimes. Have the student sort them out in different piles by KIND.
(z) Mix some quarters, nickels and dimes. Have the student sort them out in different piles.
(aa) Continue with this lesson until the student can comfortably sort and classify objects by color, shape, size, or kind.
(bb) Repeat the diagnostic test.

## © Diagnostic K6.4 Predict and Extend Existing Patterns

To pass, the student should be able to predict and extend existing patterns, correctly and with confidence.
"Predict and extend the following patterns."

(NOTE: These are beads of three different colors.)

(NOTE: These are beads of three different sizes.)

(NOTE: These are beads of three different shapes.)
If the diagnostic fails, then do the Lesson \& Exercise.

## Lesson \& Exercise

## In this lesson the student predicts and extends existing patterns using concrete materials.

(a) Set up a simple pattern with beads of two different colors.
"Here is a pattern of two colors. I shall predict and extend this pattern."

(b) Predict the pattern.
"The red bead is followed by a blue bead. The blue bead is followed by a red bead."
(c) Extend the pattern by adding more beads..
"We shall extend the pattern per the rule above."
(d) Set up a simple pattern with beads of two different sizes.
"Here is a pattern of two sizes. I shall predict and extend this pattern."

(e) Predict the pattern.
"The small bead is followed by a large bead. The large bead is followed by a small bead."
(f) Extend the pattern by adding more beads..
"We shall extend the pattern per the rule above."
(g) Set up a simple pattern with beads of two different shapes.
"Here is a pattern of two shapes. I shall predict and extend this pattern."

(h) Predict the pattern. "The triangular bead is followed by a square bead. The square bead is followed by a triangular bead."
(i) Extend the pattern by adding more beads.. "We shall extend the pattern per the rule above."
(j) Place some pennies and dimes alternating in a row. Have the student extend this pattern using additional coins.
(k) Place a quarter, a dime, and a nickel in that order. Have the student extend this pattern using additional coins.
(l) Continue with this lesson until the student can comfortably predict and extend existing patterns, correctly and with confidence.
(m) Repeat the diagnostic test.

## Diagnostic K6.5 Create Patterns

To pass, the student should be able to use concrete objects to create patterns easily with confidence.
"Create three different patterns."
(a) With objects of two different kinds
(b) With objects of three different kinds

If the diagnostic fails, then do the Lesson \& Exercise.

## Lesson \& Exercise

In this lesson the student uses concrete objects to create a pattern.
(a) Get concrete objects of two different kinds.
"I am going to make a pattern with erasers and paper clips."
(b) Create the pattern.
"The pattern is an eraser followed by a paper clip, which then repeats."

(c) Get concrete objects of three different kinds.
"I am going to make a pattern with erasers, paper clips and chalk."
(d) Create the pattern.
"The pattern is a sequence of eraser, paper clip, and chalk, which then repeats."

(e) Make a little more complex pattern.
"I am going to make a more complex pattern with erasers, paper clips and chalk."
(f) Create the pattern.
"The pattern is a sequence of paper clip, chalk, paper clip, eraser, which then repeats."
(g) Have the student create three different patterns with objects of two different kinds.
(h) Have the student create three different patterns with objects of three different kinds.
(i) Have the student create three different patterns with objects of four different kinds.
(j) Continue with this lesson until the student can comfortably create patterns with confidence.
(k) Repeat the diagnostic test.

## © Diagnostic K6.6 Transfer Patterns between Mediums

To pass, the student should be able to transfer patterns from one medium to another with easy confidence.
"Create a pattern with sound."
"Express the SAME pattern using physical movement."
"Express the SAME pattern using concrete objects."
If the diagnostic fails, then do the Lesson \& Exercise.

## Lesson \& Exercise

In this lesson the student transfers patterns from one medium to another (for example, actions, sounds, or concrete objects).
(a) Explain the purpose of this lesson.
"I am going to transfer patterns from one medium to another."
(b) Generate an action pattern.
"Here is an action pattern:
WALKING WITH A SHOE ON ONE FOOT ONLY."
(c) Express the same pattern using sound.
"We may convert it into a sound pattern as follows:
TIK, TOK, TIK, TOK, TIK, TOK, TIK, TOK."
(d) Explain the same pattern using concrete objects as follows.
"We may convert it into a pattern of physical objects as follows:"

(e) Provide another example as follows.

Action pattern:
"Here is an action pattern:
Strike___Strike $\qquad$ Strike-strike-strike $\qquad$ Strike $\qquad$ Strike-strikestrike $\qquad$ ."

Sound pattern:
"Here is the same pattern transferred to sound medium: Drum____Drum____Drum-Drum-Drum____Drum___ Drum____Drum-DrumDrum $\qquad$ ."

Concrete objects:
"Here is the same pattern expressed with pennies."
(f) Have the student create a pattern with sound.
(g) Have the student express the same pattern using physical movement.
(h) Have the student express the same pattern using concrete objects.
(i) Have the student transfer the following pattern to action, sound and concrete objects;
(1)

(2) La-La $\qquad$ La-La_La $\qquad$ La-La $\qquad$ La La-La $\qquad$ La-La $\qquad$ La-La_La
(3) La $\qquad$ La-La $\qquad$ La-La_La $\qquad$ La $\qquad$ La-La $\qquad$ La-La_La
(j) Continue with this lesson until the student can transfer patterns from one medium to another with easy confidence.
(k) Repeat the diagnostic test.

## Diagnostic K6.7 Locate Known and Unknown Numbers

To pass, the student should be able to locate known and unknown numbers on a number line.
"Use a number line to find the following."
(a) A number that is 4 more than 9
(b) A number that is 4 less than 9

If the diagnostic fails, then do the Lesson \& Exercise.

## Lesson \& Exercise

In this lesson the student locates known and unknown numbers on a number line from 0 to 10 or more (for example, finding what number you are on if you move 2 numbers forward or 3 numbers back).
(a) Start the lesson.
"In this lesson we are going to locate known and unknown numbers on a number line."
(b) Draw a number line.
"Here is a number line from 0 to 10 . We may extend it to the right by adding more numbers if we want."

(c) Find a number, which is more than another number.
"Let's find a number which is 2 more than 5.
(a) Place the pencil on 5.
(b) Count 2 spaces to the right.
(c) The pencil is now on 7.
"The number 7 is 2 more than $5 . "$
(d) Find a number, which is less than another number.
"Let's find a number which is 3 less than 7.
(a) Place the pencil on 7.
(b) Count 3 spaces to the left.
(c) The pencil is now on 4.
"The number 4 is 3 less than $7 . "$
(e) Help the student with the following:
(1) Draw a NUMBER LINE from 0 to 10.
(2) Indicate the directions left and right.
(3) Find a number that is 3 more than 3.
(4) Find a number that is 4 less than 9.
(f) Have the student:
(5) Find a number that is 5 more than 3.
(6) Find a number that is 5 less than 6.
(7) Find a number that is 3 less than 9.
(8) Find a number that is 4 more than 9.
(9) Find a number that is 2 less than 11.
(10) Find a number that is 5 more than 8.
(11) Find a number that is 7 more than 7.
(12) Find a number that is 6 less than 13.
(13) Find a number that is 8 more than 6.
(14) Find a number that is 6 more than 8.
(g) Continue with this lesson until the student can comfortably locate known and unknown numbers on a number line.
(h) Repeat the diagnostic test.

## © Diagnostic K6.8 Represent Missing Quantities

To pass, the student should be able to see that symbols can be used to represent missing or unknown quantities.
"Fill out the missing NUMBERS in the boxes?"
(a) $5,6 . \square, 8$
(b) $2, \square 4,5$
(c) $\square, 9, \square, 11$
"Replace the symbol \#, \$ or *, by missing numbers."
(a) 7 \# 910 \# 12 \# \# 15
(b) $1314 \$ \$ 17 \$ 19 \$$
(c) 57 * * * * * * 6465

If the diagnostic fails, then do the Lesson \& Exercise.

## Lesson \& Exercise

In this lesson the student learns that symbols can be used to represent missing or unknown quantities (for example, fill in the missing number in 5, 6, 0,8, ).
(a) Start the lesson.
"In this lesson we are going to look at number patterns."
" $1,2,3,4,5,6,7,8,9$, etc., is a number pattern because each number is one more than the previous number."

Show the following picture.
"The numbers 4, 5, 6, 7, 8 follow the number pattern."

(b) Show the following numbers.
"Some numbers are missing in the number pattern below."

$$
12479
$$

(c) Show how the missing numbers may be represented.
"We may put boxes to represent the missing numbers as follows.

$$
\mathbf{1 2} \mathbf{2} \square \square \square \mathbf{7} \square \mathbf{9}
$$

"You may find the missing numbers by counting and then filling in the boxes. In this case the missing numbers are $3,5,6$, and 8 respectively."

## 123456789

(d) Show how the missing numbers may be represented by other symbols
"We may use some other symbol to represent the missing numbers as follows.

## 1, \#, 3, 4, \#, 6, \#, \#, 9

"In this example the missing numbers are $2,5,7$, and 8.
(e) Help the student fill out the missing NUMBERS in the boxes?
(a) $5,6 . \square, 8$
(c) $3, \square 5, \square$
(e) $\square, 9, \square, 11$
(b) $2, \square 4,5$
(d) $\square, 8,9,10$
(f) 11, 12, $\square, \square$
(f) Have the student replace the symbols by missing numbers.
(a) 7 \# 9 \# 12 \# \# 15
(b) $1314 \$ \$ 17 \$ 19 \$$
(c) 57 * * * * * * 6465
(g) Continue with this lesson until the student can see that symbols can be used to represent missing or unknown quantities.
(h) Repeat the diagnostic test.

## Diagnostic K6.9 Determine if Two Groups are Equal

To pass, the student should be able to use one-to-one matching to determine if two groups are equal.
"Make two piles of pennies from the following pairs of numbers and show if they are equal."
(a) 6,6
(b) 5, 7
(c) 10,10

If the diagnostic fails, then do the Lesson \& Exercise.

## Lesson \& Exercise

In this lesson the student uses one-to-one matching to determine if two groups are equal.
(a) Start the lesson.
"In this lesson we are going to determine if two groups are equal."
(b) Make two columns with 5 pennies in each next to each other. "Here are two columns of pennies. Let's see if they are equal."
(c) Match the pennies one-to-one.
"We shall match each penny from the first column with a corresponding penny in the other column."
"All the pennies match one-to-one. Therefore, these two groups are EQUAL."
(d) Make two columns containing 5 and 8 pennies respectively. "Here are two columns of pennies. Let's see if they are equal."
(e) Match the pennies one-to-one.
"When you match the one-to-one, some pennies are left in the second column. Therefore, these two groups are NOT equal."
(f) Help the student make two piles of pennies from following pairs of numbers to see if they are equal.
(a) 6,6
(d) 5,7
(g) 10,0
(b) 3,4
(e) 2,2
(h) 6,6
(c) 1,1
(f) 8,3
(i) 0,5

(g) Have the student make two piles with equal number of pennies.
(h) Have the student make two piles with unequal number of pennies.
(i) Have the student circle the pairs that are EQUAL.
(a) 5,5
(d) 5,7
(g) 0,10
(b) 3,4
(e) 7,4
(h) 6,6
(c) 1,1
(f) 2,2
(i) 0,0
(j) Continue with this lesson until the student can use one-to-one matching to determine if two groups are equal.
(k) Repeat the diagnostic test.

## SUMMARY

This is the sixth of the seven levels of the Troubleshooting Guide for KINDERGARTEN MATH. The Troubleshooting Guide for Kindergarten introduces the concept of UNIT, and explores ways to measure length, weight, capacity and time. It further develops the concept of counting into the concepts of addition and subtraction.

The Kindergarten troubleshooting guide is divided into the following levels:

## (1) ORIENTATION \& SPATIAL SENSE

Orientation and Spatial Sense forms the foundation of the subject of GEOMETRY. It introduces the elements of space and how they relate to us.

## (2) NUMBERS \& PLACE VALUES

Numbers and Place Values form the foundation of the subject of ARITHMETIC. It introduces a system of whole numbers to represent quantities in a simple manner.

## (3) UNITS \& FRACTIONS

Units \& Fractions addresses ways to represent quantities, which cannot be represented by whole numbers.

## (4) COUNTING \& MEASUREMENTS

Counting \& Measurements provides ways to determine the various magnitudes. It helps to bring familiarization with the use of numbers.

## (5) NUMBERS \& OPERATIONS

Numbers \& Operations introduces the basic operations with numbers and how such operations may be executed with skill.

## (6) PATTERNS \& RELATIONAL SENSE

Patterns and Relational Sense forms the foundation of the subject of ALGEBRA. It is a study of patterns underlying numbers, and quantitative relationships.

## (7) DATA ANALYSIS \& PROBABILITY

Data Analysis \& Probability shows how to display quantitative comparisons graphically. It introduces the estimation of likelihood.

Though these lessons are designed for the kindergarten level, these diagnostic actions may be used for students at higher grades to help discover and resolve missing basics.

## GLOSSARY

[For additional words refer to the glossary at the end of Level K1.]
Pattern A pattern comes about because of some repeating characteristic, such as color, shape, size, etc. Different types of repetitions generate different patterns regardless of what is being repeated.

Relation
A relation is an existing connection; or a significant association between or among things. Because of a relation, if something is present then we may expect another thing to be present as well. A pattern is made up of relations. To see a pattern is to have a sense of relations among things.

A number line is a pattern that presents relations among numbers. With the help of this pattern we may locate a missing or hidden number.

A symbol may be used in place of a missing or hidden number in a pattern. The position of that symbol in that relationship may then help discover that number.

